Optimization

Sometimes we are asked to maximize or minimize something that is restricted us to a certain amount. This is called **optimization**. We look at some examples that will help you master optimization.

Example: Two positive numbers when multiplied together give 36 and their sum is a **minimum**. Find the two numbers and give the minimum sum.

1. Write down the information given and draw a picture (if necessary) labeling all parts of it.

   Let x and y be the two numbers. We know that the product of the two numbers is 36 or xy = 36. xy = 36 is called the **constraint**. A **constraint** is what we are limited to.

   Let S be the sum of x and y. We know that the sum has to be a maximum. Its equation is given by S = x + y. S = x + y is called the **primary function**. The primary function is the function that is being maximized or minimized.

2. Take the constraint and solve for either variable of your choice and plug result into primary function.

   Solving for y in the constraint yields $y = 36x^{-1}$. Plugging into the constraint gives us $S = x + 36x^{-1}$.

3. Take the derivative of new primary function found in step 2.

   Take derivative of S gives us $S' = 1 - 36x^{-2}$.

4. Set the derivative equal to zero and solve for the critical numbers.

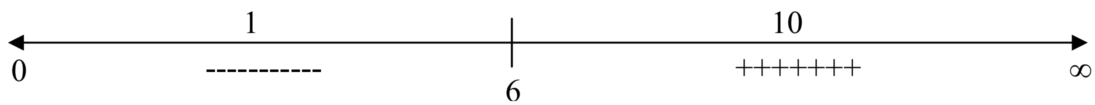
   $0 = 1 - 36x^{-2} \rightarrow 0 = 1 - \frac{36}{x^2} \rightarrow 0 = x^2 - 36 \rightarrow 0 = (x-6)(x+6) \rightarrow x = \pm 6$

   Because the problem require the numbers to be positive, we must throw out the value x = -6. Therefore we use x = 6 as the critical number.

5. Verify the value found in step 4 is a max or min according to the problem.

   Use the First Derivative Test to verify that the value found is a maximum.

$S' = 1 - \frac{36}{x^2}$



6. Find the other value by plugging into the constraint.

   Using xy = 36, we get y = 6 as the value.

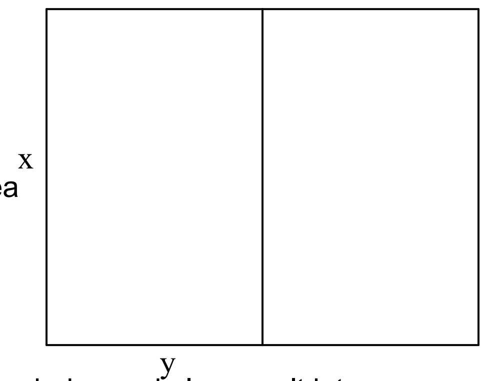
7. Write the answer to the question.

   Make sure to refer back to the question to see what they are asking for. This just asked for the two numbers. So the numbers are x = 6, y = 6 and the minimum sum is 6 + 6 = 12.

Example: A farmer has 1600 ft of fencing to enclose two rectangular corrals. Find the dimensions of the corral that give the maximum area and give the maximum area.

1. Write down the information given and draw a picture (if necessary) labeling all parts of it.

Since we are told that the farmer has 1600 ft of fencing, then we know that will be related to perimeter of the of corral which is given by $P = 2(x) + 2(2y)$ which in turn is $1600 = 2x + 4y$, the constraint. We also are told that we need to maximize the area of the corral, so its formula is given by $A = 2xy$, which is the primary function.



2. Take the constraint and solve for either variable of your choice and plug result into primary function.

Solving for x in the constraint yields $800 – 2y = x$. Plugging into the constraint gives us $A = 2(800 – 2y)y = 1600y – 4y^2$.

3. Take the derivative of new primary function found in step 2.

Take derivative of S gives us $A' = 1600 – 8y$.

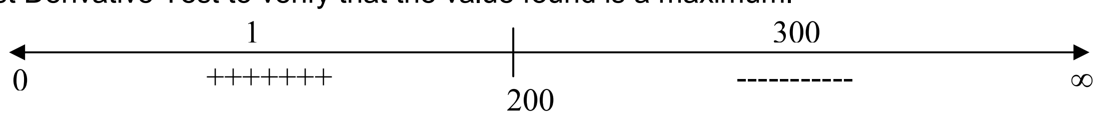
4. Set the derivative equal to zero and solve for the critical numbers.

$0 = 1600 – 8y \rightarrow 8y = 1600 \rightarrow y = 200$

5. Verify the value found in step 4 is a max or min according to the problem.

Use the First Derivative Test to verify that the value found is a maximum.

$A' = 1600 – 8y$



6. Find the other value by plugging into the constraint.

Using $1600 = 2x + 4y$, we get $x = 400$ as the value.

7. Write the answer to the question.

Make sure to refer back to the question to see what they are asking for. This just asked for the dimensions of the corral. So the dimension of the corral is 400 ft x 200 ft. The maximum area is given as 160,000 ft^2.