Final Exam Review

I. Evaluate:

1. Find the domain of the following functions:

a) $f(x) = \frac{x}{x^3-4x}$ **b)** $y = 3x^3 - 2x + x$ **c)** $y = \sqrt{4-x^2}$ **d)** $y = \frac{\sqrt{4-2x}}{x^2-25}$ **e)** $f(x) = \sqrt{\frac{20-5x}{2-x}}$

$f(x) = \frac{x}{x(x+2)(x-2)}$

$D_x: (-\infty, -2) \cup (-2, 0) \cup (0, 2) \cup (2, \infty)$

$D_x: (-\infty, \infty)$

$4 - x^2 \geq 0$

$(2-x)(2+x) \geq 0$

$x = 2, -2$

$\boxed{D_x: (-2, 2)}$

$(x+5)(x-5) = 0$

$x \neq -5, 5$

$4 - 2x \geq 0$

$-2x \geq -4$

$x \leq 2$

$\boxed{D_x: (-\infty, -5) \cup (-5, 2]}$

$\frac{20-5x}{2-x} \geq 0$

$20 - 5x = 0$ $\quad 2 - x = 0$

$x = 4$ $\quad x = 2$

$+ \quad - \quad +$

$2 \quad 4$

$\therefore \boxed{(-\infty, 2) \cup [4, \infty)}$

2. Determine the symmetry of the following relations, determine whether a function or not, and also tell whether odd or even?

vert. line test

a) $y = x^2 - x$ **b)** $f(x) = -5/x$ **c)** $x^2 + y^2 = 9$ **d)** $y^2 = x$

a) x-axis symm

$y \Rightarrow -y$

$-y = x^2 - x$

$y = -x^2 + x$ ∅

y-axis symm

$x \Rightarrow -x$

$y = (-x)^2 - (-x)$

$y = x^2 + x$ ∅

origin symm

$(x \to -x)\ (y \to -y)$

$-y = (-x)^2 - (-x)$

$-y = x^2 + x$

∅ $y = -x^2 - x$

Vert. line test

Prove that $f(x)$ is a function.

Even

$f(-x) = f(x)$

$y = (-x)^2 - (-x)$

$y = x^2 + x$ ∅

odd

$f(-x) = -f(x)$

$y = (-x)^2 - (-x)$

$y = x^2 + x$ ∅

b) x-axis symm

$(x, y) \Rightarrow (x, -y)$

$-y = -\frac{5}{x} \Rightarrow y = \frac{5}{x}$ ∅

y-axis symm

$(x, y) \Rightarrow (-x, y)$

$y = -\frac{5}{-x}$, $y = \frac{5}{x}$ ∅

origin symm

$(x, y) \Rightarrow (-x, -y)$

$-y = -\frac{5}{-x} \Rightarrow y = \frac{-5}{x}$

Origin symm ✓

- It is a function

- Even

$f(-x) = $

$f(x) = $

Odd ✓

$f(-x) = $

$f(-x) = \frac{5}{x} = -f(x)$ ✓

c) Circle. $r = 3$.

✓ X-axis symm

✓ y-axis symm

✓ Origin symm

Not a function by vertical line test

Even ✓

$f(-x)$ $(-x)^2 + y^2 = $

$x^2 + y^2 = f(x)$

d) X-axis symm

$(x, y) \Rightarrow (x, -y)$

$(-y)^2 = x$

$y^2 = x$ ✓

- It is not a function

$-f(-x) = y^2 = $

$= -f(x)$

odd funct

3. Determine the equation of the line parallel to $4x - 2y = 8$, and goes through the point (2,3); perpendicular to the line and through the point (2,3)?

$\parallel$ = $-2y = -4x + 8$

$y = 2x - 4$ $\quad m = 2$

$(y - 3) = 2(x - 2)$

$\boxed{y = 2x - 1}$

$\perp$ = $m = 2$ $\quad m_\perp = -\frac{1}{2}$

$(y - 3) = -\frac{1}{2}(x - 2)$

$\boxed{y = -\frac{1}{2}x + 4}$

III. Evaluate:

1. Find the inverse and prove:

a) $F(x) = \frac{2x+4}{3-x}$ **b)** $y = 4\log_3(2x-1) + 5$ **c)** $f(x) = 5\ln(3x+2) - 5$ **d)** $y = e^{2x+4} - 2$ **e)** $y = 3\sin(2x-5) + 2$

a)

$x \leftrightarrow y$

$x = \frac{2y+4}{3-y}$

$(3-y)(x) = 2y+4$

$3x - yx = 2y+4$

$3x - 4 = 2y + yx$

$3x - 4 = y(2+x)$

$\frac{3x-4}{2+x} = y$ (inverse)

$f(g(x)) = f\left(\frac{3x-4}{2+x}\right)$

$= \frac{2\left(\frac{3x-4}{2+x}\right)+4}{3-\left(\frac{3x-4}{2+x}\right)}$

$= \frac{2(3x-4)+4(2+x)}{3(2+x)-(3x-4)}$

$= \frac{10x}{10} = x$

b)

$x \leftrightarrow y$

$x = 4\log_3(2y-1)+5$

$\left(\frac{x-5}{4}\right) = \log_3(2y-1)$

$3^{\left(\frac{x-5}{4}\right)} = (2y-1)$

$\frac{3^{\left(\frac{x-5}{4}\right)}+1}{2} = y$

$f(g(x)) = 4\log_3\left(2\left(\frac{3^{\left(\frac{x-5}{4}\right)}+1}{2}\right)-1\right)+5$

