Derivatives

I. Basic rules of derivatives

Basic rule: f(x) = Ax^n (A = number & n = exponent)

To take derivative, you can use the following notations:

$$f'(x); \frac{dy}{dx}; y'$$

For example, if we were to use one notation (i.e. f'(x)), then the general rule would be as follows: f'(x) = An x^{n-1}

(Bring the exponent down and multiply by the number A, then subtract 1 from the exponent.)

Examples:

1) f(x) = x^2 f '(x) = 2x^{2-1} = 2x

2) $f(x) = 4\sqrt{x} = (rewrite) 4x^{\frac{1}{2}}$

$$f'(x) = 4\left(\frac{1}{2}x^{\frac{1}{2}-\frac{2}{2}}\right) = 2x^{\frac{-1}{2}} = \frac{2}{x^{\frac{1}{2}}} = \frac{2}{\sqrt{x}}$$

***Note*: this method only applies if the variable is in numerator, so if the variable is in denominator, move it up and make it negative.**

Example:

$$f(x) = \frac{2}{x^3}$$

$$f'(x) = 2x^{-3} = (-3)(2)x^{-3-1} = -6x^{-4} = \frac{-6}{x^4}$$

When many terms added or subtracted:
Do each one individually: for example,

$$f(x) = 4x^2 - 2x - \frac{1}{x} + \frac{1}{\sqrt{x}}$$

$$\text{Re } write : f(x) = 4x^2 - 2x - x^{-1} + x^{\frac{-1}{2}}$$

$$f'(x) = 8x - 2 - (-1)x^{-1-1} - \frac{1}{2}x^{\frac{-1}{2}-\frac{2}{2}}$$

$$f'(x) = 8x - 2 + \frac{1}{x^2} - \frac{1}{2x^{\frac{3}{2}}}$$

***Rule: Derivative of any number is always 0!!!!**

f(x) = x^2 + 10
f '(x) = 2x + 0 = 2x

If terms are not added or subtracted use:

<u>Product rule</u>: when two terms with x's are multiplied together.

<u>Quotient rule</u>: when two terms with x's are divided.

<u>Product rule example</u>:

$$f(x) = (2x^5 + x^6)(3x^2 - \sqrt{x})$$

f(x) = U V
f '(x) = U 'V + V 'U ← *****Product rule*****

It states that do derivative of the first terms times the second term without any changes, plus the derivative of the second term times the first without any changes.

<u>Example 1</u>:

$$f(x) = (2x^5 + x^6)(3x^2 - \sqrt{x})$$

$$f'(x) = (10x^4 + 6x^5)(3x^2 - x^{\frac{1}{2}}) + (6x - \frac{1}{2}x^{\frac{-1}{2}})(2x^5 + x^6)$$

$$f'(x) = (10x^4 + 6x^5)(3x^2 - x^{\frac{1}{2}}) + (6x - \frac{1}{2x^{\frac{1}{2}}})(2x^5 - x^6)$$

<u>Example 2</u>:

$$f(x) = (\sin x + 1)(1 - \cos x)$$

u v

$$f'(x) = \cos x(1 - \cos x) + (-(-\sin x))(\sin x + 1)$$

u' v v' u

$$f'(x) = \cos - \cos^2 x + \sin^2 x + \sin x$$

<u>Quotient Rule:</u>

$$f(x) = \frac{2x+1}{1-x} = \frac{u}{v} \qquad Rule = \frac{vu' - uv'}{v^2} = \frac{LodHi - HidLo}{lo^2}$$

Example 1:

$$f(x) = \frac{2x+1}{1-x} = \frac{u}{v}$$

$$f'(x) = \frac{(1-x)(2)-(2x+1)(-1)}{(1-x)^2}$$

$$f'(x) = \frac{2-2x+2x+1}{(1-x)^2} = \frac{3}{(1-x)^2}$$

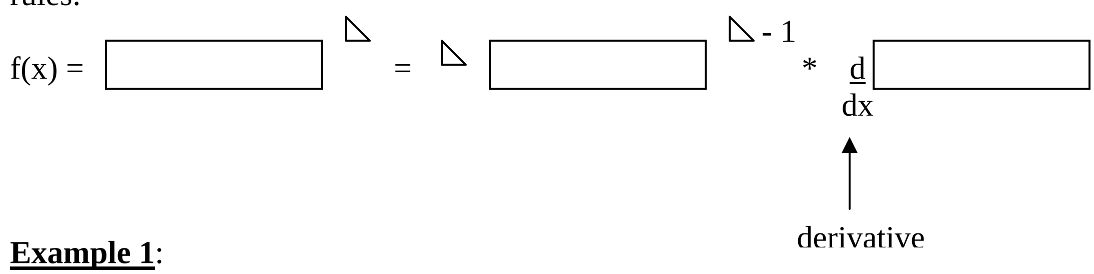
Example 2:

$$f(x) = \frac{\sin x}{1+\tan x}$$

$$f'(x) = \frac{lodHi - Hidlo}{lo^2} = \frac{(1+\tan x)(\cos x)-(\sin x)(\sec^2 x)}{(1+\tan x)^2}$$

$$f'(x) = \frac{\cos x + \tan x \cos x - \sin x \sec^2 x}{(1+\tan x)^2}$$

Chain Rule: whenever terms are complex and don't fall under normal differentiation rules:



