<u>Phase Transformations Rules for Sine and Cosine</u>

<u>Definitions:</u>

The graphs of $y = A\sin(Bx + C)$ *and* $y = A\cos(Bx + C)$ *have*

$$Amplitude = |A|$$

$$Period = \frac{2\pi}{|B|}$$

$$Frequency = |B|$$

$$Phase\ shift = -\frac{C}{B}$$

The interval for the fundamental cycle can be found by solving the inequality

$$0 \le Bx + C \le 2\pi$$

*The **guidepoints** can be found by dividing this interval into four equal parts.*

*<u>Amplitude (A)</u>: the **highest** and **lowest** possible values for the graph. Usually, these two oscillate between a maximal value of 1 and a minimal value of –1. If there is a multiplier in front of sine or cosine, multiply it by 1 and –1 to get new maximum and minimum values.*

<u>Period (T)</u>: This is the length that it takes for the graphs to make one complete cycle. Its formula is given above.

<u>Frequency (f):</u> Whenever there is a coefficient in front of x, it alters the period. If the coefficient in front is a whole number, then it makes the cycle repeat more than once between 0 and 2π.

*Note**: I recommend that you have the basic trigonometric graphs understood so that you can use these rules effectively. If you don't know them, then these rules won't help.***

Example:

Graph

$$y = 4\sin(2x + \pi).$$

1. *Calculate the **amplitude**.*

   Maximum point = 4

   Minimum point =-4

2. *Calculate the **period**.*

$$Period = T = \frac{2\pi}{|B|}$$

$$= \frac{2\pi}{2}$$

$$= \pi$$

   Thus, we can expect this graph to repeat at every π.

3. *Calculate the **phase shift** by setting Bx + C = 0.*

$$Bx + C = 0$$

$$2x + \pi = 0$$

$$2x = -\pi$$

$$x = -\frac{\pi}{2}$$

   We will need the above x-value to verify the value we get in the Step 5 below.

