

Law of Sines and Cosines

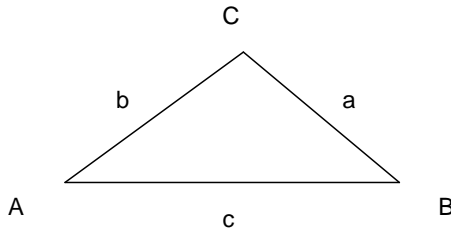
I. Law of Sines:

These identities are used to find missing pieces of oblique triangles. The law of sines can only be applied if the following things are given:

- S- Side A-Angle –

SSA If two sides and the angle not included between them are given,
ASA If two angles and the side between them are given,
SAA Two angles and one side that is not included in the angles.

For the following examples use the following triangle:



A. Examples-

1. **A = 40; B = 20; a = 2.**

This is a SAA problem which means that we can use the law of sines:

$$\frac{\sin A}{a} = \frac{\sin B}{b} \rightarrow \frac{\sin 40}{2} = \frac{\sin 20}{b} \rightarrow \text{cross multiply} \rightarrow b \sin 40 = 2 \sin 20$$

$$b = \frac{2 \sin 20}{\sin 40} \rightarrow b = 1.0642$$

2. $A = 35; B = 15; c = 5.$

This is ASA problem which uses the law of sines:

The trick to this type is that you have to find the third angle, C:

We know that the sum of the triangles is 180; therefore:

$$A + B + C = 180 \rightarrow 35 + 15 + C = 180 \rightarrow 50 + C = 180 \rightarrow C = 130$$

We can then use the law of sines $\rightarrow \frac{\sin B}{b} = \frac{\sin C}{c} \rightarrow \frac{\sin 15}{b} = \frac{\sin 130}{5}$

$$5 \sin 15 = b \sin 130 \quad b = \frac{5 \sin 15}{\sin 130} = 1.689 \cong 1.69$$

$$\frac{\sin A}{a} = \frac{\sin C}{c} \quad \text{same steps as above to get } 3.74$$

3. $a = 6; b = 8; A = 35.$

This, SSA, is the one you got to watch for, when the angle is not included in the sides:

Use the law of sines as you normally would,

$$\frac{\sin B}{b} = \frac{\sin A}{a} \rightarrow \frac{\sin B}{8} = \frac{\sin 35}{6} \rightarrow 6 \sin B = 8 \sin 35 \rightarrow \sin B = \frac{8 \sin 35}{6}$$

$$\sin B = .7648$$

***Note- If this quantity is greater than 1 or less than negative one there is no solution, and the law of sine cannot be applied. This is due to the fact that sine's max and min are 1 and -1, respectively.**

$$\sin^{-1}(\sin B) = \sin^{-1}(.7648)$$

$B = 49.88$, and in the Quadrant II
(where sin is also pos) 130.11

II. Law of Cosines:

If you encounter the following situations, you must use the law of cosines:

SAS if given two sides and an included angle,

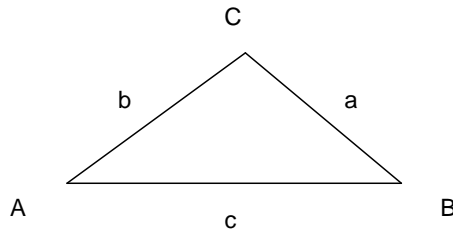
SSS if given all three sides,

$$a^2 = b^2 + c^2 - 2(b)(c) \cos A$$

$$b^2 = a^2 + c^2 - 2(a)(c) \cos B$$

$$c^2 = b^2 + a^2 - 2(b)(a) \cos C$$

Using the same triangle:



A. Examples:

1. **b = 4; c = 2; A = 30**

This is a SAS type:

So begin with the appropriate identity: since a is missing

$$\begin{aligned} a^2 &= b^2 + c^2 - 2(b)(c) \cos A \rightarrow a^2 = 4^2 + 2^2 - 2(4)(2)\cos 30 \rightarrow \text{simplify} \rightarrow 20 - 16(.866) \rightarrow \\ &= 6.14 \rightarrow a^2 = 6.14 \rightarrow a = 2.47 \end{aligned}$$

From here you could use the law of sines with a, b, & A, or continue using the law of cosines to find angle B using sides b,c, & a.

2. a = 4; b = 3; c = 6.

SSS in this problem, so use the sides to find the angles,

$$b^2 = a^2 + c^2 - 2(a)(c) \cos B$$

$$3^2 = 4^2 + 6^2 - 2(4)(6) \cos B$$

$$9 = 16 + 36 - 48 \cos B$$

$$9 = 52 - 48 \cos B$$

$$-43 = -48 \cos B$$

$$*\frac{43}{48} = \cos B \text{ *note the value (43/48) should be between 1 and -1.}$$

$$\text{Cos}^{-1} \frac{43}{48} = 26.38 = B$$

Then again choose what identity and apply it to solve for the remaining angles.

The trick is knowing when to use what. If you remember law of cosines is SSS & SAS, then use the law of sine at all other situations involving oblique triangles.