$= 4\log_3\left(3^{\left(\frac{x-5}{4}\right)}\right)+5$

$= 4\left(\frac{x-5}{4}\right)+5$

$= x-5+5$

$= x$ ✓

c)

$x \leftrightarrow y$

$x = 5\ln(3y+2)-5$

$\frac{x+5}{5} = \ln(3y+2)$

$e^{\left(\frac{x+5}{5}\right)} = 3y+2$

$\frac{e^{\left(\frac{x+5}{5}\right)}-2}{3} = y = g(x)$

$f(g(x)) = f\left(\frac{e^{\left(\frac{x+5}{5}\right)}-2}{3}\right)$

$= 5\ln\left(3\left(\frac{e^{\left(\frac{x+5}{5}\right)}-2}{3}\right)+2\right)-5$

$= 5\left(\frac{x+5}{5}\right)-5$

$= x$ ✓

d)

$x \leftrightarrow y$

$x = e^{2y+4}-2$

$x+2 = e^{2y+4}$

$\ln(x+2) = 2y+4$

$\frac{\ln(x+2)-4}{2} = y/g(x)$

$f(g(x)) = e^{2\left(\frac{\ln(x+2)-4}{2}\right)+4}-2$

$= (x+2)-2$

$= x$ ✓

e)

$x \leftrightarrow y$

$x = 3\sin(2y-5)+2$

$\left(\frac{x-2}{3}\right) = \sin(2y-5)$

$\arcsin\left(\frac{x-2}{3}\right) = 2y-5$

$\frac{\arcsin\left(\frac{x-2}{3}\right)+5}{2} = y$

$f(g(x)) = f\left(\frac{\arcsin\left(\frac{x-2}{3}\right)+5}{2}\right)$

$y = 3\sin\left(2\left(\frac{\arcsin\left(\frac{x-2}{3}\right)+5}{2}\right)-5\right)+2$

$y = 3\sin\left(\arcsin\frac{x-2}{3}+5-5\right)+2$

$y = 3\sin\left(\arcsin\left(\frac{x-2}{3}\right)\right)+2$

$y = 3\left(\frac{x-2}{3}\right)+2$

$y = x$ ✓

2. Find f(g(x)) & its domain:

a) $f(x) = \sqrt{4-2x}$ and $g(x) = \frac{1}{x^2-9}$

$f(g(x)) = \sqrt{4 - 2\left(\frac{1}{x^2-9}\right)}$

$= \sqrt{4 - \frac{2}{x^2-9}}$

$= \sqrt{\frac{4(x^2-9)-2}{x^2-9}}$

$= \sqrt{\frac{4x^2-36-2}{x^2-9}}$

$= \sqrt{\frac{4x^2-38}{x^2-9}}$

$= \frac{\sqrt{4x^2-38}}{\sqrt{x^2-9}}$

$4x^2 = 38$

$x^2 = \frac{38}{4} = \frac{19}{2}$

$x = \pm\sqrt{19/2} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \pm\frac{\sqrt{38}}{2} \approx 3.08 \cong 3.1$

$\frac{\left(x+\frac{\sqrt{38}}{2}\right)\left(x-\frac{\sqrt{38}}{2}\right)}{(x+3)(x-3)} \geq 0$

$+ \quad -\frac{\sqrt{38}}{2} \quad - \quad -3 \quad + \quad 3 \quad - \quad \frac{\sqrt{38}}{2} \quad +$

$\boxed{D_{f(g(x))}: \left(-\infty, -\frac{\sqrt{38}}{2}\right) \cup (-3,3) \cup \left(\frac{\sqrt{38}}{2}, \infty\right)}$

Sorry - my bad. didn't mean to make them so close to each other!

b) ~~f(x)~~ g(x) $= \frac{\cos 4x}{2}$ and ~~g(x)~~ f(x) $=$ arcos $2x$

$f(g(x)) = \arccos \cancel{2}\left(\frac{\cos 4x}{\cancel{2}}\right)$

$= \arccos(\cos 4x)$

$= 4x$

$D_{f(g(x))}: [-1, 1]$

Trigonemtry:

1. **Change the following angles into radian measures and tell me the reference angles:**

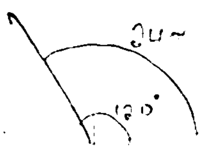
a) 135 b) 210 c)-30 d) -90

a.) $3\pi/4$ b.) $5\pi/3$ c.) $-\pi/6$ d.) $3\pi/2$

2. **Change the following radian measures into degrees:**

a) 5π/3 b) 5π/6 c) 4π/3 d)- π/6

a.) 300° b) 150° c.) 240° d.) 330°



3. **Find the Area, A, of a sector with an arclength of 24π and a central angle of 120 degrees.**

$C = 2\pi r$ $\quad 3(24\pi) = 2\pi r$ $\quad \frac{72}{2} = r$ $\quad r = 36.$ $\quad A = \pi r^2 = \pi(36)^2 = \cancel{1296\pi/3} = \boxed{432\pi}$

4. Find the values of all six trigonometric functions if the point (-4,-3) is a point on the terminal side of angle θ

$\tan\theta = \frac{3}{4}$ $\quad \csc\theta = \frac{-5}{3}$

$\sin\theta = \frac{-3}{5}$ $\quad \sec\theta = \frac{-5}{4}$

$\cos\theta = \frac{-4}{5}$ $\quad \cot = \frac{4}{3}$

5. Find the following

a) tan 33π/4

$\boxed{1}$

b) cos π/6

$\boxed{\sqrt{3}/2}$

c) sin 6π

$\boxed{0}$

d) sin 3π/2

$\boxed{-1}$

e) cot θ = 2/5 ; sin θ < 0 find cos θ

-5, -2

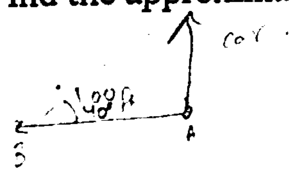
$\cos\theta = \frac{-5}{\sqrt{29}}$

e) csc θ = - 4/3; tan θ > 0; find cos θ.