4. *Calculate the **fundamental cycle**.*

$$0 \le Bx + C \le 2\pi$$

$$0 \le 2x + \pi \le 2\pi$$

$$-\pi \le 2x \le \pi$$

$$-\frac{\pi}{2} \le x \le \frac{\pi}{2}$$

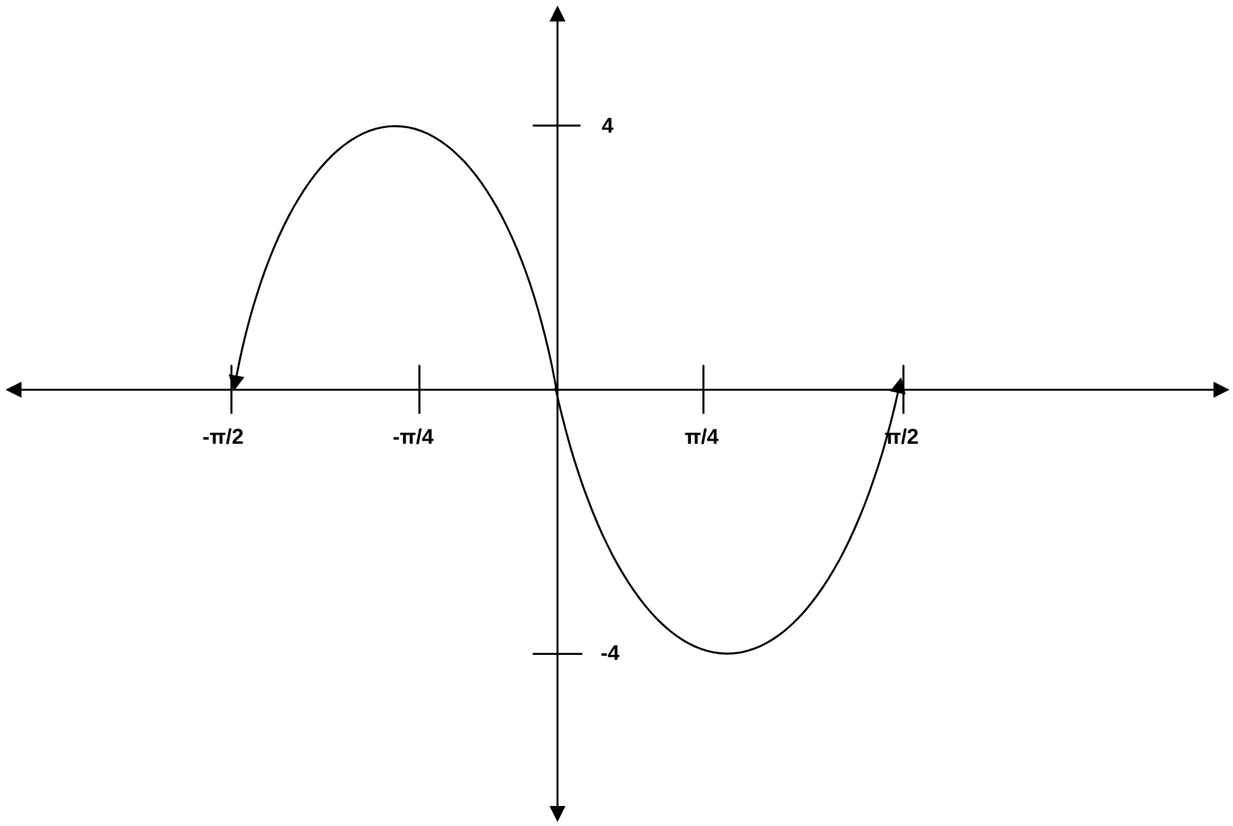
   *Note: **You are usually asked this to find the interval to graph the above equation on**.*