Example 1:

$$f(x) = (2x+1)^3$$

$$f'(x) = 3(2x+1)^{3-1} * \frac{d}{dx}(2x+1)$$

$$f'(x) = 3(2x+1)^2(2) = 6(2x+1)^2$$

Example 2:

$$f(x) = \sqrt{x^2-2x} \leftarrow rewrite \rightarrow f(x) = (x^2-2x)^{\frac{1}{2}}$$

$$f'(x) = \frac{1}{2}(x^2-2x)^{\frac{1}{2}-\frac{2}{2}} \bullet \frac{d}{dx}(x^2-2x) = \frac{1}{2}(x^2-2x)^{\frac{-1}{2}} \bullet (2x-2)$$

$$f'(x) = \frac{2x-2}{2(x^2-2x)^{\frac{1}{2}}} = \frac{2(x-1)}{2(x^2-2x)^{\frac{1}{2}}} = \frac{(x-1)}{(x^2-2x^{\frac{1}{2}})}$$

For trig terms: derivative rules only apply when taking sin, cos, tan,…of x only. So, if it is anything other than the 4 trig terms, use chain rule as follows.

Example 1: f(x) = sin10x f'(x) = cos10x * (10) = 10cos10x

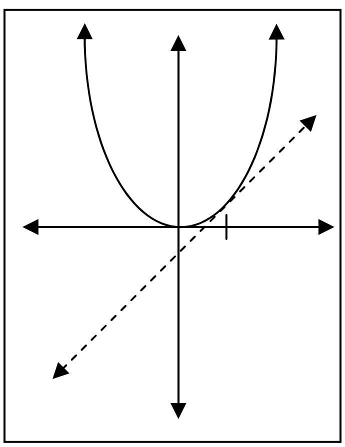
Example 2: f(x) = tan(x/2) f'(x)= $\sec^2$(x/2) * (1/2) = (1/2)$\sec^2$(x/2)

Example 3: f(x) = sec ($4x^2$) f'(x)= sec ($4x^2$)tan($4x^2$) * der ($4x^2$) = sec($4x^2$)tan($4x^2$)(8x)
= (8x sec($4x^2$)tan($4x^2$)

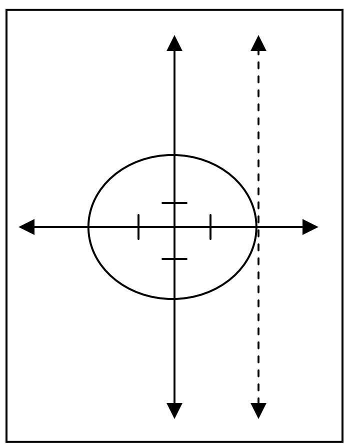
<u>Basics behind chain rule</u>: take derivative of outside term, then multiply by the derivative of inside term.

<u>II. Tangent Line</u>

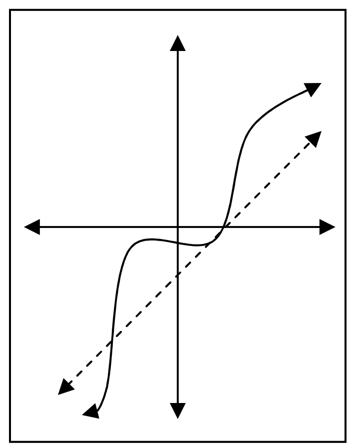
A. **Definition**: a line that touches the function at one point and one point only.



Tangent line at x = 1 touches curve once only at x = 1



Tangent line at x = 2 (doesn't have to be a function, can be just relation)



Not tan line b/c touches graph at 2 places. Called secant line.

B) Steps to finding equation of a tangent line:

1) First find slope: done by finding 1st derivative, then plugging in the point of interest.

Example: y = x^2 find tangent line at x = 2

$$\frac{dy}{dx} = 2x$$

So, plug in x = 2 to the derivative equation, then = 2(2) = 4 ← slope of the tangent line.

2) Use either **the point-slope formula** or **slope intercept formula** to get equation.

a) **Point-Slope**: $y - y_1 = m(x - x_1)$

$y - 4 = 4(x - 2)$

→ Calculated slope from step 1

$y - 4 = 4x - 8$

$\boxed{y = 4x - 4}$ → equation of tangent line at x = 2.

***if they only give you x-value, plug into original equation and solve for y:

$y = x^2$

$y = 2^2 = 4$

So, therefore pt (2,4)

b) **Slope intercept formula**

$y = mx + b$

point (2, 4)

→ m = 4 from step 1

$4 = 4(2) + b$

$4 = 8 + b$

$-4 = b$

Therefore, y = 4x – 4 (equation of tangent line at x = 2