Calculus One: Test III Review

I. Find the derivative of each function.

1. $y=\dfrac{(x^2-x)^{3/2}(e^{x^2+1})}{\sqrt{2x-1}}$

$\ln y = \ln\left(\dfrac{(x^2-x)^{3/2}(e^{x^2+1})}{\sqrt{2x-1}}\right)$

$= \ln(x^2-x)^{3/2} + \ln(e^{x^2+1}) - \ln(2x-1)^{1/2}$

$= \frac{3}{2}\ln(x^2-x) + x^2+1 - \frac{1}{2}\ln(2x-1)$

$\dfrac{1}{y}\dfrac{dy}{dx} = \dfrac{d}{dx}\left[\ln(x^2-x) + x^2+1 - \frac{1}{2}\ln(2x-1)\right]$

$\dfrac{1}{y}\dfrac{dy}{dx} = \dfrac{2x-1}{x^2-x} + 2x - \dfrac{1}{\cancel{2}}\left(\dfrac{\cancel{2}}{2x-1}\right)$

$\dfrac{1}{y}\dfrac{dy}{dx} = \dfrac{2x-1}{x^2-x} + 2x - \dfrac{1}{2x-1}$

$\boxed{\dfrac{dy}{dx} = \left[\dfrac{2x-1}{x^2-x} + 2x - \dfrac{1}{2x-1}\right] \times \dfrac{\sqrt{2x-1}}{(x^2-x)^{3/2}(e^{x^2+1})}}$

2. $y=2x^{1/2x}$

$\ln y = \ln(2x^{1/2x})$

$= \dfrac{1}{2x}\ln 2x$

$= \dfrac{1}{2}\ln 2x + \dfrac{1}{2x}\cdot\dfrac{\cancel{2}}{2x}$

$\dfrac{1}{y}\dfrac{dy}{dx} = \dfrac{\ln 2x}{2} + \dfrac{1}{2x^2}$

$\dfrac{dy}{dx} = \left(\dfrac{\ln 2x}{2} + \dfrac{1}{2x^2}\right)\cdot 2x^{1/2x} = \boxed{(\ln 2x)\cancel{x}^{1/2x} + \dfrac{\cancel{x}^{1/2x}}{x^2}}$

3. $e^{2y}=(\sin x)(\arcsin e^{2y})$

$2e^{2y}\dfrac{dy}{dx} = (\sin x)\left(\dfrac{2e^{2y}}{\sqrt{1-e^{4y}}}\cdot\dfrac{dy}{dx}\right) + \cos x\,(\arcsin e^{2y})$

$\dfrac{dy}{dx}2e^{2y} - (\sin x)\left(\dfrac{2e^{2y}}{\sqrt{1-e^{4y}}}\cdot\dfrac{dy}{dx}\right) = \cos x\,(\arcsin e^{2y})$

$\dfrac{dy}{dx}\left(2e^{2y} - (\sin x)\left(\dfrac{2e^{2y}}{\sqrt{1-e^{4y}}}\right)\right) = \cos x\,(\arcsin e^{2y})$

$\dfrac{dy}{dx} = \dfrac{\cos x\,(\arcsin e^{2y})}{2e^{2y} - (\sin x)\left(\dfrac{2e^{2y}}{\sqrt{1-e^{4x}}}\right)}$

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4. $y = \ln \frac{\left(x^2 - e^{x^3}\right)^{1/2}}{\left(25 - e^{-x}\right)^2}$

$$y = \ln(x^2 - e^{x^3})^{1/2} - \ln(25 - e^{-x})^2$$

$$= \frac{1}{2}\ln(x^2 - e^{x^3}) - 2\ln(25 - e^{-x})$$

$$= \frac{1}{2}\frac{2x - 3x^2e^{x^3}}{x^2 - e^{x^3}} - 2\left(-(-e^{-x})\right)$$

$$\frac{dy}{dx} = \frac{1}{2}\left(\frac{2x - 3x^2e^{x^3}}{x^2 - e^{x^3}}\right) - 2e^{-x}$$

II. Evaluate each integral.

1. $\int \left(6x^5 + 4x - \frac{1}{3}x^{1/3} + \frac{1}{\sqrt{x}} - \frac{1}{x^2} + \frac{1}{x}\right)dx$

$$= x^6 + 2x^2 - \frac{1}{3}\left(\frac{3}{2}x^{2/3}\right) + 2x^{1/2} - (-x^{-1}) + \ln x + C$$