5. *Divide the interval into four equal parts by dividing the **period** by 4.*

$$\frac{T}{4} = \frac{\pi}{4}$$

   Thus, π/4 is the interval to use on the x-axis.

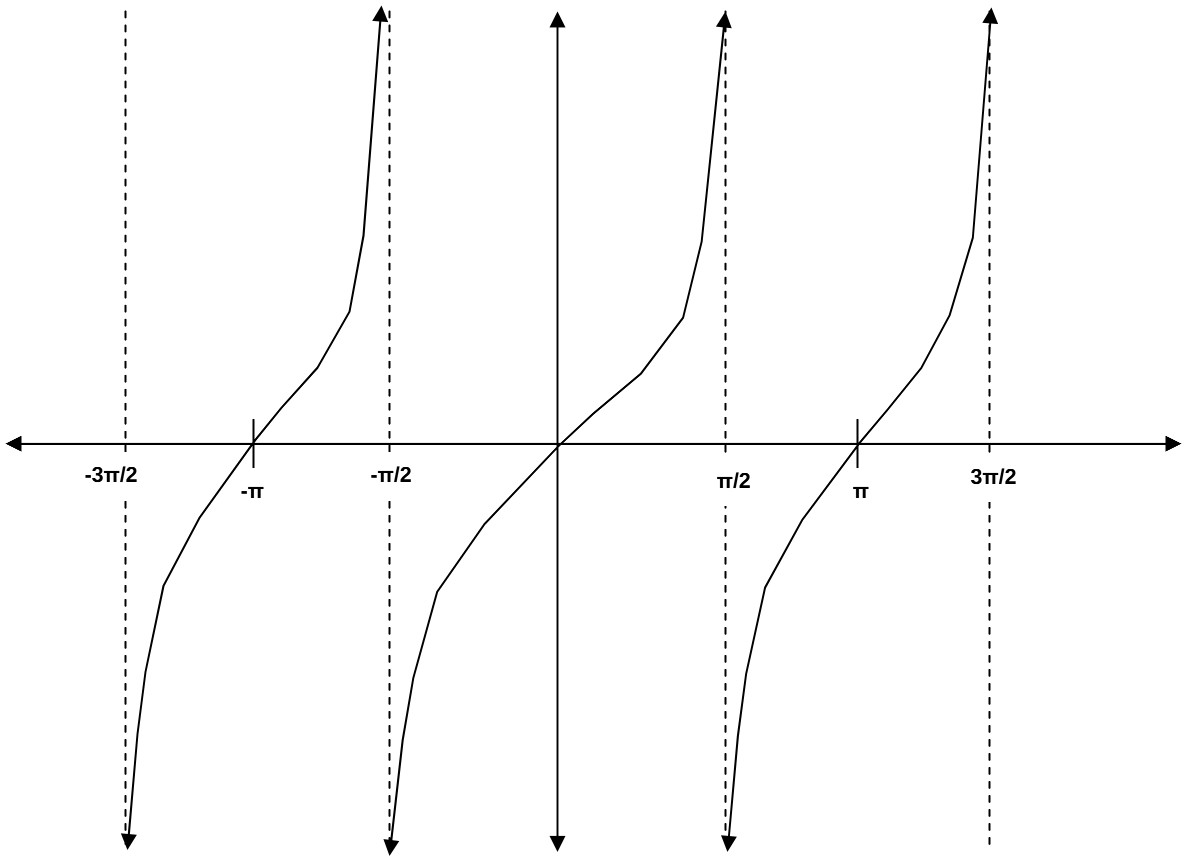
6. Sketch the graph using the information from Steps 1-5.



