Log and e Review

1. **Condense the following log expressions using the log rules:**

   a) $\frac{1}{2}\log_2(2-x) - \frac{3}{4}\log_2(3+x) - \log_2(2+x^2)$

$$= \frac{1}{4}\left[2\log_2(2-x) - 3\log_2(3+x) - 4\log_2(2+x^2)\right]$$

$$= \frac{1}{4}\left[\log_2(2-x)^2 - \left(\log_2(3+x)^3 + \log_2(2+x^2)^4\right)\right]$$

$$= \frac{1}{4}\left[\log_2(2-x)^2 - \log_2(3+x)^3(2+x^2)^4\right]$$

$$= \frac{1}{4}\log_2\frac{(2-x)^2}{(3+x)^3(2+x^2)^4} \Rightarrow \log_2\sqrt[4]{\frac{(2-x)^2}{(3+x)^3(2+x^2)^4}}$$

   b) $3\ln(2x) + 6\ln(4x+1) - 9\ln(2x) - \ln(1-3x)$

$$= \ln(2x)^3 - \ln(4x+1)^6 - \ln(2x)^9 - \ln(1-3x)$$

$$= \ln 8x^3 - \left(\ln(4x+1)^6 + \ln 512x^9 + \ln(1-3x)\right)$$

$$= \ln 8x^3 - \ln(4x+1)^6 512x^9(1-3x)$$

$$= \ln\left[\frac{8x^3}{(4x+1)^6 512x^9(1-3x)}\right]$$

2. **Use log rules to expand the following expressions:**

   a) $\log_3\sqrt[3]{\frac{(1-2x)^4(x-1)}{(x+3)^2}}$

$$= \log_3\left[\frac{(1-2x)^4(x-1)}{(x+3)^2}\right]^{1/3}$$

$$= \frac{1}{3}\left[4\log_3(1-2x) + \log_3(x-1) - 2\log_3(x+3)\right]$$

$$= \frac{4}{3}\log_3(1-2x) + \frac{1}{3}\log_3(x-1) - \frac{2}{3}\log_3(x+3)$$

   b) $\log_4\sqrt[3]{\frac{(1-x)^3(x+1)}{(x-2)^5}}$

$$= \log_4\left[\frac{(1-x)^3(x+1)}{(x-2)^5}\right]^{1/3}$$

$$= \frac{1}{3}\left[\log_4(1-x)^3 + \log_4(x+1) - \log_4(x-2)^5\right]$$

$$= \frac{1}{3}\left[3\log_4(1-x) + \log_4(x+1) - 5\log_4(x-2)\right]$$

$$= \log_4(1-x) + \frac{1}{3}\log_4(x+1) - \frac{5}{3}\log_4(x-2)$$

3. **Use the log rules to evaluate without using a calculator (show your work!):**

a) $(1/9)^{x-1} = 27^{2x-1}$

$\left(\frac{1}{3^2}\right)^{x-1} = \left(3^3\right)^{2x-1}$

$\left(3^{-2}\right)^{x-1} = 3^{6x-3}$

$3^{-2x+2} = 3^{6x-3}$

$-2x+2 = 6x-3$

$-8x = -5$

$x = 5/8$

b) $e^{-x^2} = e^{-3x-4}$

$-x^2 = -3x-4$

$-x^2+3x+4=0$

$-x^2+4x-1x+4=0$

$-x(x-4)-(x-4)$

$(x-4)(-x-1)=0$

$x-4=0 \quad -x-1=0$

$x=4 \quad x=-1$

c) $\log_{1/4} 64$

$\log_{1/4} 64 = x$

$\frac{1}{4}^x = 64$

$\left(4^{-1}\right)^x = 4^3$

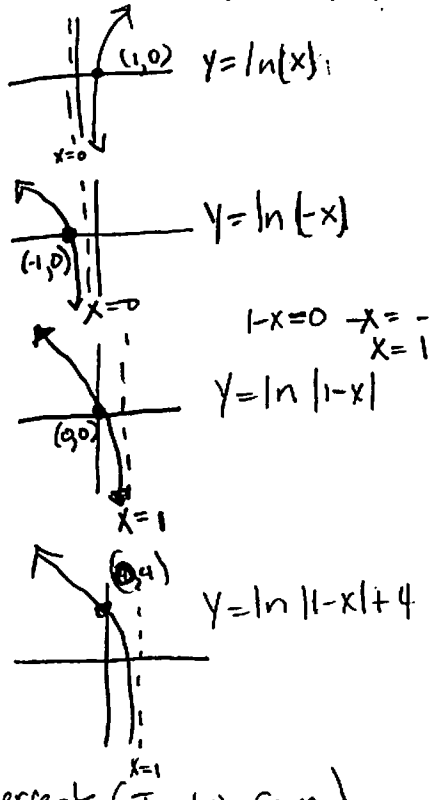
$4^{-x} = 4^3$

$-x = 3$

$x = -3$

4. **Graph using transformation rules:**

a) $Y = \ln(1-x) + 4$



Intercepts (Just in Case)

