

Domain

- I. **Polynomials:** Any equation that lacks a denominator or a radical (square root sign) has a domain of all real numbers, which means the domain is defined from negative infinity to positive infinity.

Ex:

$$y = x^3 - 2x + 3$$

$$y = -2x$$

- not $x = \text{any } \#$ why?

- II. **Denominators:** Equations with denominators can become undefined if the denominator is equal to zero. Therefore, to find the domain of the function, you must set whatever is in the denominator not equal to zero and solve for the given variable:

Ex: $y = \frac{2x}{x^3 - 4x^2}$ $y = \frac{3}{x^2 - 2x - 15}$ $y = \frac{5x + 3}{4x^2 + 16}$

$$X^3 - 4X^2 \neq 0$$

$$X^2 - 2X - 15 \neq 0$$

what's different about this one?

$$X^2(X - 4) \neq 0$$

$$(X - 5)(X + 3) \neq 0$$

$$X^2 \neq 0 \quad X - 4 \neq 0$$

$$X - 5 \neq 0 \quad X + 3 \neq 0$$

$$X \neq 0 \quad X \neq -4$$

$$X \neq 5 \quad X \neq -3$$

III. Radicals:

A. You should always remember that square roots always have to have a positive number inside; otherwise, you enter the world of complex numbers. So to find the domain of a radical(with the power of x), you set whatever is under the square root sign greater than or equal to zero:

Ex: $y = \sqrt{2 - 4x}$

1. $2 - 4x \geq 0$

2. Solve for x

$$-4x \geq -2$$

$$\frac{-4x}{-4} \leq \frac{-2}{-4} \text{ * note*}$$

$$x \leq -\frac{1}{2}$$

*note- the direction of the inequality flipped upon mult or dividing by a negative number.

$$y = \frac{1}{\sqrt{x+5}}$$

1. $x+5 > 0$ * note*

2. Solve for x

$$x + 5 - 5 > 0 - 5$$

$$x > -5$$

*note- no equal sign b/c in denomin.

B. If the radical has a power of 2 or more, you have to approach it slightly differently. The first step is to find cut points by setting the argument underneath the radical equal to zero, and solving for the variable. Cut points are the zeros or roots of the equation; therefore, they are neither greater than or less than zero. Plot these points on a number line, and pick test points in between the cut points and plug them into the original equation. The results that are positive then are your domain.

Ex:

$$Y = \frac{12}{\sqrt{x^2 - 4}}$$

1. $X^2 - 4 > 0$ IS THE STATEMENT TO BE TESTED*

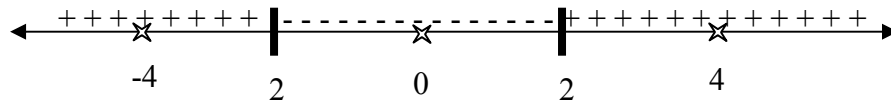
NOTE- THERE IS NO EQUAL SIGN B/C FROM DENOMINATOR

2. $X^2 - 4 = 0$ IS HOW YOU FIND THE CUT POINTS.

$$(X+2)(X-2) = 0$$

$$X=2; X=-2.$$

3. SET UP A NUMBER LINE AND USE TEST POINTS:



USE A NUMBER TO THE LEFT AND RIGHT OF EACH CUT POINT.

***PLUG INTO THE GENERAL EQUATION OF $x^2 - 9 > 0$ ***

$$(-4)^2 - 9 > 0 \qquad (0)^2 - 9 > 0 \qquad (4)^2 - 9 > 0$$

$$16 - 9 > 0 \qquad 0 - 9 > 0 \qquad 16 - 9 > 0$$

$$7 > 0 \text{ true} \qquad -9 > 0 \text{ false} \qquad 7 > 0 \text{ true}$$

THEN TAKE THE REGIONS OF THE NUMBER LINE THAT GAVE TRUE STATEMENTS: $(-\infty, -2)$ and $(2, \infty)$. ***note that you read left to right like a sentence***