Note: ***This graph keeps going in either direction repeating a cycle at every π. By knowing the basic sine graph, I know that the first point is on the axis, the second point is at the maximal value, the third point back on the axis, and the fourth at the minimal value. If the coefficient A is negative, the graph is reflected about the x-axis. Again, the same rules apply.***

Phase Transformations Rules for Tangent, Cotangent, Secant, and Cosecant

Tangent and Cotangent:
Note: ***For explanation, I will use the tangent graph for applying transformation rules. Cotangent is similar to that of tangent. Again, the transformations are difficult if you don't know the basic graphs. The basic tangent graph is shown below.***



Example:
Graph

$$y = \tan\left(2x - \frac{\pi}{2}\right).$$

1. Calculate the ***fundamental cycle****. In the basic tangent graph, the cycle is from* ***–π/2 to π/2****.*

$$-\frac{\pi}{2} \le Bx + C \le \frac{\pi}{2}$$

$$-\frac{\pi}{2} \le 2x - \frac{\pi}{2} \le \frac{\pi}{2}$$

$$0 \le 2x \le \pi$$

$$0 \le x \le \frac{\pi}{2}$$

2. *Calculate the* ***period****. In the basic tangent graph, the period is* ***π****.*

$$T = \frac{\pi}{|B|}$$
$$= \frac{\pi}{2}$$

3. *Divide the period into two equal parts to get the halfway point.*

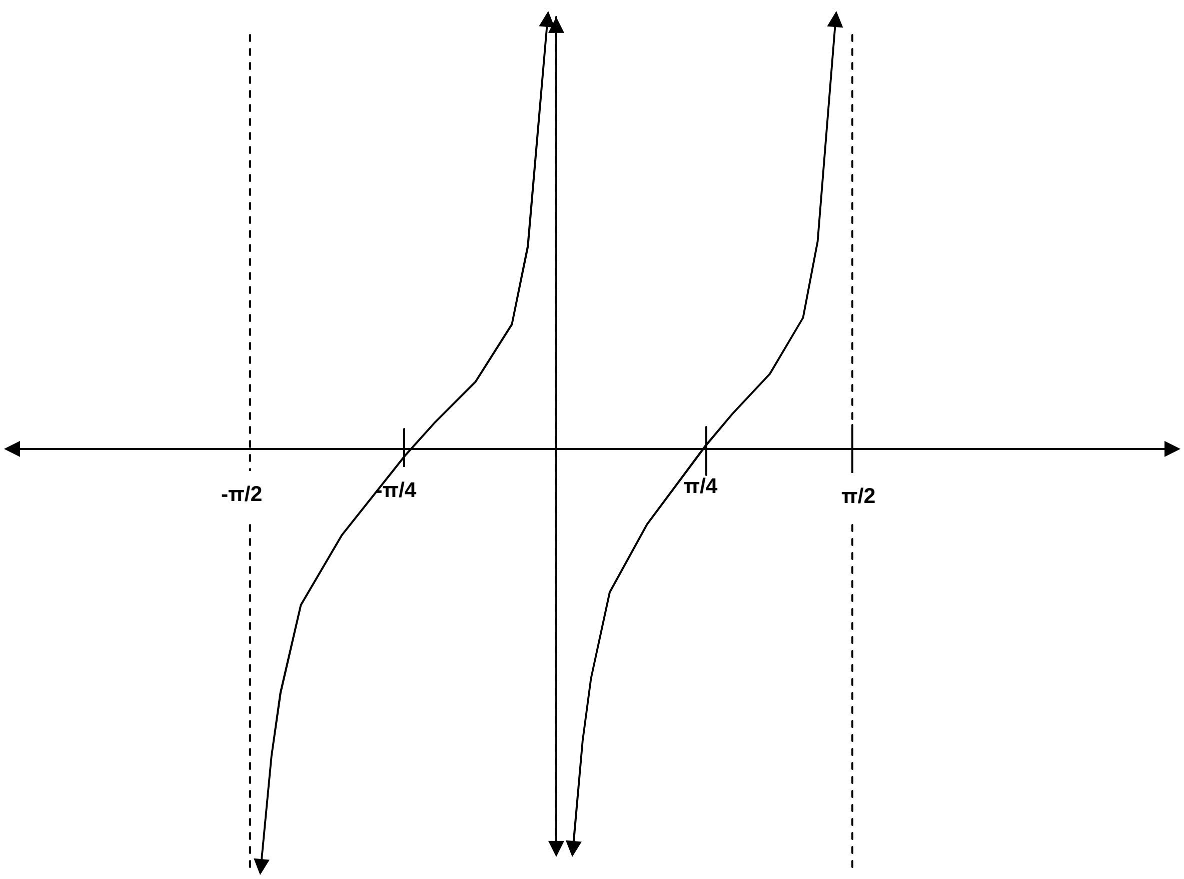
$$\frac{T}{2} = \frac{\frac{\pi}{2}}{2}$$
$$= \frac{\pi}{4}$$

4. *Calculate the points of the graph using the interval and the halfway point.*

$$0 + \frac{\pi}{4} = \frac{\pi}{4} \rightarrow \text{point on the axis for graph}$$
$$\frac{\pi}{4} + \frac{\pi}{4} = \frac{\pi}{2} \rightarrow \text{asymptote}$$
$$\frac{\pi}{2} + \frac{\pi}{4} + = \frac{3\pi}{4} \rightarrow \text{point on the axis for graph}$$

