<u>Sum to Product Identities</u>

I. Identities: I shall use sin and cos to explain the concepts. The sum to product identities can allow you to find theta values that are usually found with a calculator.

A. Sine and Cosine:

sin (A + B) = sin A cos B + cos A sin B
sin (A - B) = sin A cos B - cos A sin B

***note- try to memorize patterns- sine identity is the one that keeps the same sign, and sine alternates with cosine.**

1. ex: Find sin 75 without using a calculator:

a) first thing is to determine what angles(30,45,60,90,…) trig you can break down 75 (angles that have **<u>memorized</u>** values of sine, cosine,…):

30 + 45 = 75

b) then use the appropriate identity:

sin (30 + 45) = sin 30 cos 45 + cos 30 sin 45

c) now using your **<u>memorized</u>** quantities substitute**:**

$$\textbf{sin (30 + 45) = (1/2)}\left(\frac{\sqrt{2}}{2}\right) + \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right)$$

$$= \frac{\sqrt{2}}{4} + \frac{\sqrt{6}}{4}$$

$$= \frac{\sqrt{2}+\sqrt{6}}{4}$$

B. cos(A+ B) = cos A cos B – sin A sin B

cos(A - B) = cos A cos B + sin A sin B

*__note-__ the pattern here is no alternation(2 cos then 2 sin), and the sign of the identity is the opposite of + and -.

1. Find cos π/12 = cos 15:

a) same angles as the previous example, but this time it is a difference: **45 – 30 = 15**

cos(A - B) = cos A cos B + sin A sin B

b) cos(45 - 30) = cos 45 cos 30 + sin 45 sin 30

c) cos(45 - 30) = $\left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) + \left(\frac{\sqrt{2}}{2}\right)(1/2)$

d) $\frac{\sqrt{6}}{4} + \frac{\sqrt{2}}{4} = \frac{\sqrt{2}+\sqrt{6}}{4}$

C. Tangent: tan (A+B) = $\frac{\tan A + \tan B}{1 - \tan A \tan B}$

tan (A-B) = $\frac{\tan A - \tan B}{1 + \tan A \tan B}$

By now you should see it is just memorization and plugging into the formula.