

Rules for Dividing Polynomials

When dividing polynomials, the problem may be given to you in many ways like the following below:

$$\frac{\square}{\triangle} \quad \text{or} \quad \square + \triangle \quad \text{or} \quad \square \text{ divided by } \triangle$$

Always begin rewriting the problem in long division form:

$$\triangle \overline{) \square} \longrightarrow \text{Always top term/first term}$$

↓

Always bottom term/second term

Example:

$$(x^5 + 2x^3 - 5x) \div (x^2 - 1)$$

1. Rewrite the problem in long division form.

$$x^2 - 1 \overline{) x^5 + 2x^3 - 5x} \quad ?$$

2. To figure out '?', divide the highest term under by the highest term outside division sign.

$$\frac{x^5}{x^2} = x^3$$

3. Just like regular long division, multiply result by every term outside the division sign.

$$\begin{array}{r} x^3 \\ x^2 - 1 \overline{) x^5 + 2x^3 - 5x} \\ \underline{x^5 - x^3} \end{array}$$

4. Change the signs of the terms that were multiplied out.

$$\begin{array}{r} x^3 \\ x^2 - 1 \overline{) x^5 + 2x^3 - 5x} \\ \underline{-x^5 + x^3} \end{array}$$

5. Combine like terms.

$$\begin{array}{r} x^3 \\ x^2 - 1 \overline{) x^5 + 2x^3 - 5x} \\ \underline{-x^5 + x^3} \end{array}$$

$$3x^3 - 5x$$

6. Repeat the process again (Steps 2-5) with this as a new problem until division is done.

$$\begin{array}{r}
 \boxed{(2)} \quad \frac{3x^3}{x^2} = 3x \\
 \begin{array}{r}
 x^2 - 1 \overline{) x^5 + 2x^3 - 5x} \\
 \underline{-x^5 + x^3} \\
 3x^3 - 5x \\
 \underline{-3x^3 + 3x} \\
 -2x
 \end{array}
 \end{array}$$

$$\begin{array}{r}
 \overline{) x^5 + 2x^3 - 5x} \\
 \underline{-x^5 + x^3} \\
 3x^3 - 5x \\
 \boxed{(4)} \quad \underline{-3x^3 + 3x} \\
 -2x
 \end{array}$$

$$\begin{array}{r}
 \overline{) x^5 + 2x^3 - 5x} \\
 \underline{-x^5 + x^3} \\
 3x^3 - 5x \\
 \boxed{(5)} \quad \underline{-3x^3 + 3x} \\
 -2x
 \end{array}$$

Since the highest power left behind is one and the highest power of the divisor is two, then $-2x$ is a remainder because you can't divide x^2 to get another term. Hence

$$(x^5 + 2x^3 - 5x) \div (x^2 - 1) = (x^3 + 3x)R(-2x) = x^3 + 3x + \frac{-2x}{x^2 - 1}$$