$\sqrt{7}$, -3, 4

$\cos\theta = \frac{-\sqrt{7}}{4}$

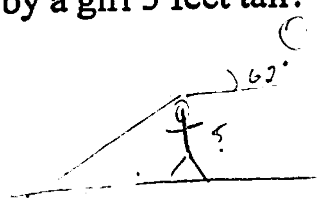
6. If you are standing at a point on the lake, let's call it point A, and you know that there is a restroom 100 ft due west, point B. Your car is at some distance due north of the restrooms. You guesstimate the angle made between your path to the restroom and the location of the car to be about 40 degrees. Find the approximate distance between the restroom and the cars.



$$\cos 40^\circ = \frac{100}{x}$$

$$X = 130.5 \text{ ft}$$

7. If the sun is at an angle of elevation of 62 degrees, how long a shadow will be cast by a girl 5 feet tall?



$$\tan(62^\circ) = \frac{5}{x}$$

$$x = \frac{5}{\tan(62^\circ)}$$

$$x = 3.19 \text{ ft}$$

I. **Divide the following functions(use synthetic division when applicable):**

1. $2x^3 + x^2 - 5x + 2$ div by $x + 2$

$$\begin{array}{rrrr|l} 2 & 1 & -5 & 2 & -2 \\ & -4 & 6 & -2 & \\ \hline 2 & -3 & 1 & 0 & \end{array}$$

$\boxed{= 2x^2 - 3x + 1}$

2. $x^6 + 3x^5 - 2x^4 - 6x^3 + x^2 + 3x + 2$ div by $x^2 + 3x$

$x^4 \cancel{-3x} - 2x^2 + 1$

$x^2+3x \,\big)\, x^6 + 3x^5 - 2x^4 - 6x^3 + x^2 + 3x + 2$

$- \; x^6 + 3x^5$

$-2x^4 - 6x^3 + x^2 + 3x + 2$

$- \; -2x^4 - 6x^3$

$x^2 + 3x + 2$

$x^2 + 3x$

2

$\boxed{= x^4 - 2x^2 + 1 + \frac{2}{x^2+3x}}$

3. $x^5 + 2x^2 - 3$ div by $x - 2$

$$\begin{array}{rrrrrr|l} 1 & 0 & 0 & 2 & -3 & & 2 \\ & 2 & 4 & 8 & 20 & & \\ \hline 1 & 2 & 4 & 10 & 17 & & \end{array}$$

$\boxed{= x^4 + 2x^3 + 4x^2 + 10 + \frac{17}{x-2}}$

II. **Graph the following functions:** Tell me the zeros, the y-int, the vertical asymptotes, horizontal asymptotes, oblique asymptotes, the behavior on either side of the asymptotes, and sketch the following functions.

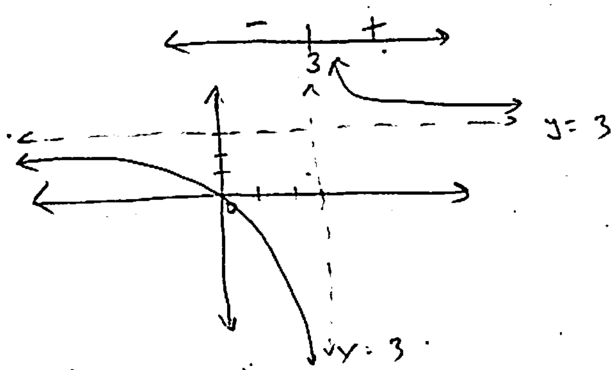
1. $y = \dfrac{9x}{3x-9} = \dfrac{3x}{x-3}$

x-int : $x = 0$

y-int : $y = 0$

Vertical asymp : $x = 3$

$y = \dfrac{9x}{3x-9} \cdot \dfrac{\frac{1}{x}}{\frac{1}{x}} = \dfrac{9}{3 - \frac{9}{x}} = 3$



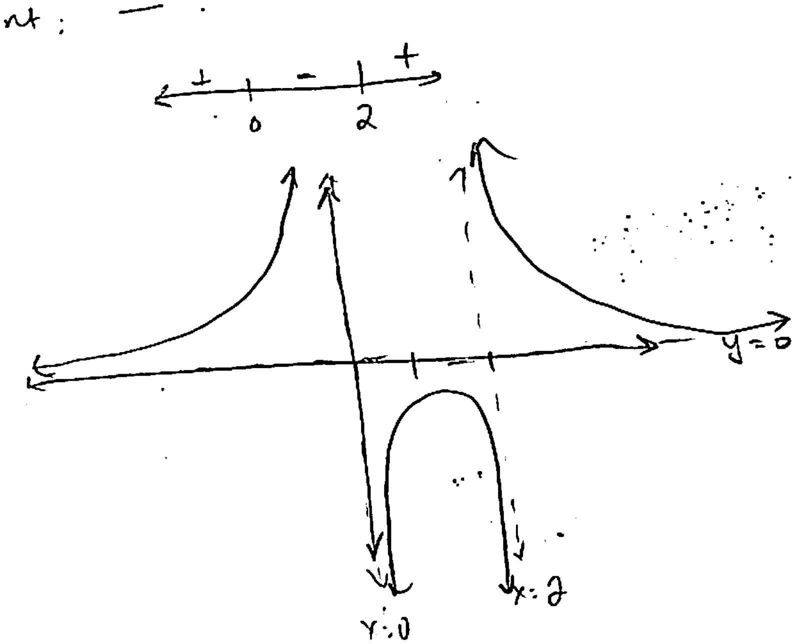
2. $y = \dfrac{1}{x^2 - 2x} = \dfrac{1}{x(x-2)}$