$$= x^6 + 2x^2 - \frac{1}{2}x^{2/3} + 2x^{1/2} + \frac{1}{x} + \ln x + C$$

2. $\int \frac{u-1}{u^2+9}\,du$

$$\int \frac{u}{u^2+9}\,du - \int \frac{1}{u^2+9}\,du$$

$$\Downarrow \qquad \qquad \Downarrow$$

$W = u^2 + 9$ $\qquad \frac{1}{3}\arctan\frac{u}{3} \quad + \quad C$

$$\frac{dw}{du} = 2u$$

$$dw = 2u\,du$$

$$\frac{dw}{2} = u\,du$$

$$\frac{1}{2}\int \frac{dw}{w}$$

$$\frac{1}{2}\ln(w)$$

$$= \boxed{\frac{1}{2}\ln(u^2+9) - \frac{1}{3}\arctan\frac{u}{3} + C}$$

3. $\int (3x-4)^5\,dx$

$u = 3x-4$

$du = 3\,dx$

$\frac{du}{3} = dx$

$\frac{1}{3}\int u^5\,du = \frac{1}{3}\frac{u^6}{6} + C = \frac{1}{3}\frac{(3x-4)^6}{6} + C = \boxed{\frac{(3x-4)^6}{18} + C}$

4. $\int x \ln x\,dx$

ILATE

$uv - \int v\,du$

$u = \ln x \qquad v = \frac{x^2}{2}$

$du = \frac{1}{x} \qquad dv = x$

$\ln x\left(\frac{x^2}{2}\right) - \int \frac{x^2}{2}\, \frac{1}{x}\,dx = \ln(x)\frac{x^2}{2} - \int \frac{x}{2}\,dx$

$= \ln(x)\frac{x^2}{2} - \frac{1}{2}\int x\,dx$

$= \ln(x)\frac{x^2}{2} - \frac{1}{2}\frac{x^2}{2}$

$= \boxed{\frac{x^2}{2}\ln(x) - \frac{x^2}{4}}$

5. $\int x^2 e^{4x^3}\,dx$

$u = 4x^3$

$\frac{du}{dx} = 12x^2$

$du = 12x^2\,dx$

$\frac{du}{12} = x^2\,dx$

$\frac{1}{12}\int e^u\,du$

$\frac{1}{12}e^u + c$

$\boxed{\frac{1}{12}e^{4x^3} + c}$

6. $\int_1^e \frac{dx}{x(\ln x)^2}$ Can't evaluate $\ln(0)$

$u = \ln x$

$du = \frac{1}{x} dx$

$\int_1^e \frac{1}{u} du = [\ln u] = \ln[\ln x]_1^e = \ln[e] - \ln[0]$

7. $\int_{-\pi/6}^{\pi/6} \frac{\arctan 2x}{1+4x^2} dx$

Rewrite the integral

$u = \arctan 2x$

$\frac{du}{dx} = \frac{2}{1+4x^2} \Rightarrow du = \frac{2}{1+4x^2} \cdot dx \Rightarrow \frac{1}{2}\int u \, du$

$\Rightarrow \frac{1}{2} u^2 + c$

$= \left[\frac{1}{2}(\arctan^2 2x)\right]_{-\pi/6}^{\pi/6}$

8. $\int \frac{2x+1}{(x+5)^3} dx$

$u = x+5$

$du = dx$

$\int \frac{2x}{(x+5)^3} dx + \int \frac{1}{(x+5)^3} dx$

$\int \frac{du}{u^3} dx = \frac{u^{-2}}{-2} + c = \frac{-1}{2u^2}$

$= \frac{-1}{2(x+5)^2}$

$u = x+5$

$du = dx$

$u - 5 = x$

$2\int \frac{x \, du}{u^3} = 2\int \frac{u-5}{u^3}$

$= 2\left[\int \frac{u}{u^3} du - \int \frac{5}{u^3} du\right] = 2\frac{u^{-1}}{-1} - 5\frac{u^{-2}}{-2}$

u^{-2}

$= \frac{-2}{x+5} + \frac{5}{2(x+5)^2}$

$= \boxed{-\frac{2}{x+5} + \frac{5}{2(x+5)^2} - \frac{1}{2(x+5)^2}}$

9. $\int \frac{dx}{\sqrt{16-6x-x^2}}$

Complete the square:

$16-6x-x^2 \rightarrow -x^2-6x+16$

$-(x^2+6x-16)$

$-((x^2+6x+9)-16-9)$

$-((x+3)^2-25)$

$-(x+3)^2+25$

$25-(x+3)^2$

$\int \frac{dx}{\sqrt{25-(x+3)^2}}$

$a=5 \quad u=x+3 \quad du=dx$

$\int \frac{du}{\sqrt{a^2-u^2}} = \arcsin\frac{u}{a}+C = \arcsin\frac{x+3}{5}+C$

10. $\int \frac{dx}{4x^2+16x+41} =$

~~Complete the square~~

Complete Square:

$\left[4(x^2+4x+41/4)\right]$

$\left[4(x+2)^2+41/4-16\right]$

$\left[4(x+2)^2-23/4\right]$

$\begin{array}{r} 64 \\ 41 \\ \hline 23 \end{array}$

$\frac{41}{4} - \frac{64}{4}$

$\frac{41}{4} - \frac{64}{4}$

$\int \frac{dx}{4(x+2)^2-23/4} =$

11. $\int x \arctan x \, dx$