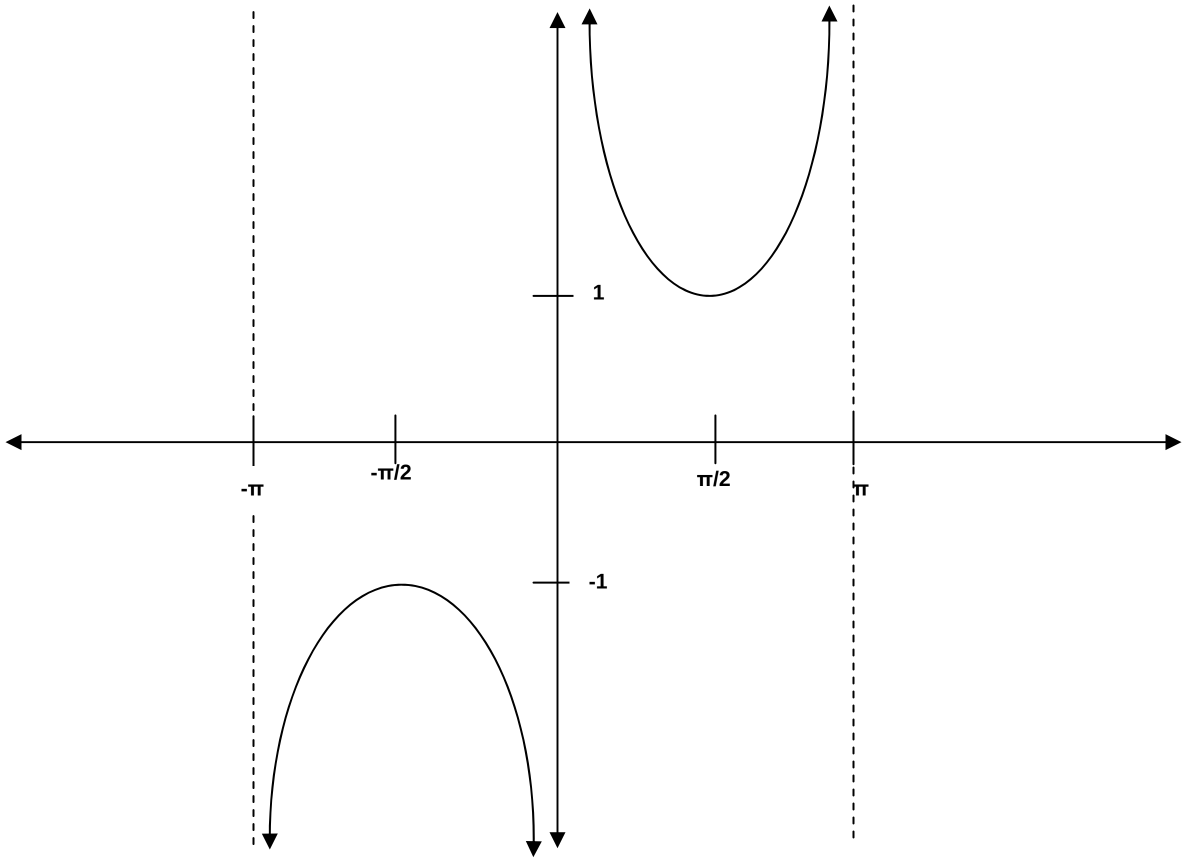
Note: To get other points, repeat Step 4 above for every ***π/2*** *period.*

5. *Draw the graph using the information from Steps 1-4.*



Secant and Cosecant:

Note: ***For explanation, I will use the cosecant graph for applying the transformation rules. Cosecant is similar to that of secant. Again, the transformations are difficult if you don't know the basic graphs. The basic cosecant graph is shown below.***



Example:

Graph

$$y = -2\csc(2x - \pi).$$

*1. Calculate the **fundamental cycle**. In the basic tangent graph, the cycle is from **–π/2 to π/2**.*

$$-\pi \le Bx + C \le \pi$$
$$-\pi \le 2x - \pi \le \pi$$
$$0 \le 2x \le 2\pi$$
$$0 \le x \le \pi$$

*2. Calculate the **period**. In the basic cosecant graph, the period is **2π**.*

