<u>Relative Extrema</u>

You can tell a lot about the behavior of the graph by analyzing **first derivatives**.
1. First derivatives tell you the **slope** of a **tangent line**.
2. If you set the first derivative equal to zero, you can obtain the possible maxs and mins of your graph. These points are called **critical points**.

<u>Example—Relative Extrema:</u>

Find the relative extrema of $f(x) = x^3 + 6x^2 - 15x$.

1. Find the first derivative.

$$f'(x) = 3x^2 + 12x - 15$$

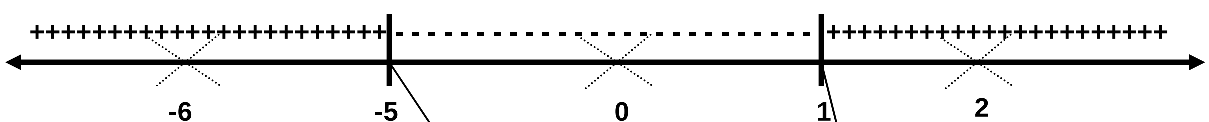
*2. Set derivative equal to zero and solve for the **critical numbers**.*

*Note: Remember, a critical number is **an x-value for which f´(x) = 0** or **f´(x) is undefined (DNE)**.*

$$\underline{f'(x) = 0}$$
$$3x^2 + 12x - 15 = 0$$
$$3(x^2 + 4x - 5) = 0$$
$$3(x+5)(x-1) = 0$$
$$(x+5)(x-1) = 0$$
$$x = -5, x = 1$$

$$\underline{f'(x) = DNE}$$
$$none$$

3. Determine if each of the x-values found in Step two are possible maxs or mins by drawing a number line and test points on each side of the critical numbers. In this example, the test points will be denoted by the dotted x's.



Test Point	***Sign of f´(x)***
-6	***(+)***
0	***(-)***
2	***(+)***

Sign change from (+) to (-); therefore, x = -5 is a relative max.

Sign change from (-) to (+); therefore, x= 1 is a relative min.

Example:

Find the relative extrema of $y = x^2(x-3)$.

1. Find the first derivative.

$$y = x^3 - 3x^2$$

$$\frac{dy}{dx} = 3x^2 - 6x$$

*2. Set derivative equal to zero and solve for the **critical numbers**.*

*Note: Remember, a critical number is **an x-value for which dy/dx = 0** or **dy/dx is undefined (DNE)**.*

$$\frac{dy}{dx} = 0$$

$$3x^2 - 6x = 0$$

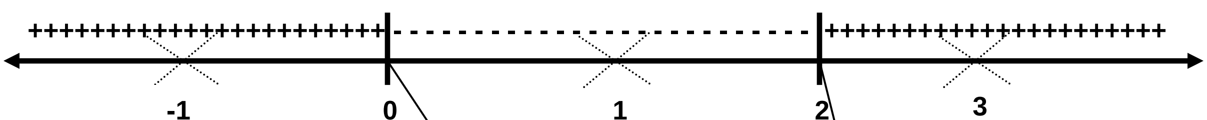
$$3x(x-2) = 0$$

$$x = 0, x = 2$$

$$\frac{dy}{dx} = DNE$$

$$none$$

3. Determine if each of the x-values found in Step two are possible maxs or mins by drawing a number line and test points on each side of the critical numbers. In this example, the test points will be denoted by the dotted x's.



Test Point	*Sign of f´(x)*
-1	***(+)***
1	***(-)***
2	***(+)***

Sign change from (+) to (-); therefore, x = 0 is a relative max.

Sign change from (-) to (+); therefore, x= 2 is a relative min.

*4. To find the **y-values** of the relative extrema, plug the **x-values** (critical numbers verified as maxs or mins) into the original equation.*

$$y = 0^3 - 3(0)^2$$
$$= 0 - 0$$
$$= 0$$
$$(0,0) \text{ is a relative max}$$

$$y = 2^3 - 3(2)^2$$
$$= 8 - 12$$
$$= -4$$
$$(2,-4) \text{ is a relative min}$$

*There could ask the intervals in which f(x) is **increasing** or **decreasing**.*
*--If there is a (+) sign, then f(x) is **increasing** towards or away for the critical points.*
*--If there is a (-) sign, then f(x) is **decreasing** towards or away from the critical point.*

In this example:
--Increasing intervals: (-∞ ,0) and (2,∞)
--Decreasing intervals: (0,2)