Points of Inflection

When finding relative extrema, we find the first derivative. When we find the second derivative, we find **possible points of inflection**. A possible point of inflection actually becomes a **point of inflection** (POI) when the concavity changes on either side of it.

Example:

Find the points of inflection of $y = x^2(x-3)$.

1. *Find the first derivative.*

$$y = x^3 - 3x^2$$

$$\frac{dy}{dx} = 3x^2 - 6x$$

Note: We know the following information from the first derivative:

1. *(0, 0) is a **relative max**,*
2. *(2,-4) is a **relative min**,*
3. *f(x) is **increasing** on the intervals (-∞ ,0) and (2,∞), and*
4. *f(x) is **decreasing** on the intervals: (0,2).*

2. *Find the second derivative.*

$$\frac{dy}{dx} = 3x^2 - 6x$$

$$\frac{d^2y}{dx^2} = 6x - 6$$

3. *Set derivative equal to zero and solve for the **critical numbers**.*

*Note: Remember, a critical number is **an x-value for which $d^2y/dx^2 = 0$** or **d^2y/dx^2 is undefined (DNE)**.*

$$\frac{d^2y}{dx^2} = 0$$

$$6x - 6 = 0$$

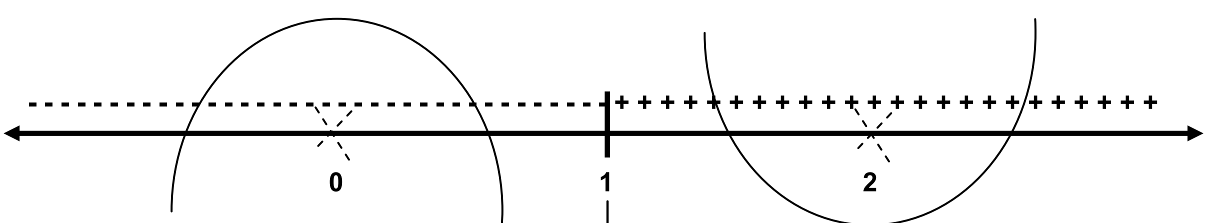
$$6x = 6$$

$$x = 1$$

$$\frac{d^2y}{dx^2} = DNE$$

$$none$$

4. *Determine if each of the x-values found in Step three are possible points of inflection by drawing a number line and test points on each side of the critical numbers. In this example, the test points will be denoted by the dotted x's.*



Test Point	***Sign of f´´(x)***
0	***(-)***
2	***(+)***

Sign change from (-) to (+); therefore, x = 1 is a point of inflection.

*5. To find the **y-values** of the points of inflection, plug the **x-values** (critical numbers verified as points of inflection) into the original equation.*

$$\begin{aligned} y &= 1^3 - 3(1)^2 \\ &= 1 - 3 \\ &= -2 \end{aligned}$$

(1,–2) is a point of inflection

*There could ask the intervals in which f(x) is **concave up** or **concave down**.*
*--If there is a (+) sign, then f(x) **concaves up** towards or away for the critical points.*
*--If there is a (-) sign, then f(x) **concaves down** towards or away from the critical points.*

In this example:
--Concave down intervals: (-∞ ,1)
--Concave up intervals: (1,∞)