CALCULUS TEST II REVIEW

These are questions that I believe to be important. Treat this like a test, don't use your notes, and see how you do. If you can't do any part of this, go to the corresponding section and find problems like the ones you cannot do.

1. Find the absolute extrema $f(x) = x - 12x^{1/3}$ on the interval $-27 \le x \le 1$:

$$f(x) = x - 12x^{1/3}$$

$$f'(x) = 1 - \frac{12}{3}x^{-2/3} = 0$$

$$1 - \frac{4}{x^{2/3}} = 0$$

$$\frac{x^{2/3} - 4}{x^{2/3}} = 0$$

$$x^{2/3} - 4 = 0$$

$$\left(x^{2/3}\right)^3 = (4)^3$$

$$\sqrt{x^2} = \sqrt{64}$$

$$x = \pm 8 \quad \text{\&} \quad x = 0$$

*make

$x = 0$ is CP b/c $f(0)$ is defined & $f'(0)$ is undefined.

Test endpoints

$f(-27) = 9$

$f(1) = -11$

$f(-8) = 16$

$\cancel{f(8) = -16}$

$f(0) = 0$

Since $f(-8)$ gives largest y value it is an absolute max.

Since $f(8)$ gives smallest y-value it is an absolute min

2. Find all intervals of increasing, decreasing, concave up, concave down, relative extrema, and points of inflection of the function: $F(x) = \sin\theta + \sqrt{3}\cos\theta$ between $0 \le x \le 2\pi$. Give reasons for your conclusions.

$$f(x) = \sin\theta + \sqrt{3}\cos\theta$$

$$f'(x) = \cos\theta - \sqrt{3}\sin\theta = 0$$

$$\cos\theta = \sqrt{3}\sin\theta$$

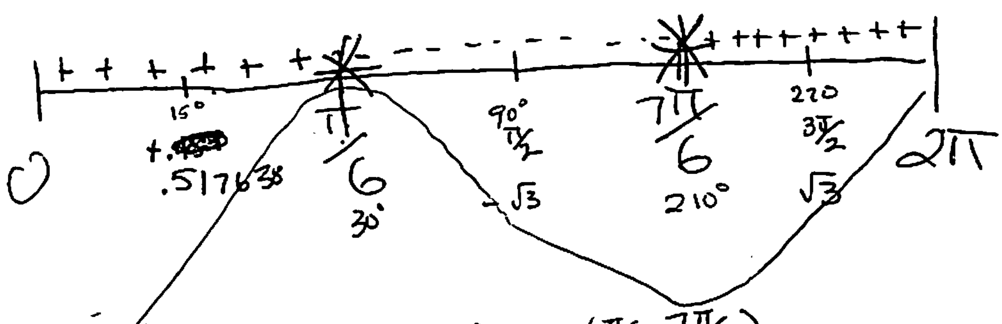
$$\frac{\cos\theta}{\sin\theta} = \sqrt{3}$$