horizontal asymp : $y = 0$

Vert. asymp : $x = 0$, $x = 2$

x-int : —

y-int : —



2. $4x^3+3x^2+8x+6=0$

$x^2(4x+3)+2(4x+3)=0$

$(x^2+2)(4x+3)=0$

$x=-3/4$ & $\sqrt{2}\,i$

3. $-x^4+8x^2-15=0$

$x^4-8x^2+15=0$

$(x^2-5)(x^2-3)=0$

$x=\pm\sqrt{5},\ \pm\sqrt{3}$

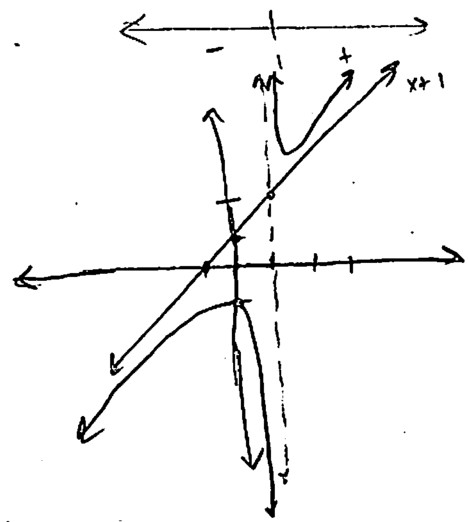
$$\begin{array}{r} x+1 \\ x-1\overline{)\,x^2+0x+1} \\ \underline{-(x^2-x)} \\ +x \\ \underline{-(x-1)} \\ +2 \end{array}$$

2. $y = \dfrac{x^2+1}{x-1}$

x-int : —

y-int : $(0,-1)$

Slant asymp. : $x+1$



III. **Find the roots to the following equations: Be sure and test the root using the Factor theorem prior to using the Rational Root Theorem:**

1. $x^4+7x^3+17x^2+17x+6=0$

$$\begin{array}{r|rrrrr} -1 & 1 & 7 & 17 & 17 & 6 \\ & \downarrow & -1 & -6 & -11 & -6 \\ \hline & 1 & 6 & 11 & 6 & 0 \end{array}$$

$$\begin{array}{r|rrrr} -1 & 1 & 6 & 11 & 6 \\ & \downarrow & -1 & -5 & -6 \\ \hline & 1 & 5 & 6 & 0 \end{array}$$

$x^2+5x+6=0$

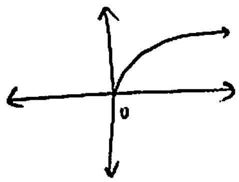
$(x+3)(x+2)=0$

$$\boxed{(x+1)^2(x+3)(x+2)}$$

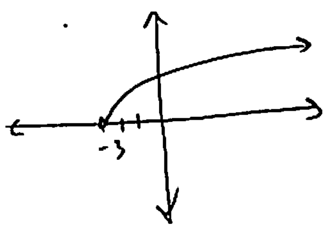
IV. **Sketch the graph of the following equations using transformation rules:**