$$T = \frac{2\pi}{|B|}$$
$$= \frac{2\pi}{2}$$
$$= \pi$$

3. *Divide the period into two four parts.*

$$\frac{T}{4}=\frac{\pi}{4}$$

4. *Calculate the points of the graph using the interval.*

$$0+\frac{\pi}{4}=\frac{\pi}{4}\rightarrow \text{point on the axis for graph}$$

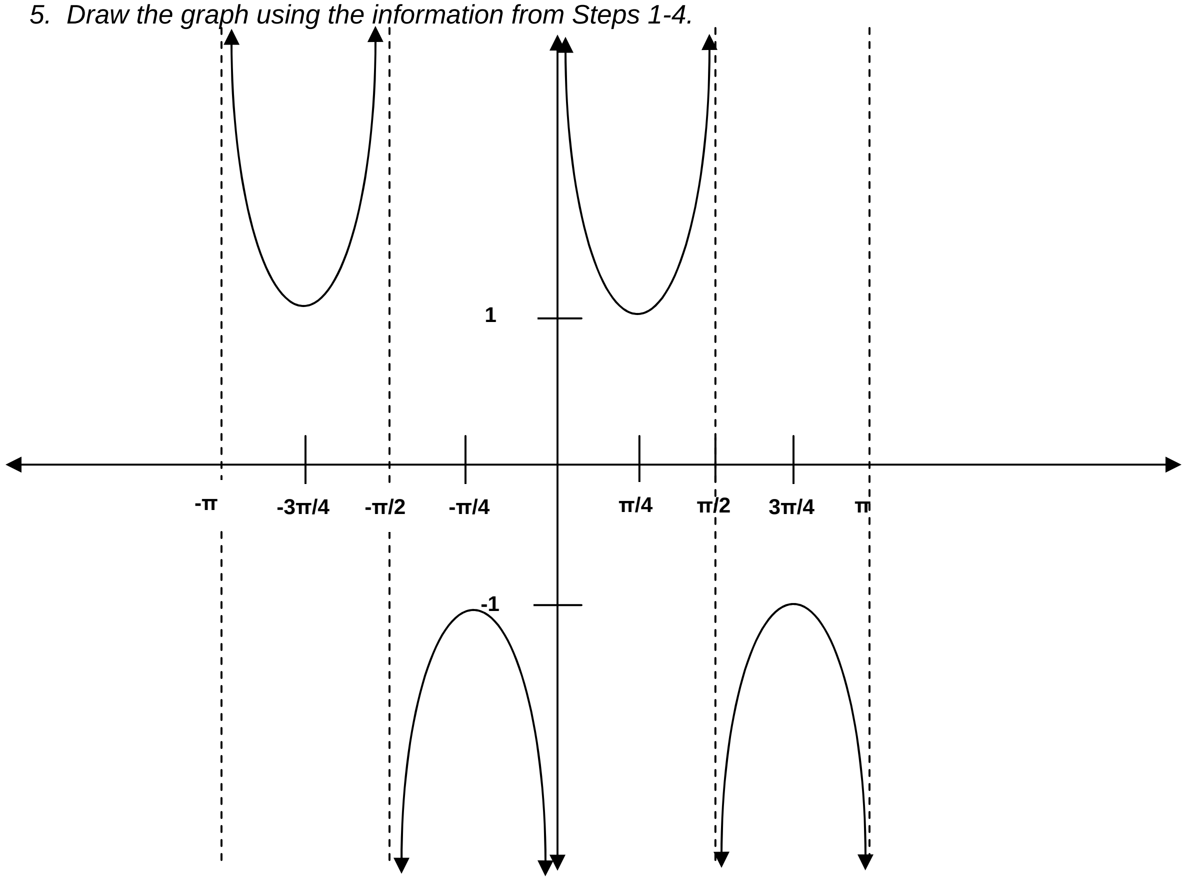
$$\frac{\pi}{4}+\frac{\pi}{4}=\frac{\pi}{2}\rightarrow \text{asymptote}$$

$$\frac{\pi}{2}+\frac{\pi}{4}+=\frac{3\pi}{4}\rightarrow \text{point on the axis for graph}$$

$$\frac{3\pi}{4}+\frac{\pi}{4}=\pi\rightarrow \text{asymptote}$$

*Note: To get other points, repeat Step 4 above for every **π** period.*

5. *Draw the graph using the information from Steps 1-4.*



Note: ***Since the multiplier in front of cosecant is negative, then the graph is reflected about the x-axis.***