$$\cot\theta = \sqrt{3}$$

$$\text{arcot}\cot\theta = \text{arcot}\sqrt{3}$$

θ's btw 0 & 2π that have cot of $\sqrt{3}$ $\quad \theta = \pi/6,\ 7\pi/6$

1st der test tells slope INC or DEC



∴ INC: $(0, \pi/6) \cup (7\pi/6, 2\pi)$ $\qquad$ DEC $(\pi/6, 7\pi/6)$

relative max @ $(\pi/6, 2)$ $\qquad$ relative min @ $(7\pi/6, -2)$

$f(\pi/6) = \sin\pi/6 + \sqrt{3}\cos\pi/6$

$\frac{1}{2} + \sqrt{3}\cdot\frac{\sqrt{3}}{2}$

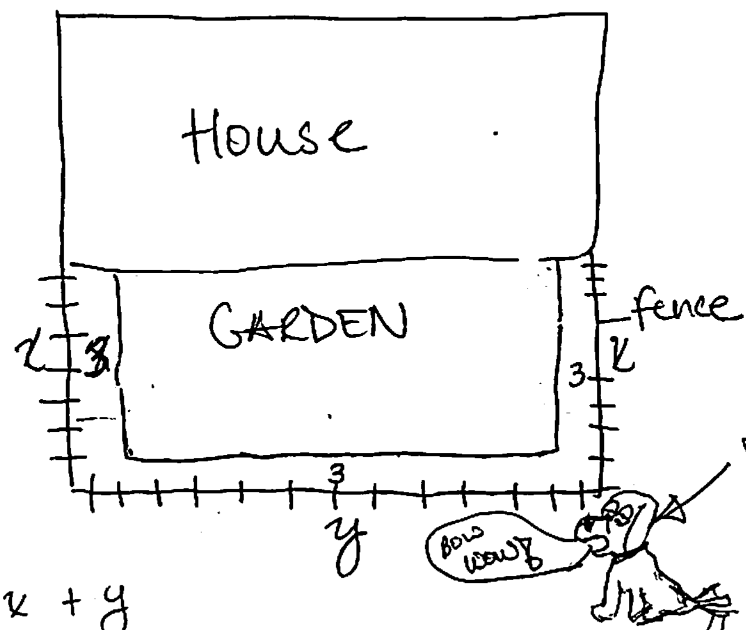
$\frac{1}{2} + \frac{3}{2} = \frac{4}{2} = 2$

$f(7\pi/6) = \sin\frac{7\pi}{6} + \sqrt{3}\cos\frac{7\pi}{6}$

$-\frac{1}{2} + \sqrt{3}\left(-\frac{\sqrt{3}}{2}\right)$

$-\frac{1}{2} - \frac{3}{2} = -\frac{4}{2} = -2$

3. Farmer Joe wants to make his back yard purty. So he decides to make a flower garden in his back yard. Joe has a big guard dog that gets into everything, so he decides he wants an electric fence to keep Rex out. He finds out the fencing is expensive and he can only afford 172 ft of it. To maximize the area of the garden, he decides plant the flowers to where one side of the garden is adjacent to the house and doesn't need fencing. He decides that he must have at least 3 feet of walkway around the garden so he can get around. What is the maximal are of the garden and what are its dimensions?



$P_{outside} = 2x + y$

$172 = 2x + y$

$172 - 2x = y$

$172 - 85 = y$

$86 = y$

Dimensions $\boxed{86 \times 42.5}$

$Area_G = (86-6)(\cancel{42.5}\,43-3)$

$(81)(39.5)$

$\cancel{3199.5}$

$\boxed{3200\ ft^2}$

$A_G = (y-6)(x-3)$

$A_G = (172-2x-6)(x-3)$

$(166-2x)(x-3)$

$A' = -2(x-3) + (1)(166-2x)$

$-2x+6+166-2x$

$-4x+172=0$

$-4x=-172$

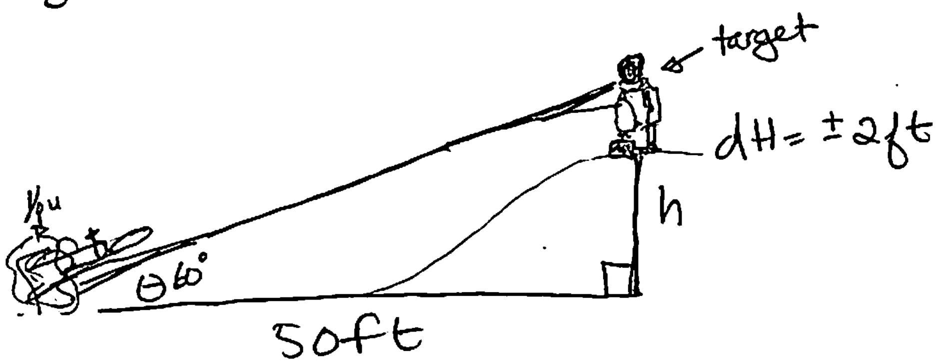
$\cancel{42.5}$

$x = 43$

maximal value

43 50

4. A student standing 50 ft from his Calculus teacher has a water balloon launcher in his hand. The student is at an angle of elevation to the teacher of 60 degrees. The allowable error in height between you and your target is about + or – 2 ft, what is the maximal allowable error in the angle measurement in order for the student to exact revenge.



$$\tan\theta = \frac{h}{50}$$

$$\sec^2\theta \, d\theta = \frac{1}{50}\, dh$$

$$d\theta = \frac{1}{50} \frac{dh}{\sec^2\theta}$$

$$d\theta = \frac{1}{50} \frac{2}{\sec^2 60}$$

$$d\theta = \frac{1}{25} \frac{1}{4} = \frac{1}{100}$$

got to be pretty accurate on angle measurement otherwise all is for not.