1. $y = \sqrt{x+3}$

Parent : $f(x) = \sqrt{x}$

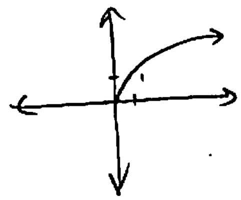


• move to the left 3 units

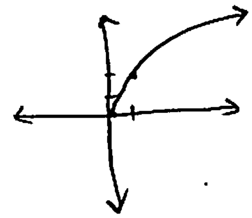


2. $y = 2\sqrt{x} + 6$

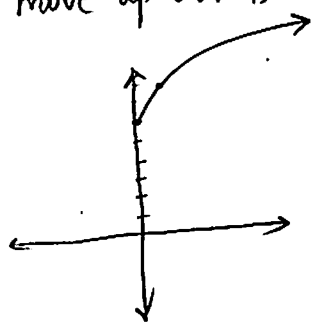
Parent : $f(x) = \sqrt{x}$



• Vertical stretch by a factor of 2

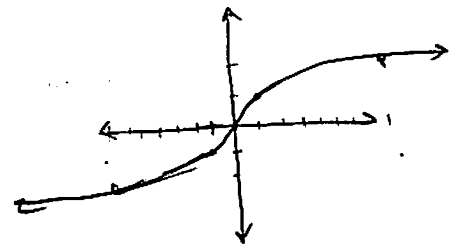


• move up 6 units

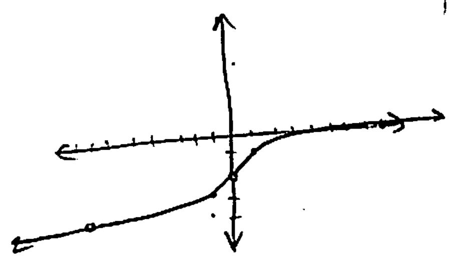


3. $y = \sqrt[3]{x} - 2$

Parent : $f(x) = \sqrt[3]{x}$



• move down 2 units



Additional problems

#1 Solve

$\frac{1}{2}\log_4 x = \log_4 9$

$\log_4 x^{1/2} = \log_4 9$

$4^{\log_4 9} = x^{1/2}$

$9 = x^{1/2}$

$\boxed{x = \pm 3}$

#2 Solve: $\log_4 x + \log_4 (x-6) = 2$

$\log_4 (x)(x-6) = 2$

$x^2 - 6x = 16$

$x^2 - 6x - 16 = 0$

$(x-8)(x+2) = 0$

$\boxed{x = 8}$ $\boxed{x = -2}$

3 Solve $\log_2 6 + \log_2 x - \log_2 (x+2) = 2$

$\log_2 \frac{6x}{x+2} = 2$

$\frac{6x}{x+2} = 4$

$6x = 4x + 8$

$2x = 8$

$\boxed{x = 4}$

Simplify

① $\ln e^{10}$

$\boxed{10}$

② $\log .001$

$\boxed{-3}$

③ $y = \log 10^{2t+3}$

$\boxed{2t+3}$

find inverse:

