

Limit Identities Involving Sine and Cosine

Please go ahead and memorize these:

$$*\lim_0 \frac{\sin^*}{*} = *\lim_0 \frac{*}{\sin^*} = 1$$

$$*\lim_0 \frac{1 - \cos^*}{*} = 0$$

***Note- the asterisk is placed there to signify that it can be something other than just x, as long as it shows up on the bottom as well.**

A.

$$*\lim_0 \frac{\sin^*}{*} = 1$$

Look for a sin in the problem, **If the argument of the sine approaches zero**, see if you can make the denominator look like the argument of sine. This will then allow you to separate this part of it out, and allow the use of the identity.

1. Example-

$$*_x \lim_0 \frac{\sin 4x^2 + 2x}{x} = ?$$

get "0"/0

The first step is to plug into the equation, and you

Since a trig term is involved, you cannot factor or do any algebra to simplify.

*The objective is to get the denominator to equal the argument of the sin term.

$$\begin{aligned} *_x \lim_0 \frac{\sin 4x^2 + 2x}{x} \left(\frac{4x^2 + 2x}{4x^2 + 2x} \right) &\rightarrow \\ \left(\frac{4x^2 + 2x}{x} \right) \frac{\sin 4x^2 + 2x}{4x^2 + 2x} &\rightarrow \\ \left(*_x \lim_0 \frac{4x^2 + 2x}{x} \right) \left(*_x \lim_0 \frac{\sin 4x^2 + 2x}{4x^2 + 2x} \right) &\rightarrow \\ \left(\frac{0}{0} \right) (1) \rightarrow *_x \lim_0 4x + 2 &\rightarrow 2 \end{aligned}$$

If you multiply the top and bottom by the argument of sine, you use the identity.
 Separate it out, and then evaluate the other part of the problem.

B.

$$\lim_{x \rightarrow 0} \frac{1 - \cos^2 x}{x} = 0$$

Same principle as above, but different identity:

1. Example:

$$\begin{aligned} \lim_{x \rightarrow 0} \frac{\sin^2 4x}{x} &= \frac{1 - \cos^2 4x}{x} = \frac{(1 - \cos 4x)(1 + \cos 4x)}{x} \\ &= \left(\lim_{x \rightarrow 0} \frac{1 - \cos 4x}{x} \right) \left(\lim_{x \rightarrow 0} (1 + \cos 4x) \right) = > \\ &= \left(\lim_{x \rightarrow 0} \frac{1 - \cos 4x}{4x} \right) 4 \left(\lim_{x \rightarrow 0} (1 + \cos 4x) \right) = > \\ &= 0 \left(\lim_{x \rightarrow 0} (1 + \cos 4x) \right) = 0 \end{aligned}$$

The natural tendency might be to try to get the previous identity to work, but you will soon see that you will get nowhere, so then go for the other identity... It may not be obvious if you forgot your trig identities. Once converted, separate out the identity part, and show that it indeed fits the identity by multiplying the top and bottom by 4. Then it really doesn't matter what the second part is, (unless it is what? One exception what is it?) the first part makes the whole thing approach zero.

***Again, the trick is to get the denominator to look like your argument of sine or cosine, and to separate it out.**

***Make sure that the denominator and the argument are approaching zero.**