<u>Differentials</u>

Differentials show **rates of change**:

$$\frac{dy}{dx} \approx \frac{\Delta y}{\Delta x}$$

If this is true, then situations can arise in which **an equation, an x-value, and the change in x (Δx)** is given.

<u>Example One:</u>
Find dy (Δy) given

$$y = 1\text{-}2x^2,\ x = 1,\ \text{and } dx = \Delta x = -0.1.$$

1. Find dy/dx by differentiating y.

$$\frac{dy}{dx} = -4x$$

2. Plug into the relationship mentioned above.

$$\frac{\Delta y}{\Delta x} \approx \frac{dy}{dx}$$

$$\frac{\Delta y}{\Delta x} = -4x$$

3. Solve for Δy.

$$\Delta y = -4x(\Delta x)$$

4. Substitute the given values into the equation to solve for Δy.

$$\Delta y = -4(1)(-0.1)$$

$$= 0.4$$

Before this technique, Δy can be obtained by using the following formula:

$$\Delta y = f(x+\Delta x) - f(x).$$

Lets do the previous example using the above formula.

$$y = 1\text{-}2x^2,\ x = 1,\ \text{and } dx = \Delta x = -0.1$$

*1. Calculate **f(x)** and **f(x+Δx)**.*

$$f(x+\Delta x) = 1 - 2(x+\Delta x)^2$$
$$= 1 - 2\left(x^2 + 2x(\Delta x) + (\Delta x)^2\right) \qquad\qquad y = f(x) = 1 - 2x^2$$
$$= 1 - 2x^2 - 4x(\Delta x) - 2(\Delta x)^2$$

2. Use the formula above and the information given to calculate Δy. *Simplify*

$$\Delta y = f(x+\Delta x) - f(x)$$
$$= \left(1 - 2x^2 - 4x(\Delta x) - 2(\Delta x)^2\right) - \left(1 - 2x^2\right)$$
$$= 1 - 2x^2 - 4x(\Delta x) - 2(\Delta x)^2 - 1 + 2x^2$$
$$= -4x(\Delta x) - 2(\Delta x)^2$$
$$= -4(1)(-0.1) - 2(-0.1)^2$$
$$= 0.4 - 0.02$$
$$= 0.38$$

Note: Comparing the differential method (done in Example One with calculus) with the one done in Example Two, the answers are approximately close.

Differentials can also be used to approximate values of functions.

Example Two:

Use differentials to approximate $\sqrt{4.5}$.

1. Find the basic function. In this example the basic function is

$$f(x) = \sqrt{x} = x^{1/2}.$$

2. Find the derivative of f(x)—the basic function found in Step 1.

$$f(x) = \sqrt{x} = x^{1/2}$$

$$f'(x) = \frac{1}{2}x^{-1/2} = \frac{1}{2x^{1/2}} = \frac{1}{2\sqrt{x}}$$

3. Plug into the relationship for differentials.

$$\frac{\Delta y}{\Delta x} \approx \frac{dy}{dx}$$

$$\frac{\Delta y}{\Delta x} = \frac{1}{2\sqrt{x}}$$

4. Solve for Δy.

$$\Delta y = \frac{1}{2\sqrt{x}}\Delta x$$

5. Since this is a square root function, pick the closest integer that is a perfect square to 4.5 and assign it to x. In this example, the closest perfect square is 4. Take x and subtract it from the number given in the problem to get dx or Δx. In this example, ***Δx = 4.5-4 = 0.5***

6. Plug in x and Δx to get a value for Δy.

$$\Delta y = \frac{1}{2\sqrt{4}}(0.5)$$

$$= \frac{0.5}{4} = \frac{1/2}{4} = \frac{1}{8}$$

The conclusion is by increasing x from 4 to 4.5, then the change in y will be from 2 (square root of 4) to 2 + 1/8 or 17/8. Therefore,

$$\sqrt{4.5} \approx 2.125$$

Calculator answer: 2.1232
Our answer: 2.125