④ $f(x) = e^{4x+8}$

$x \leftrightarrow y$

$x = e^{4y+8}$

$\ln x = 4y+8$

$\boxed{\frac{\ln x - 8}{4} = y^{-1}}$

⑤ find function so that $(f \circ g)(x) = (g \circ f)(x) = x$

for $f(x) = e^{\sqrt{x}}$

$x = e^{\sqrt{y}}$ $x \leftrightarrow y$

$\ln x = \sqrt{y}$

$\boxed{(\ln x)^2 = y}$

$f(g(x)) = e^{\sqrt{(\ln x)^2}}$

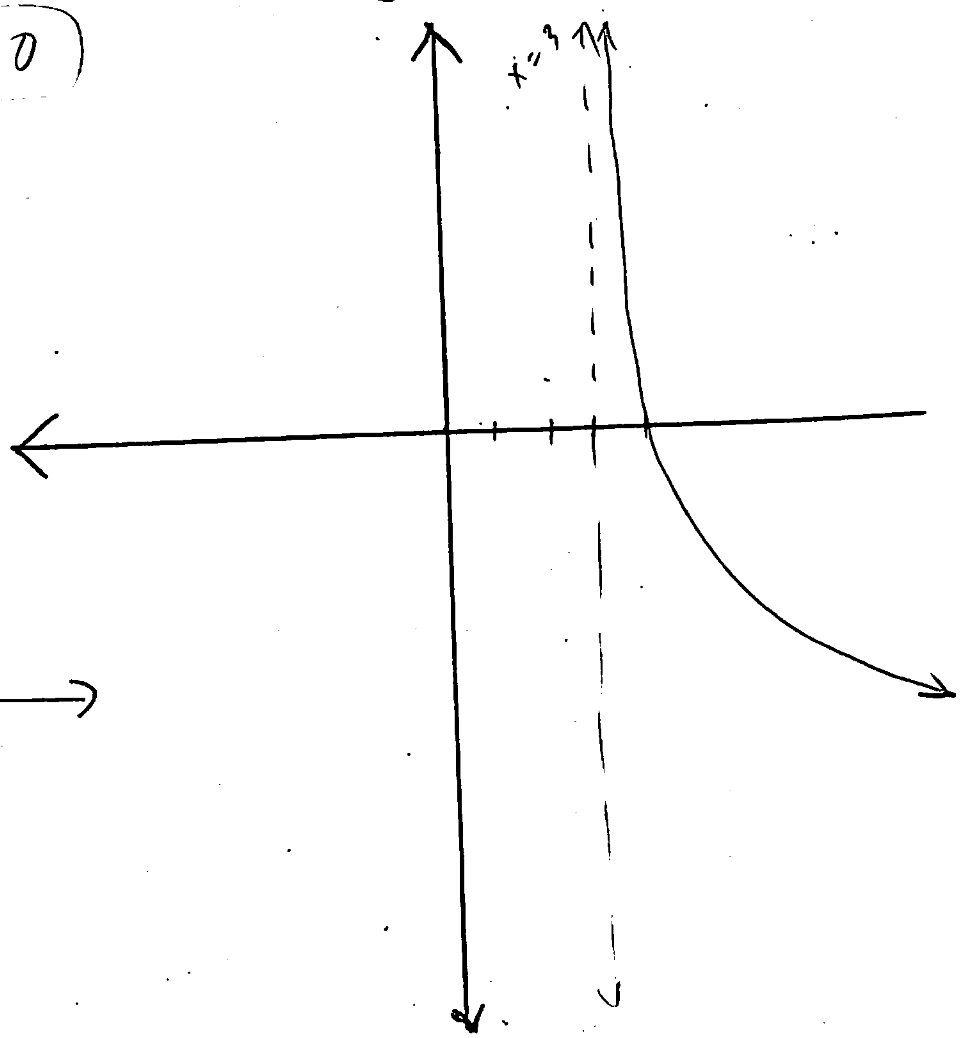
$f(g(x)) = e^{\ln x}$

$= x$

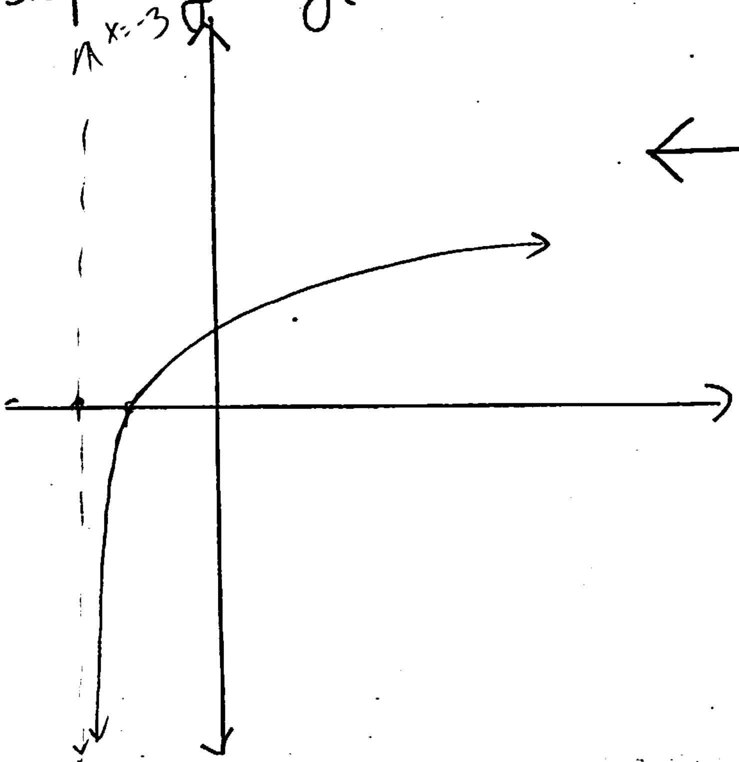
Use change of base

$\log_4 7 = \frac{\ln 7}{\ln 4} = \boxed{1.40}$

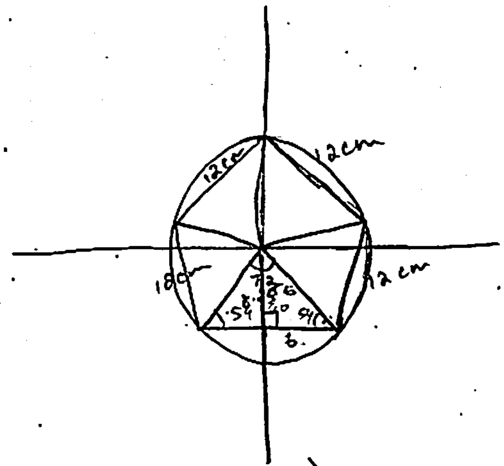
Graph $y = \log_{\cancel{4}}(x-3)$



graph $y = \log(x+3)$



8. Find the area of a regular pentagon that is inscribed in a circle with sides of 12 cm.



$$A = 10\left(\frac{1}{2}(b)(h)\right)$$

$$= 10\left(\frac{1}{2}(6)(8.26)\right)$$

$$= \boxed{247.8 \text{ cm}^2}$$

8. **Perform the indicated operations:**

a) $(4\cos\theta + 3)(\cos\theta - 2)$

$= 4\cos^2\theta + 3\cos\theta - 8\cos\theta - 6$

$= 4\cos^2\theta - 5\cos\theta - 6$

b) $(\sin 3\theta - 1)^2$

$= \sin^2 3\theta - 2\sin 3\theta + 1$

Factor:

c) $\cos^2\theta - 2\cos\theta + 1$

$(\cos\theta - 1)(\cos\theta - 1)$

d) $\sec^2\theta + 5\sec\theta + 6$

$(\sec\theta + 2)(\sec\theta + 3)$

Simplify:

e) $3/\sin\theta - 4/\cos\theta$

$$\frac{3}{\sin\theta} - \frac{4}{\cos\theta}$$

$$= \frac{3\cos\theta}{\sin\theta\cos\theta} - \frac{4\sin\theta}{\sin\theta\cos\theta}$$

$$= \boxed{\frac{3\cos\theta - 4\sin\theta}{\sin\theta\cos\theta}}$$

f) $1 + 1/\tan^2\theta$

$$1 + \frac{1}{\tan^2\theta}$$

$$= \frac{\tan^2\theta + 1}{\tan^2\theta} = \frac{\sec^2\theta}{\tan^2\theta} = \frac{\frac{1}{\cos^2\theta}}{\frac{\sin^2\theta}{\cos^2\theta}}$$

$$= \boxed{\csc^2\theta}$$

g) $\dfrac{4\cos^2\theta - 1}{2\cos^2\theta - 5\cos\theta - 3}$

$x^2 - 5x - 6$

$(x - 6)(x + 1)$ $\frac{}{2}$ $\frac{}{2}$

$$= \frac{\cancel{(2\cos\theta + 1)}(2\cos\theta - 1)}{\cancel{(2\cos\theta + 1)}(\cos\theta - 3)}$$

$$= \boxed{\frac{2\cos\theta - 1}{\cos\theta - 3}}$$

h) $\dfrac{8\cos^2 5\theta}{2\cos^2 5\theta - 4\cos 5\theta}$

$$= \frac{\cancel{\cos 5\theta}(8\cos 5\theta)}{\cancel{\cos 5\theta}(2\cos 5\theta - 4)}$$

$$= \frac{8\cos 5\theta}{2(\cos 5\theta - 2)}$$

$$= \boxed{\frac{4\cos 5\theta}{\cos 5\theta - 2}}$$

9.Find the Amplitude, Frequency, Phaseshifts,and Period of the following functions:

a) $y = \cos 3x$

$y = a\cos x$

$|b| = 1$

Period : $\frac{2\pi}{3}$

freq: $b = 3$

Ph. Shift : None.