4. Apply Rolle's theorem and Mean to the following functions if applicable. If not applicable, state the reason why:

a) $F(X) = x^2 - 2x$ [0,2]

Cont ; diff

R: $f(0) = 0$ $f(2) = 0$

equal

$\therefore f'(x)$ has to have 1c which sets der = 0.

$f'(x) = 2x - 2 = 0$

$2x = 2$

$\boxed{x = 1}$

cont ; diff

MVT: $\therefore f'(c) = \frac{f(b) - f(a)}{b - a}$

$\frac{0 - 0}{2 - 0} = 0$

$f'(x) = 2x - 2$

$f'(c) = 2c - 2 = 0$

$2c = 2$

$\boxed{c = 1}$

b) $F(X) = x^{2/3} - 1$ [-8,8]

$f(x)$ is cont. $[-8, 8]$

but not diff $(-8, 8)$

$f'(x) = \frac{2}{3} x^{-1/3} = \frac{2}{3x^{1/3}}$ & $f'(0)$ is und

$\therefore$ neither theorem is applicable

c) $F(X) = \sin 2x$ $[\pi/6, \pi/3]$

Cont ; diff

R: $f(\pi/6) = \sin \frac{2\pi}{6}$

$\sin \frac{\pi}{3} = \frac{\sqrt{3}}{2}$

$f(\pi/3) = \sin \frac{2\pi}{3}$

$\frac{\sqrt{3}}{2}$

$f'(x) = 2\cos 2x$

$f'(c) = 2\cos 2c = 0$

$\cos 2c = 0$

$2c = \arccos 0$

$2c = \frac{\pi}{2}, \frac{3\pi}{2}$

$c = \frac{\pi}{4}, \frac{3\pi}{4}$ outside range

$c = \pi/4$

MVT: cont ; diff

$f'(c) = \frac{f(b) - f(a)}{b - a}$

$2\cos 2c = \frac{\frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{2}}{\pi/3 - \pi/6}$

$2\cos 2c = 0$

$c = \pi/4$

* Note I just happened to pick endpoints that equal each other when plugged into function. DOESN'T MEAN ALWAYS = 0 ON MVT. *

d) $F(X) = \sec x$ $[0, \pi]$

f(x) is not cont @ $[0, \pi]$

$f(x) = \sec x$

$= \frac{1}{\cos x}$ so wherever $\cos x = 0$ the funct. is undef.

$\therefore \cos x = 0$ @ $\pi/2, \frac{3\pi}{2}$.

; $\pi/2$ is w/in range of inquiry.

$\therefore$ neither theorem can apply.

5. Calculate the following integrals:

a) $\int x^2 - 2x + \frac{2}{x^3} - \sqrt[3]{x^2}\,dx$

$$\frac{x^{2+1}}{2+1} - \frac{2x^{1+1}}{1+1} + \frac{2x^{-3+1}}{-3+1} - \frac{x^{2/3+3/3}}{\frac{2}{3}+\frac{3}{3}} + C$$

$$\frac{x^3}{3} - \frac{2x^2}{2} + \frac{2x^{-2}}{-2} - \frac{x^{5/3}}{5/3} + C$$

$$\frac{x^3}{3} - x^2 - \frac{1}{x^2} - \frac{3}{5}x^{5/3} + C$$

c) $\int \theta^2 + \sec^2\theta\, d\theta$

$$\frac{\theta^{2+1}}{2+1} + \tan\theta + C$$

$$\frac{\theta^3}{3} + \tan\theta + C$$

b) $\int \frac{x^2 - 1}{x^2}dx$

$$\int \frac{x^2}{x^2} - \frac{1}{x^2}dx$$

$$\int 1 - x^{-2}dx$$

$$\frac{x^{0+1}}{0+1} - \frac{x^{-2+1}}{-2+1} + C$$

$$x - \frac{x^{-1}}{-1} + C$$

$$x + \frac{1}{x} + C$$

d) $\int (3x+1)(2x-1)dx$

$$\int 6x^2 + 2x - 3x - 1$$

$$\int 6x^2 - x - 1dx$$

$$\frac{6x^{2+1}}{2+1} - \frac{x^{1+1}}{1+1} - \frac{x^{0+1}}{0+1} + C$$

$$\frac{6x^3}{3} - \frac{x^2}{2} - x + C$$

$$2x^3 - \frac{x^2}{2} - x + C$$

THIS IS A TYPICAL TEST. MAKE SURE YOU CAN DO THIS IN ONE HOUR… AND GOOD LUCK!☺!