

## Laws of Exponents

**\*\*The following are some properties for the laws of exponents.\*\***

$$a^m a^n = a^{m+n}$$

$$(a^m)^n = a^{mn}$$

$$(ab)^n = a^n b^n$$

$$\frac{a^m}{a^n} = a^{m-n} = \frac{1}{a^{n-m}}, \text{ if } a \neq 0$$

$$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}, \text{ if } b \neq 0$$

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### MULTIPLICATION

$$ax^n \cdot bx^m = (a \cdot b)x^{n+m}$$

**\*\*Multiply the coefficients and add the exponents.\*\***

Example 1: Using The Laws of Exponents

$$4x^2 \cdot 2x^3 = (4 \cdot 2)x^{2+3} = 8x^5$$

Example 2: Using The Laws of Exponents—The Missing Exponent

$$2x \cdot 5x^2 = (2 \cdot 5)x^{1+2} = 10x^3$$

**\*\*If there is not an exponent present, it is to be an understood one as the power of the base.**

Example 3: Using The Laws of Exponents—Fractional Exponents

$$\frac{2}{3}x^{\frac{1}{2}} \cdot \frac{9}{4}x^{\frac{1}{3}} = \left(\frac{2}{3} \cdot \frac{9}{4}\right)x^{\frac{1}{2}+\frac{1}{3}} = \frac{3}{2}x^{\frac{5}{6}}$$

**\*\*The same rules apply for fractions and fractional exponents.\*\***

Example 4: Using The Laws of Exponents

$$4x^2 y^5 z \cdot z^2 y^2 x^3 = (4 \cdot 1)x^{2+3} y^{5+2} z^{1+3} = 4x^5 y^7 z^3$$

Other Properties for Laws of Exponents

**\*\*The following are other properties that also apply to the laws of exponents.\*\***

1. Anything to the zero power is always equal to one.

$$x^0 = 1 \quad (2y)^0 = 1 \quad (-4)^0 = 1$$

2. Radicals and Fractional Exponents

$$\sqrt{x^3} \Leftrightarrow x^{\frac{3}{2}} \quad \sqrt{y} \Leftrightarrow y^{\frac{1}{2}} \quad (64)^{\frac{2}{3}} \Leftrightarrow \left(\sqrt[3]{64}\right)^2 \text{ or } \sqrt[3]{64^2}$$

$$\sqrt[B]{x^A} \Leftrightarrow x^{\frac{A}{B}}$$

### DIVISION

$$\frac{ax^n}{bx^m} = \left(\frac{a}{b}\right)x^{n-m}$$

**\*\*Divide the coefficients and subtract the exponents.\*\***

Example 1: Using The Laws of Exponents

$$\frac{4x^5}{2x^2} = \left(\frac{4}{2}\right)x^{5-2} = 2x^3$$

Example 2: Using The Laws of Exponents

$$\frac{3x^4}{6x} = \left(\frac{3}{6}\right)x^{4-1} = \frac{1}{2}x^3$$

Example 3: Using The Laws of Exponents

$$\frac{15x^4y^9z^{10}}{7x^2y^3z^5} = \left(\frac{15}{7}\right)x^{4-1}y^{9-3}z^{10-5} = \frac{15}{7}x^3y^6z^5$$

Example 4: Using The Laws of Exponents—The Negative Exponent

$$\frac{2x^2}{x^5} = 2x^{2-5} = 2x^{-3} = \frac{2}{x^3}$$

*\*\*This example has a negative exponent as the answer. To make the exponent positive, move the negative exponent to the denominator and change the sign.\*\**

Example 5: Using The Laws of Exponents—Negative Exponents

$$\frac{4x^4y^5z^3}{16x^3y^6z^5} = \left(\frac{4}{16}\right)x^{4-3}y^{5-6}z^{3-5} = \frac{1}{4}x^{-2}y^{-1}z^{-2} = \frac{1}{4x^2yz^2}$$

Example 6: Using The Laws of Exponents—Moving Negative Exponents

$$2x^{-2} = \frac{2}{x^2}$$

*\*\*Since the exponent is negative in the numerator, move it to the denominator to make it positive.\*\**

Example 7: Using The Laws of Exponents—Moving Negative Exponents

$$\frac{4}{x^{-3}} = 4x^3$$

*\*\*Since the exponent is negative in the denominator, move it to the numerator to make it positive.\*\**

Example 8: Using The Laws of Exponents—Moving Negative Exponents

$$\frac{2^{-1}x^2z^{-6}}{x^{-4}} = \frac{x^2y^4}{2z^6}$$

*\*\*Since there are exponents negative in both the numerator and denominator, move them so that their exponents are positive.\*\**

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RAISING A POWER TO ANOTHER POWER

Multiplication:

$$(a^m x^m y^m)^n = a^{mn} x^{mn} y^{mn}, a \rightarrow \text{constant/number}$$

*\*\*When there is a group raised to a power, multiply the exponents.\*\**

Example 1: Using The Laws of Exponents—Power To Power

$$(4x^2)^3 = 4^{1 \times 3} x^{2 \times 3} = 4^3 x^6 = 64x^6$$

Example 2: Using The Laws of Exponents—Power To Power

$$(3xy^2z^3)^2 = 3^{1 \times 2} x^{1 \times 2} y^{2 \times 2} z^{3 \times 2} = 3^2 x^2 y^4 z^6 = 9x^2 y^4 z^6$$

Example 3: Using The Laws of Exponents—Power To Power

$$(2x^2y)^{-1} = 2^{1 \times -1} x^{2 \times -1} y^{1 \times -1} = 2^{-1} x^{-2} y^{-1} = \frac{1}{2x^2y}$$

Division:

$$\left( \frac{a^n x^n}{b^n y^n} \right)^m = \frac{a^{mn} x^{mn}}{b^{mn} y^{mn}}$$

**\*\*Same as multiplication, multiply the exponents.\*\***

Example 1: Using The Laws of Exponents—Power To Power

$$\left( \frac{2x^2}{y} \right)^2 = \frac{2^{1 \times 2} x^{2 \times 2}}{y^{1 \times 2}} = \frac{2^2 x^4}{y^2} = \frac{4x^4}{y^2}$$

Example 2: Using The Laws of Exponents—Power To Power

$$\left( \frac{3xy}{z} \right)^3 = \frac{3^{1 \times 3} x^{1 \times 3} y^{1 \times 3}}{z^{1 \times 3}} = \frac{3^3 x^3 y^3}{y^3} = \frac{27x^3 y^3}{y^3}$$

Example 3: Using The Laws of Exponents—Power To Power

$$\left( \frac{5x^2y}{z^3} \right)^{-2} = \frac{5^{1 \times -2} x^{2 \times -2} y^{1 \times -2}}{z^{3 \times -2}} = \frac{5^{-2} x^{-4} y^{-2}}{y^{-6}} = \frac{z^6}{5^2 x^4 y^2} = \frac{z^6}{25x^4 y^2}$$