

## Area between two curves

Basic formula:

$$\int_A^B \text{Top} - \text{Bottom}$$

where A & B are the endpoints for the region that they meet.

Example:  $y = -x^2 + 4x + 2$        $y = x + 2$

Steps:

- 1) Set them equal to each other to get endpoints of region:

$y = y$  at the points of intersection.  
Therefore, we get the following;

$$-x^2 + 4x + 2 = x + 2$$

- 2) Simplify the equation, set it equal to zero, and solve.

$$\begin{aligned}x^2 - 3x &= 0 \\x(x-3) &= 0 \\x = 0 &\quad (x-3) = 0 \\&\quad\quad\quad x = 3\end{aligned}$$

Therefore the region of interest lies between 0 and 3. So only plot between these points. (\*Note: if there are more than two curves, just graph and look for a region where all three curves intersect.)

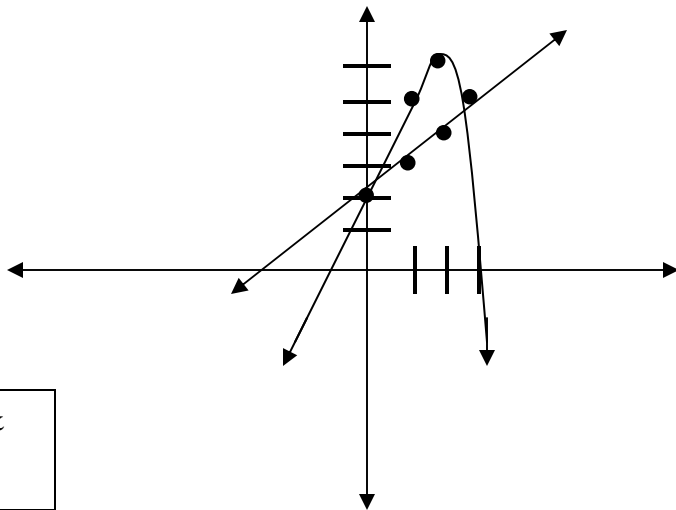
- 3) Graph

$$y = -x^2 + 4x + 2$$

x	y
0	2
1	5
2	6
3	5

$$y = x + 2$$

x	y
0	2
1	3
2	4
3	5



\*The graph tells us which is on top & which is on bottom for our formula.

\* If they don't ask you to sketch it, you can find out which is on top by plugging in a number between  $x = 0$  and  $3$  (which are the endpoints) into the equations. The one with the higher answer is on top.

$$y = -x^2 + 4x + 2 \qquad y = x + 2$$

Test  $x = 1$

$$y = -(1)^2 + 4(1) + 2 \qquad y = 1 + 2$$

$$y = 5 \qquad y = 3$$

Therefore,  $y = -x^2 + 4x + 2$  has to be on top, since  $5$  is greater than  $3$ .

4) Setup the integral and proceed with the integration part.

$$\int_0^3 (-x^2 + 4x + 2) - (x + 2) dx$$

Top
Bottom

Remember the bottom equation needs to be in parenthesis.

Remember smaller number on bottom.

$$\int_0^3 -x^2 + 4x + 2 - x - 2 dx$$

$$\int_0^3 -x^2 + 3x dx$$

$$\left. \frac{-x^3}{3} + \frac{3x^2}{2} \right|_0^3$$

$$\left( \frac{-(3)^3}{3} + \frac{3(3)^2}{2} \right) - \left( \frac{-(0)^3}{3} + \frac{3(0)^2}{2} \right)$$

$$\frac{-27}{3} + \frac{27}{2} = \frac{-54}{6} + \frac{81}{6} = \frac{27}{6} = \frac{9}{2}$$

\* For the answer, you should get a positive number because we are looking for an area, and there is no such thing as a negative area. So, you better make sure that you get a positive numerical value for your answer.