Cut Points

I. General:

When dealing with inequalities, watch for:

exponents greater than one,
& numerator & denominator.

If you encounter either of these situations, you must use cut points:

To obtain your cut points, find the zeros of either numerator denominator, or of whatever is under radical(if finding domain).

You then must set up a number line with the cut points on it. Then pick points on either side of the cut points as test points and plug them into the function so we can determine the behavior of the function on either side of the zeros. Remember zeros are on the axis, and we are trying to determine whats happening on either side of the zeros.

If positive answer, denote by plus signs; if neg, put minus signs. After analyzing the number line, pick the range of numbers that make your inequality true.

II. Examples:

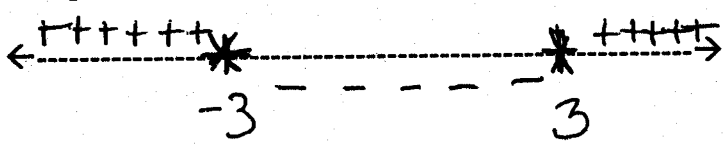
1. Find the domain of $f(x) = \sqrt{x^2 - 9}$?
You should remember that all terms under radical should be positive; therefore,

$$x^2 - 9 > 0$$

Find your cut points:

Factor the expression to yield x = 3, -3;

Set up the number line and do test points:



try,
$-4 : (-4)^2 - 9$ = pos answer
$0: (0)^2 - 9$ = neg ans
$4: (4)^2 - 9$ = pos answer.

We can then conclude that the region we are interested in, $x^2 - 9 > 0$, lies between:

$(-\infty,-3)$ U $(3, \infty)$. (U means that the two answer sets are combined)

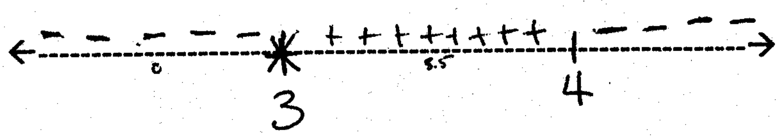
2. $\frac{x+1}{3x-6} \langle 2$ When confronted with a numerator/denominator, make sure that you have greater or less than zero. In this problem, this means to bring the two to the other side. Before finding the cut points you must find a common denominator and combine into one simple expression.

$$\frac{x+2}{3x-9} - 2 < 0 ---> \frac{x+2-2(3x-9)}{3x-9} ---> \frac{x+2-6x+18}{3x-9} ---> \frac{-5x+20}{3x-9} < 0$$

$$-5x + 20 = 0 ---> 20 = 5x ---> x = 4$$

$$3x - 9 = 0 ---> 3x = 9 ---> x = 3$$

Then set up a number line and plug in test points. . .



remember to plug into the simplified expression of $\frac{-5x+20}{3x-9} < 0$.

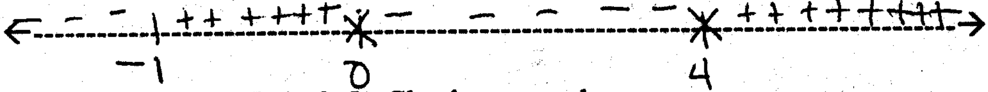
Check my numbers…. $\therefore (-\infty -3) \cup (4,\infty)$

3. $3x^3 - 9x^2 - 12x > 0$ Factor out the greatest common term, 3x

$3x(x^2 - 3x - 4) > 0$ then factor the quadratic,

$3x(x-4)(x+1) > 0$ your cut points are where these equal zero, so go off to the side and set them equal to zero and solve, You should get x = 0, 4, -1 as your cut points.

Set up the number line and its business as usual…



I will choose $x = -2, -.5, 1, \& 5$. Check my numbers.

Therefore the range that I am interested, where $3x(x - 4)(x + 1) > 0$ is,

$(-1,0) \cup (5,\infty)$.