<u>**Rolle's Theorem**</u>

<u>Conditions for **Rolle's Theorem**</u>:

1. The function must be ***continuous*** *on the interval [a,b].*

Note: This also includes the endpoints a and b. Use the domain rules you've learned in the past.

2. The function must be ***differentiable*** *on the open interval (a,b).*

Note: This also says that its derivative must be ***continuous*** *on its interval, excluding the endpoints. Again, use the domain rules you've learned in the past.*

3. ***f(b) = f(a)***

Note: This says that the original function evaluated at the endpoints must be equal.

If all three conditions are satisfied, then there exists at c for which f´(c) = 0.

<u>Example:</u>

Determine if Rolle's Theorem applies to the following function. If not, tell why.

$$f(x) = x^2 - 3x + 2, \qquad [1,2]$$

<u>Analysis:</u>

1. The function f(x) is continuous for all x because it is a polynomial and by the domain rules, all polynomials are continuous.

2. The function f(x) is differentiable for all x because its derivative f´(x) = 2x –3, is continuous since it's also a polynomial.

3. f(b) = f(a)

$$f(2) = f(1)$$
$$(2)^2 - 3(2) + 2 = (1)^2 - 3(1) + 2$$
$$0 = 0 \quad \checkmark$$

<u>Conclusion:</u>

Since all three conditions are met, Rolle's Theorem does apply and we can set f´(c) = 2c-3 = 0 and solve for c.

$$f'(c) = 0$$
$$2c - 3 = 0$$
$$2c = 3$$
$$c = \frac{3}{2}$$

Mean Value Theorem

Conditions for **The Mean Value Theorem**:

1. *The function must be **continuous** on the interval [a,b].*
   Note: This also includes the endpoints a and b. Use the domain rules you've learned in the past.
2. *The function must be **differentiable** on the open interval (a,b).*
   *Note: This also says that its derivative must be **continuous** on its interval, excluding the endpoints. Again, use the domain rules you've learned in the past.*

If both conditions are satisfied, then there exists a c for which

$$f'(c) = \frac{f(b) - f(a)}{b - a}.$$

Example:

Determine if the Mean Value Theorem applies to the following function. If not, tell why.

$$f(x) = x^2, \qquad [-2,1]$$

Analysis:

1. The function f(x) is continuous for all x because it is a polynomial and by the domain rules, all polynomials are continuous.

2. The function f(x) is differentiable for all x because its derivative, f´(x) = 2x, is continuous since its also a polynomial.

Conclusion:

Since all two conditions are met, the Mean Value Theorem does apply and we can set

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

and solve for c.

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

$$2c = \frac{f(1) - f(-2)}{1 - (-2)}$$

$$2c = \frac{1 - 4}{1 - (-2)}$$

$$2c = \frac{-3}{3} = -1$$

$$2c = -1$$

$$c = -\frac{1}{2}$$

The final answer is

$$f'\left(-\frac{1}{2}\right) = -1.$$