b) $y = -4\sin(3x + \pi/4)$

$|a| = 4$

Period : $\frac{2\pi}{3}$

freq : $= 3$

Ph. Shift :

$3x + \frac{\pi}{4} = 0$

$3x = -\frac{\pi}{4}$

$x = -\frac{3\pi}{4}$

Phase Shift left $\left(-\frac{3\pi}{4}\right)$

c) $y = 2\cos 1/4x$

$|a| = 2$

Period $\frac{2\pi}{1/4} = 8\pi$

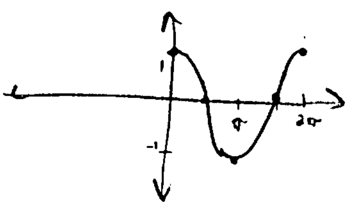
freq : $1/4$

Phase Shift : None.

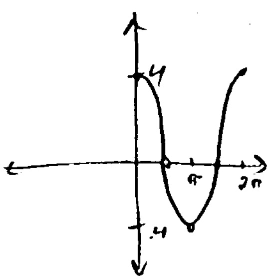
10. Sketch the following graphs:

a) $y = 4\cos x$

Parent $y = \cos x$

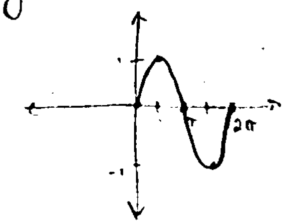


Stretch to 4 units.



b) $y = -1 + 2\sin(x/2 + \pi/4)$

$y = \sin x$



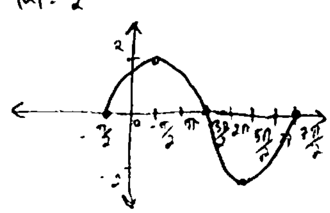
- Period changes to 4π
- Ph. Shift to left $-\frac{\pi}{2}$

$\frac{x}{2} + \frac{\pi}{4} = 0$

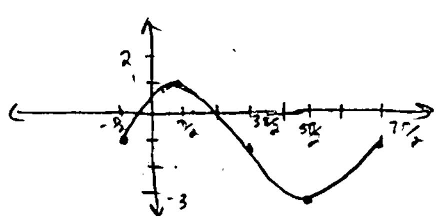
$\frac{x}{2} = -\frac{\pi}{4}$

$x = -\frac{\pi}{2}$

$|a| = 2$



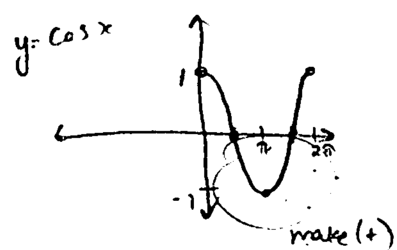
· move down 1 unit

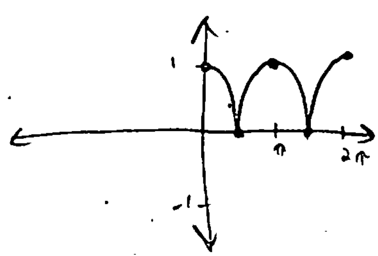


c) $y = |\cos x|$

absolute value makes all (−) values positive

$y = \cos x$

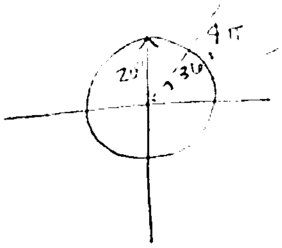




Trig:

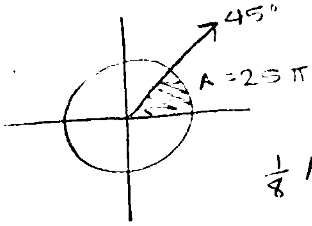
1) Find the central angle θ of a sector w/a radius of 20" & arc of 4π inches.

circumference = 40π



$\boxed{\theta = 36^\circ}$

2) Find the radius of a sector w/a central angle of 45° & an area of 25π



$r = ?$

$A = \pi r^2$

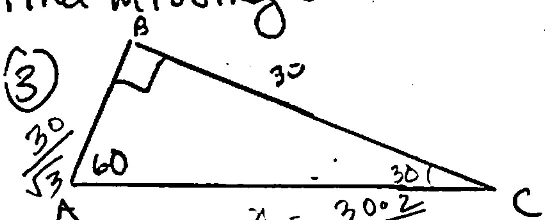
$\frac{1}{8} A = 25\pi$

$A = 200\pi$

$\boxed{r = 10\sqrt{2}}$

Find missing sides & x's

3)

