Implicit Differentiation

Usually, functions are solved for y in terms of x and the derivative is done explicitly. When the y's are intermingled with x's, we must use **implicit differentiation**.

Example One:

$$y^3 - y^2 - 5y - x^2 = -4$$

1. Differentiate both sides of the equation with respect to x. In this example, we just take the derivative of each term taking note that y is also a variable. Simplify.

Note: Differentiation rules still apply for all of the terms in the equation.

Special Rule: When taking the derivative of y, multiply by ***dy/dx*** *or* ***y´****.*

$$3y^2\boxed{\frac{dy}{dx}} - 2y\boxed{\frac{dy}{dx}} - 5\boxed{\frac{dy}{dx}} - 2x = 0$$

Don't forget to multiply by dy/dx or y´ every time you take the derivative of y.

2. Collect all the terms with ***dy/dx*** *or* ***y´*** *on one side of the equation. In this example, we must add 2x to both sides because it does not contain* ***dy/dx*** *or* ***y´****.*

$$3y^2\frac{dy}{dx} - 2y\frac{dy}{dx} - 5\frac{dy}{dx} = 2x$$

3. Factor out ***dy/dx*** *or* ***y´*** *out of each term.*

$$\frac{dy}{dx}\left(3y^2 - 2y - 5\right) = 2x$$

4. Solve for ***dy/dx*** *or* ***y´*** *by dividing both sides by the expression in parenthesis.*

$$\frac{dy}{dx} = \frac{2x}{3y^2 - 2y - 5}$$

Therefore,

$$\frac{dy}{dx} = \frac{2x}{3y^2 - 2y - 5}.$$

Example Two:

$$3\left(x^2 + y^2\right)^2 = 100xy$$

1. Differentiate both sides of the equation with respect to x. In this example, we must use the ***Chain Rule*** *on the left hand side and the* ***Product Rule*** *on the right hand side taking note that y is also a variable. Simplify.*

Note: Differentiation rules still apply for all of the terms in the equation.

Special Rule: When taking the derivative of y, multiply by ***dy/dx*** *or* ***y´****.*

Chain Rule for Derivatives: The derivative of the outside times the derivative of the inside.

Product Rule for Derivatives: uv´ + vu´

At this step, we had to foil both expressions together to get dy/dx terms.

$$3\left[2\left(x^2 + y^2\right)\left(2x + 2y\frac{dy}{dx}\right)\right] = 100\left[(x)\left(\frac{dy}{dx}\right) + (1)(y)\right]$$

$$3\left[\left(2x^2 + 2y^2\right)\left(2x + 2y\frac{dy}{dx}\right)\right] = 100x\frac{dy}{dx} + 100y$$

$$3\left(4x^3 + 4x^2y\frac{dy}{dx} + 4xy^2 + 4y^3\frac{dy}{dx}\right) = 100x\frac{dy}{dx} + 100y$$

$$12x^3 + 12x^2y\frac{dy}{dx} + 12xy^2 + 12y^3\frac{dy}{dx} = 100x\frac{dy}{dx} + 100y$$

2. *Collect all the terms with* ***dy/dx*** *or* ***y´*** *on one side of the equation.*

$$12x^3 + 12x^2 y \frac{dy}{dx} + 12xy^2 + 12y^3 \frac{dy}{dx} = 100x \frac{dy}{dx} + 100y$$

$$12x^2 y \frac{dy}{dx} + 12y^3 \frac{dy}{dx} - 100x \frac{dy}{dx} = 100y - 12x^3 - 12xy^2$$

3. *Factor out* ***dy/dx*** *or* ***y´*** *out of each term.*

$$\frac{dy}{dx}\left(12x^2 y + 12y^3 - 100x\right) = 100y - 12x^3 - 12xy^2$$

4. *Solve for* ***dy/dx*** *or* ***y´*** *by dividing both sides by the expression in parenthesis.*

$$\frac{dy}{dx} = \frac{100y - 12x^3 - 12xy^2}{12x^2 y + 12y^3 - 100x}$$

Therefore,

$$\frac{dy}{dx} = \frac{100y - 12x^3 - 12xy^2}{12x^2 y + 12y^3 - 100x}.$$