Composite Functions

Composite functions are made by plugging one equation into every x of another equation. Usually denoted by f (g(x)) or (f o g)(x).

Plug in g(x) for every x.

- Concept is just like plugging in a number into an equation.

$f(x) = x^2 + 4x$ find $f(1) = (1)^2 + 4(1) = 1 + 4 = 5$

If a 2nd equation is given, $g(x) = x - 2$, to find f (g(x)) or (f o g)(x), plug in g (x) just like we plugged in a 1 in the above equation.

g(x)

So, we would get... $= (x-2)^2 + 4(x-2)$

Then, simplify... $= (x-2)(x-2)$
$= x^2 - 4x + 4 + 4x - 8$

$$(f \circ g)(x) = x^2 - 4$$

When we do it the other way around, we would get the following:

$$g(f(x)) \text{ or } (g \circ f)(x) = (x^2 + 4x) - 2$$
$$= x^2 + 4x - 2$$

* If you are asked to find the domain, combine the domain of the final answer with the domain of the equation inside the parenthesis. Or in another words, for f (g(x)), it's the domain of g(x) and the domain of the final answer. And for g (f(x)), it's the domain of f(x) and the domain of the final answer.

Example: $g(x) = 3x - 2$ → Domain is $(-\infty,\infty)$

$s(x) = \frac{1}{x}$ → Domain $x \neq 0$

$$g(s(x)) = 3\left(\frac{1}{x}\right) - 2$$
$$= \frac{3}{x} - 2$$

In this case, both the final answer & s(x) (inside parenthesis) have the same domain $x \neq 0$.

Example: $s(x) = \frac{1}{2}x$ $\qquad t(x) = \frac{4}{x+2}$

To find t (s(x)) or (t o s)(x)…

$$t(s(x)) = \frac{4}{\left(\frac{1}{2}x\right)+2}$$

Domain of s(x) is $(-\infty,\infty)$

$$= \frac{4}{\frac{1}{2}x+2}$$

Domain of answer is

$$\frac{1}{2}x+2 \neq 0$$

$$2\frac{1}{2}x \neq -2*2$$

$$x \neq -4$$

The combined final domain is $x \neq -4$.

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* Now, to find the other way around, meaning to find s (t(x)) or (s o t)(x)

$$\frac{1}{2}\left(\frac{4}{x+2}\right) = \frac{4}{2(x+2)} = \frac{4}{2x+4}$$

Domain t(x)

$$x + 2 \neq 0$$

$$x \neq -2$$

$$2x + 4 \neq 0$$

$$2x \neq 0$$

$$x \neq -2$$

Again, they match. The point is, if they don't match, then take both answers into account to get the final domain answer.

Determining Inverses

The inverse of a function is obtained by switching the domain with the range. In other words, switch x with your y; to obtain the inverse, switch x with y and solve for y.

$y = 2x + 10$, find inverse:

steps:

1) switch x with y

   $x = 2y + 10$

2) solve for y

   $x = 2y + 10$

   $x - 10 = 2y + 10 - 10$

   $x - 10 = 2y$

   $$\frac{x-10}{2} = \frac{2y}{2}$$

   $\frac{1}{2}x - 5 = y$ → The inverse is denoted by $F(x)^{-1}$ or y^{-1}.

When doing composite functions of inverses, they should = x
So, call 1st function $f(x) = 2x + 10$ and inverse $g(x) = \frac{1}{2}x - 5$

$(f \circ g)(x)$ should = x
or
$f(g(x))$

Let's test and see if they do equal to x.

$f(g(x)) = 2(\frac{1}{2}x - 5) + 10$
$= x - 10 + 10$
$= x$ ✓

and

$g(f(x))$ or $(g \circ f)(x)$ should also equal to x. → $\frac{1}{2}(2x + 10) - 5$
→ $x + 5 - 5$
→ x ✓