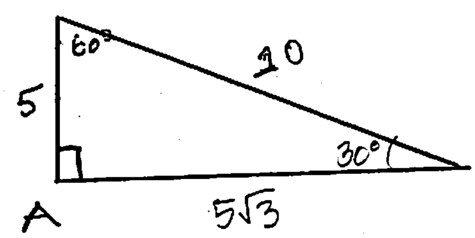


$x = \frac{30 \cdot 2}{\sqrt{3}}$

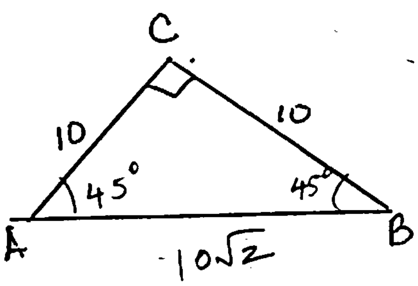
$\frac{\sqrt{3}}{2}(x) = 30$

$x = \frac{30 \cdot 2}{\sqrt{3}}$

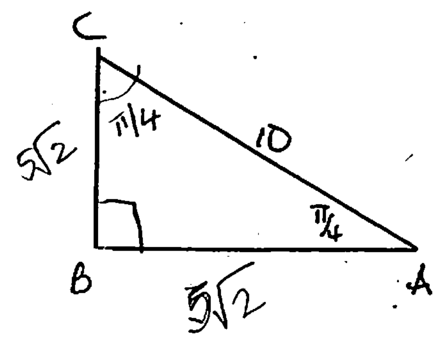
4)



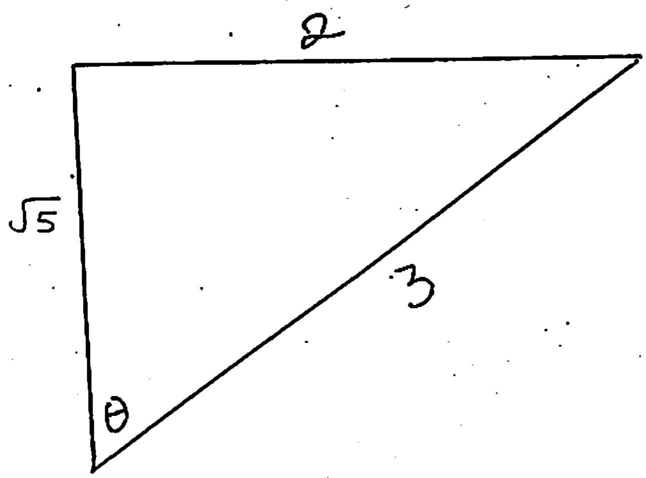
5)



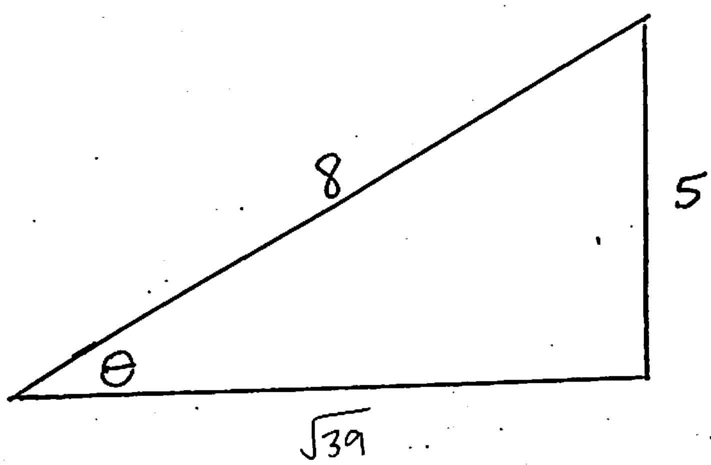
6)



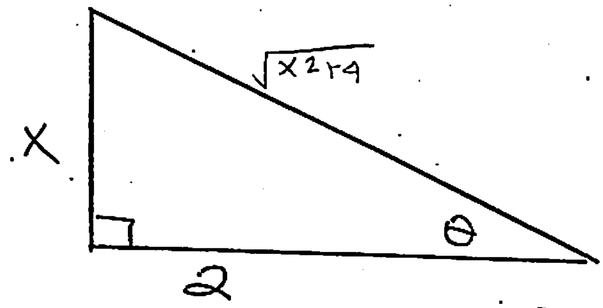
Find all 6 trigonometric functions of θ:



$\sin\theta = 2/3$
$\cos\theta = \sqrt{5}/3$
$\tan\theta = 2/\sqrt{5}$
$\csc\theta = 3/2$
$\sec\theta = 3/\sqrt{5}$
$\cot\theta = \sqrt{5}/2$

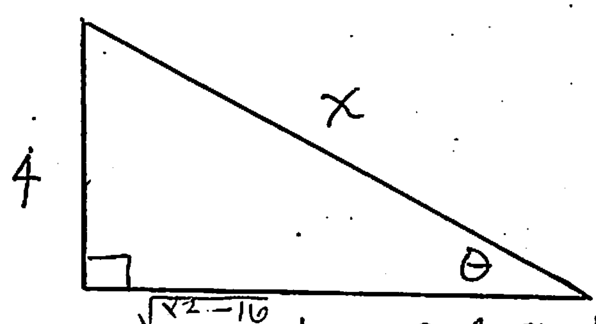


$\sin\theta = 5/8$
$\cos\theta = \sqrt{39}/8$
$\tan\theta = 5/\sqrt{39}$
$\csc\theta = 8/5$
$\sec\theta = 8/\sqrt{39}$
$\cot\theta = \sqrt{39}/5$



express csc θ in terms of x:

$\csc\theta = \dfrac{\sqrt{x^2+4}}{x}$



express x in terms of sin θ.

$\sin\theta = \dfrac{4}{x}$

$\boxed{x = \dfrac{4}{\sin\theta}}$