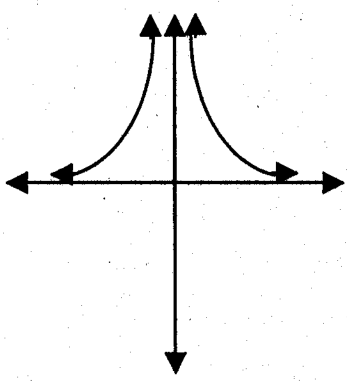
Asymptotes

I. **General definition**: an asymptote is an imaginary barrier that the function can approach, but never touches or crosses that value.

A. **Vertical asymptotes** occur in rational functions (functions with a denominator). In order to obtain the numerical value of the asymptote, simply set the denominator equal to zero and solve.

Example: $f(x) = \frac{1}{x^2}$



To find the vertical asymptote: set $x^2 = 0$

$$\sqrt{x^2} = \sqrt{0}$$

$$x = 0$$

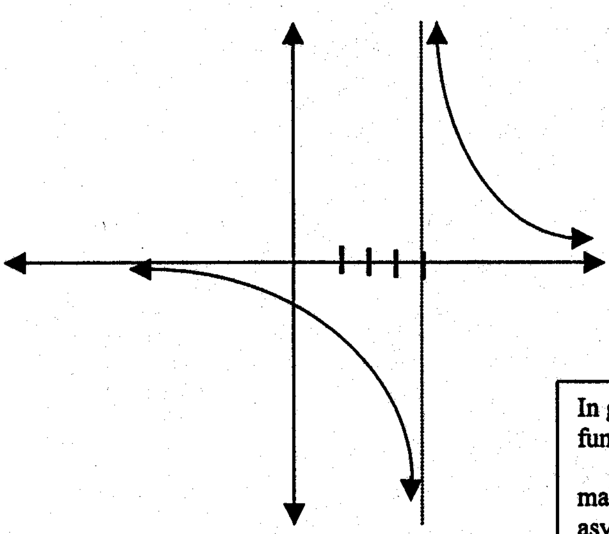
If we plug in zero for x, we would get $\frac{1}{0^2} = \infty$ or undefined.

x = 0 is the vertical asymptote

Example: $f(x) = \frac{1}{x-4}$

$x - 4 = 0$

$x = 4$ is the vertical asymptote



In general, whatever number makes the function undefined, or whatever number makes $|f(x) \to \infty|$ is the vertical asymptote.

B) Horizontal asymptotes or oblique asymptotes.

<u>Steps</u>:

1) Look at the exponents of numerator and denominator, then

   a) If the power of the top is less than the bottom, then $y = 0$ is horizontal asymptote.

Example : $\frac{x^2}{x^3 - 1} \Rightarrow$ Since the power of the top is less than the bottom,

$y = 0$ is the horizontal asymptote.

   b) If the power of the top is the same as the bottom, then the coefficients of the highest power term make up the horizontal asymptote.

Example: $\frac{4x^2}{2x^2 - 1} \Rightarrow$ Since the power of the top is the same as the bottom,

we take the cofficients of the highest power, which is

x^2, and we get $\frac{4}{2} = 2$

   c) If the power of bottom is less than the top by 1 factor, then we have oblique asymptote. In order to obtain this asymptote, we use long division, and whatever is not part of the remainder, is the oblique asymptote.

Example : $\frac{x^2 - 1}{x} \Rightarrow$ Since the power of the bottom is less than the top,

we do long division, and divide the numerator by the denominator. And we would get,

$$\begin{array}{r} x - \frac{1}{x} \\ x \overline{)\, x^2 - 1} \\ \underline{-x^2 } \\ 0 \quad \boxed{-1} \end{array}$$

$\boxed{-1}$ → Remainder

The whole answer after the long division is : $x - \frac{1}{x}$

As we can see, $\frac{1}{x}$ is the remainder, but 'x' is not part of the remainder.

So the oblique asymptote is : $y = x$.