X int: $0 = \ln(1-x)+4$

$-4 = \ln(-x)$

$e^{-4} = 1-x$

$e^{-4}-1 = -x$

$1-e^{-4} = x$

$(1-e^{-4}, 0)$

y int: $y = \ln(1-0)+4$

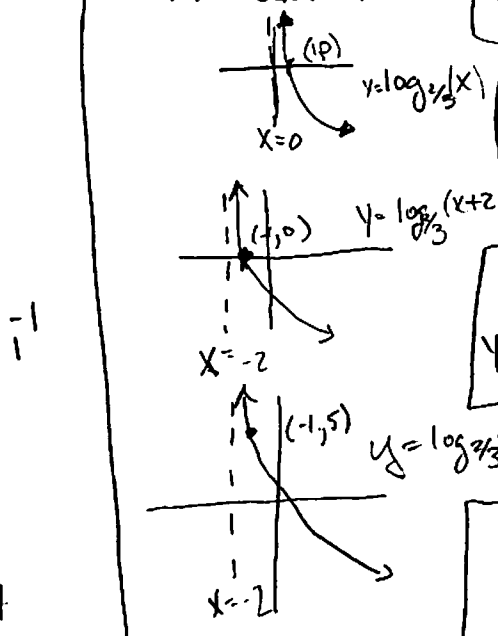
$\ln 1 + 4$

$0+4$

$y=4$

$(0,4)$

b) $y = \log_{2/3}(x+2)+5$



$y = \log_{2/3}(0+2)+5$

$y = \log_2(3/3)+5$

$(0, \log_{2/3}(1)+5)$

$0 = \log_{2/3}(x+2)+5$

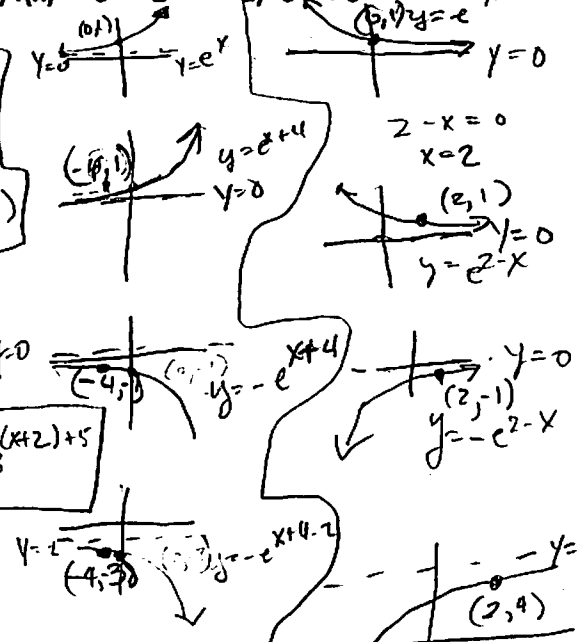
$-5 = \log_{2/3}(x+2)$

$2/3^{-5} = 2/3^{\log_{2/3}(x+2)}$

$-5 = x+2$

$x = (-7, -5/4, 0)$

c) $f(x) = -e^{x+4}-2$



$y = -e^{0+4}-2$

$y = -2-e^4$

$(0, -2-e^4)$

$0 = -e^{x+4}-2$

$2 = -e^{x+4}$

$-2 = e^{x+4}$

$\ln(-2) = x+4$

$-4+\ln(-2) = x$

$(-4+\ln(-2), 0)$

d) $-e^{2-x}+5$

$0 = -e^{2-y}+5$

$-5 = -e^{2-x}$

$\ln 5 = \ln e^{2-x}$

$\ln 5 = 2-x$

$-2+\ln 5 = -x$

$2-\ln 5 = x$

$(2-\ln 5, 0)$

5. **Solve the equations using the log & e rules:**

a) $3e^{2x} - 15 = 135$

$3e^{2x} = 150$

$e^{2x} = 50$

$2x = \ln 50$

$x = \frac{1}{2}\ln 50$

b) $8 - 4\ln 3x = 4$

$-4\ln 3x = -4$

$\ln 3x = 1$

$3x = e$

$x = \frac{e}{3}$

c) $\log_2(5-3x) = 4$

$2^{\log_2(5-3x)} = 2^4$

$5 - 3x = 16$

$-3x = 11$

$x = \frac{11}{3}$

d) $\log(x-3) + \log(x) - 1 = 0$

$\log(x-3)(x) = 1$

$\log(x^2 - 3x) = 1$

$10^{\log(x^2-3x)} = 10^1$

$x^2 - 3x = 10$

$x^2 - 3x - 10 = 0$

$x^2 - 5x + 2x - 10 = 0$

$x(x-5) + 2(x-5) = 0$

$(x-5)(x+2) = 0$

$x = 5$ $\quad x = -2$ no neg

6. **A 10,000 investment is placed in an account that is compounded continuously for 20 yrs at a rate of 12% interest. How much money is in the account when it matures?**

$A = Pe^{rt}$

$A = 10{,}000e^{.12(20)}$

$A = 110{,}231.76$

7. **How long would it take to double your initial investment in a savings account that compounds interest continuously at a rate of 10%?**

$A = Pe^{rt}$

Question $\Rightarrow$ $t = ?$ for $A = 2P$

$\frac{2P}{P} = \frac{Pe^{.1t}}{P}$

$2 = e^{.1t}$

$\ln 2 = \ln e^{.1t}$

$\ln 2 = .1t$

$t = \frac{\ln 2}{.1} = 6.93$ yrs