Limits

I. Basics:

A. What it is used for?

1. Limits are used to allow one to numerically, analytically, and graphically determine the behavior of a function at undefined points. Basically you want to see what happens to y as you approach a certain x-value. In order for the limit to exist, you should approach the desired x –value from either side getting closer and closer to that x-value, and the resulting y-values should be approaching a single number. If the left limit approaches one answer, and the right limit approaches another, then the limit does not exist. You can approach limits by evaluating them:

   a) <u>Numerically:</u> this is done by setting up a table and plugging in numbers that get closer and closer(have to be real close in the order of decimals) to the desired x-value, then if the y-values are converging to one number, then that is the limit. If different y – values emerge, the limit does not exist.

   b) <u>Graphically:</u> (Pretty self-explanatory) Graph the function, and observe the behavior at the desired point. If it looks like the function is closing in on the same y- value, then the limit exists; otherwise, it will not exist.

   c) Analytically: This process involves the use of algebra and/or calculus. This is also what you will use most often. The process entails just plugging in the x-value that the limit approaches into the equation. This should give you an answer, but if it does not, algebra may be used to rewrite the equation in a different form and re-evaluate the limit.

*Typical Notation-

$\lim_{x \to \#}$ From the left side(numbers smaller): $\lim_{x \to \#^-}$ From the right(larger): $\lim_{x \to \#^+}$

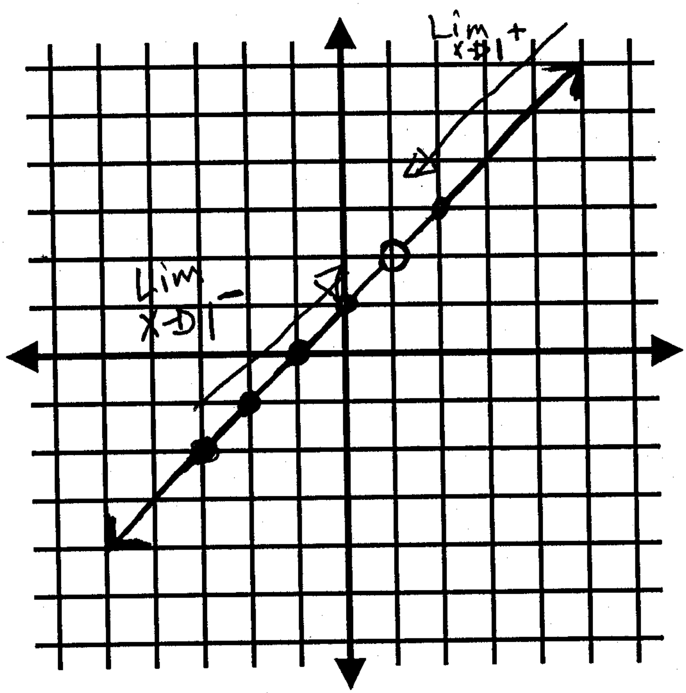
II. Example: $\lim_{x \to 1} \dfrac{x^2-1}{x-1}$

A. Numerically: Pick numbers that approach 1 from either side :

X	.9	.99	1	1.01	1.1
Y=f(x)	1.9	1.99	2	2.01	2.1

You should see that the y-value is converging to 2 from plugging in x-values larger and smaller into the equation; therefore, L = 2.

B. Graphically:



Graph the function and see that the what happens to y-values when you approach the desired x-value form either side.... You see that the y's are approaching 2; therefore, the limit is two.

C. Analytically: Just plug in the limit value and see if you get an answer, if not rewrite it and reevaluate.

$$\lim_{x\to 1} \frac{x^2-1}{x-1}$$

$$\lim_{x\to 1} \frac{(1)^2-1}{1-1} \rightarrow \text{“}\frac{0}{0}\text{”}$$

Anytime you get this or “$\frac{\infty}{\infty}$” You must rewrite the equation using algebra or L'Hopital's rule (not until CalcII)

First try to factor the numerator and denominator and then cancel what you can cancel.

$$\lim_{x\to 1} \frac{x^2-1}{x-1} \rightarrow \frac{(x+1)(x-1)}{x-1} \rightarrow x+1 \rightarrow 1+1 = 2$$

If that hadn't worked, one can try factoring out the highest power out of the top and bottom and evaluating the limit.

$$\lim_{x\to 1} \frac{x^2-1}{x-1} \rightarrow \frac{x(x - 1/x)}{x(1-1/x)}$$

Note that this is still going to give you “$\frac{0}{0}$”

This technique relies primarily on these properties:

If you have a $\#/\infty$, that equals 0;
If you have a $\#/0$, that equals ∞;
If you have $\infty/\#$, that equals ∞;
If you have $0/\#$, that equals 0;
If you have $0/0$ or ∞/∞, then you must rewrite somehow.