FACTORING OUT THE GREATEST COMMON FACTOR

Example 1: Factoring Out A Numerical Factor

$$25x^2 - 10x + 5$$

To find the common factor, look for the highest common multiple in each term. In the above expression, the common factor in each term is 5. To simplify the above expression, divide each term by 5.

$$\frac{25x^2}{5} - \frac{10x}{5} + \frac{5}{5}$$

$$= 5(5x^2 - 2x + 1).$$

Example 2: Factoring Out A Variable Factor

$$2x^5y - 10x^2y^4z - 7x^3y^2$$

To find the common factor(s) in the expression above, determine the common factor variable in each term. If all the terms have common variables, divide by the smallest power of each common variable. The only common factors in each of the above terms is x and y. Simplify the expression by dividing each term by x^2y. Notice that z is not a common factor because it is not present in all three terms.

$$\frac{2x^5y}{x^2y} - \frac{10x^2y^4z}{x^2y} - \frac{7x^3y^2}{x^2y}$$

$$= x^2y(2x^3 - 10y^3z - 7xy).$$

Example 3: Factoring By Grouping

$$7ax - 7bx + 3ay - 3by$$

To factor the above expression, look at it as two parts. Look at the first two terms as one expression, and the last two terms as another expression. Put parenthesis around the first two terms and the last two terms.

$$(7ax - 7bx) + (3ay - 3by)$$

Now, using the basic factoring rules from the above examples, the first expression can be factored by factoring out a 7 and an x.

$$7x(a - b) + (3ay - 3by)$$

Looking at the second expression, a 3 and a y can be factored out.

$$7x(a - b) + 3y(a - b)$$

Now you can factor the expression as a whole. The common factor is the expression a–b. Divide both terms by the expression a – b. The terms that are left are grouped together as one expression.

$$\frac{7x(a - b)}{a - b} + \frac{3y(a - b)}{a - b}$$

$$= (a - b)(7x + 3y).$$

Example 4: Applying The Basic Factorization Rules

$$x^3 - 6x^2 + 5x$$

Step 1: Find the greatest common term(s) in the expression above.

$$x(x^2 - 6x + 5)$$

Step 2: To factor the expression in parenthesis, find two numbers that are multipled together that results in the last term (in this case, positive one) and added together results in the middle term (in this case, negative two).

Factors	*Middle Term*	*Last Term*
+1 and +5	*+5*	*+5*
-1 and -5	*-6*	*+5*

Step 3: Break up the middle term of the expression in parenthesis into the factors that you found in step 2.

$$x(x^2 - 1x - 5x + 1)$$

Step 4: Use factoring by grouping to factor the expression in parenthesis.

$$x[(x^2 - 1x) + (-5x + 5)]$$

$$x[x(x - 1) - 5(x - 1)]$$

Step 5: Factor out the common expression inside and group the remaining terms together in parenthesis.

$$x[(x - 1)(x - 5)]$$

$$= x(x - 1)(x - 5)$$

Example 5: Applying The Basic Factorization Rules

$$y = x^4 - 8x^3 + 15x^2$$

Step 1: Find the greatest common term(s) in the expression above.

$$x^2(x^2 - 8x + 15)$$

Step 2: To factor the expression in parenthesis, find two numbers that are multipled together that results in the last term (in this case, positive fifteen) and added together results in the middle term (in this case, negative eight).

Factors	***Middle Term***	***Last Term***
+1 and +15	***+14***	***+1***
-1 and -15	***-16***	***+1***
+3 and +5	***+8***	***+15***
-3 and -5	***-8***	***+15***

Step 3: Break up the middle term of the expression in parenthesis into the factors that you found in step 2.

$$x^2(x^2 - 3x - 5x + 15)$$

Step 4: Use factoring by grouping to factor the expression in parenthesis.

$$x^2[(x^2 - 3x) + (-5x + 15)]$$

$$x^2[x(x - 3) - 5(x - 3)]$$

Step 5: Factor out the common expression inside and group the remaining terms together in parenthesis.

$$x^2[(x - 3)(x - 5)]$$

$$= x^2(x - 3)(x - 5)$$

Example 6: Applying The Basic Factorization Rules

$$10x^3 - 5x^2 - 15x$$

Step 1: Find the greatest common term(s) in the expression above.

$$5x(2x^2 - x - 3)$$

Step 2: Since there is a leading coefficient in front of x^2, take the coefficient and multiply it by the last term in the parenthesis(in this case, negative three).

$$(\text{Coefficient of } x^2) * (\text{Last term in parenthesis}) = 2 * (-3) = -6$$

Step 3: Find two numbers that when multiplied together results in the answer of step 2 (in this case, negative six) and when added results in the middle term (in this case, negative one).

Factors	***Middle Term***	***Last Term***
+1 and -6	***-5***	***-6***
-1 and +6	***+7***	***-6***
+3 and -2	***+1***	***-6***
-3 and +2	***-1***	***-6***

Step 3: Break up the middle term of the expression in parenthesis into the factors that you found in step 3.

$$5x(2x^2 - 3x + 2x - 3)$$

Step 4: Use factoring by grouping to factor the expression in parenthesis.

$$5x[(2x^2 - 3x) + (2x - 3)]$$

$$5x[x(2x - 3) + (2x - 3)]$$

Step 5: Factor out the common expression inside and group the remaining terms together in parenthesis.

$$5x[(2x - 3)(x + 1)]$$

$$= 5x(2x - 3)(x + 1)$$