$180^\circ = \pi$

$1^\circ = \frac{\pi}{180}$ radians.

$\frac{180^\circ}{\pi} = 1$ radian

Trigonometry Review

1. **Calculate the following:**
   a) **Convert the following to degrees: $15\pi/4$**

   675°

   b) **Convert the following to radians: 345**

   6.021

2. **Evaluate all six trig functions given the following:**
   a) **Csc x = -2/5 & tan < 0**

   $\cos x = \frac{\sqrt{21}}{2}$ $\quad \sec = \frac{2}{\sqrt{21}}$

   $\sin x = -\frac{5}{2}$ $\quad \cot x = -\frac{\sqrt{21}}{5}$

   $\tan x = -\frac{5}{\sqrt{21}}$

   b) **the terminal point of a ray starting at the origin is (-2,-4)**

   $\sin x = -\frac{2}{\sqrt{5}}$ $\quad \sec x = -\sqrt{5}$

   $\cos x = -\frac{1}{\sqrt{5}}$ $\quad \cot x = \frac{1}{2}$

   $\tan x = 2$

   $\csc x = -\frac{\sqrt{5}}{2}$

   $16 + 4 = \sqrt{20} = 2\sqrt{5}$

3. **Find the arclength and area of the following sector: r = 2; $\theta = 3\pi/4$.**

   $\text{Area} = \frac{\theta}{360} \times \pi r^2 \Rightarrow \frac{135}{360} \times \frac{22}{7} \times 4 = \frac{11880}{2520} = 4.714$

   $\text{Arc length} = \frac{\theta}{360} \times 2\pi r \Rightarrow \frac{135}{360} \times 2 \times \frac{22}{7} \times 2 = \frac{11880}{2520} = 4.714$

4. **Evaluate using sum and difference identities: $\sin \pi/12$**

   $\sin\frac{\pi}{12} \Rightarrow \sin\left(\frac{\pi}{3} - \frac{\pi}{4}\right)$

   $\Rightarrow \sin\frac{\pi}{3}\cos\frac{\pi}{4} - \cos\frac{\pi}{3}\sin\frac{\pi}{4}$

   $\Rightarrow \left(\frac{\sqrt{3}}{2}\cdot\frac{1}{\sqrt{2}}\right) - \left(\frac{1}{2}\cdot\frac{1}{\sqrt{2}}\right) \Rightarrow \frac{\sqrt{3}}{2\sqrt{2}} - \frac{1}{2\sqrt{2}} \Rightarrow \frac{\sqrt{3}-1}{2\sqrt{2}}$

5. **Prove the following identities:**
   a) $1 - \frac{\cos^2 x}{1+\sin x} = \sin x$ b) $\sec x - \cos x - \sin x \tan x = 0$ c) $\frac{\sin x \cos x}{\cos^2 x - \sin^2 x} = \frac{\tan x}{1 - \tan^2 x}$

(a) $\frac{1+\sin x - \cos^2 x}{1+\sin x} = LHS$

$\Rightarrow \frac{\cancel{1} + \sin x - \cancel{1} + \sin^2 x}{1+\sin x}$

$= \frac{\sin x(1+\sin x)}{(1+\sin x)}$

$= \sin x$

$= RHS$

(b) $\frac{1}{\cos x} - \cos x - \frac{\sin^2 x}{\cos x}$

$= \frac{1 - \cos^2 x - \sin^2 x}{\cos x}$

$= \frac{\cos^2 x - \cos^2 x}{\cos x}$

$= 0$

(c) RHS:-

$\frac{\frac{\sin x}{\cos x}}{1 - \frac{\sin^2 x}{\cos^2 x}}$

$\Rightarrow \frac{\frac{\sin x}{\cancel{\cos x}}}{\frac{\cos^2 x - \sin^2 x}{\cos^{\cancel{2}} x}}$

$= \frac{\sin x \cos x}{\cos^2 x - \sin^2 x}$

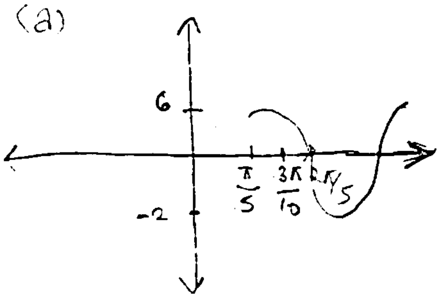
6. Use the half angle identity to evaluate sin 290:

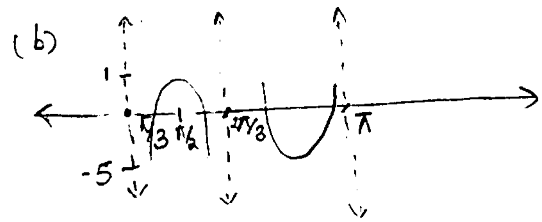
$\sin 290 = 2\sin 145 \cos 145$

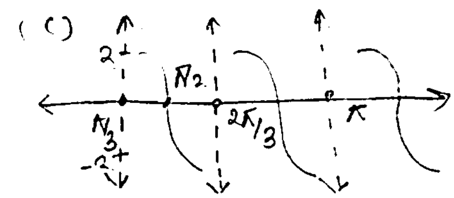
$\sin 290 = 0.8268$

7. Graph the following functions using transformation rules:

a) $Y = 4\cos(5x - \pi) + 2$ **b)** $Y = -3\csc(3x - \pi) - 2$ **c)** $y = 2\cot(3x - \pi)$







8. Find all solutions to the following trig equations:

a) $\cos 3x = 1$ **b)** $\sin(x) - 4 = \cos x - 4$ **c)** $2\sin^2 x + 3\sin x = -1$

$3x = 0, \frac{3\pi}{2}$

$\sin x = \cos x$

$\tan x = 1$

$x = \frac{\pi}{4}, \frac{5\pi}{4}, \ldots$

$\sin x(2\sin x + 3) = -1$

$\sin x = -1$ or $\sin = \frac{-3}{2}$

$x = \frac{-\pi}{2}, \frac{3\pi}{2}$

not possible as $-1 \leq \sin x \leq 1$

9. Find the exact solutions to the following trig equations [0,2π):

a) $\sin 6x = -2\sin 2x$ **b)** $\cos^2 - 2\cos x = 1$ **c)** $2\sec^2 x + \tan^2 x = 3$

$\sin(4x + 2x) = -2\sin 2x$

$\sin 4x \cos 2x + \sin 2x \cos 4x = -2\sin 2x$

$2\sin 2x \cos^2 2x + \sin 2x(1 - 2\sin^2 2x) = -2\sin 2x$

$2\sin 2x(1 - \sin^2 2x) + \sin 2x(1 - 2\sin^2 2x) = -2\sin 2x$

$2\sin 2x - 2\sin^3 2x + \sin 2x - 2\sin^3 2x = -2\sin 2x$

$-4\sin^3 2x + 3\sin 2x = -2\sin 2x$

$-4\sin^3 2x + 5\sin 2x = 0$

$\sin 2x(-4\sin^2 2x + 5) = 0$

$\sin 2x = 0$

$x = 0, \frac{\pi}{2}, \pi$

$= \cos x(\cos x - 2) = 1$

either

$\cos x = 1$ or $\cos x - 2 = 1$

$\cos x = 1$

$x = 0, \frac{3\pi}{2}$

or $\cos x - 2 = 1$

$\cos x = 3$

which is not possible as $-1 \leq \cos x \leq 1$

$= 2(\tan^2 x + 1) + \tan^2 x = 3$

$2\tan^2 x + 2 + \tan^2 x = 3$

$3\tan^2 x = 1$

$\tan^2 x = \frac{1}{3}$

$\tan x = \frac{1}{\sqrt{3}}$

$x = \frac{\pi}{6}, \frac{7\pi}{6},$

10. Determine the missing pieces of the following non right triangles:

a) C = 102.3, B = 28.8, b = 27.4 **b) C =15°15', a = 6.25, b = 2.15**

(a) $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$

$\frac{\sin A}{a} = 0.175 = \frac{0.977}{c} \Rightarrow \frac{\sin 48.9}{a} = 0.175$

$c = \frac{0.977}{0.175} = 5.58$ $\quad a = 4.30$

$A+B+C = 180$

$A = 48.9$

(b) $\frac{\sin a}{A} = \frac{\sin b}{B} = \frac{\sin c}{C}$

$= \frac{\sin a}{6.25} = \frac{\sin b}{2.15} = \frac{0.261}{c}$

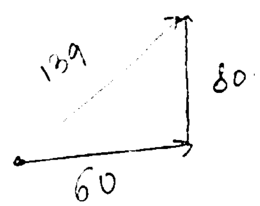
$c^2 = a^2+b^2 - 2ab\cos C$

$c^2 = 43.685 - 25.940 \Rightarrow c = 4.21$

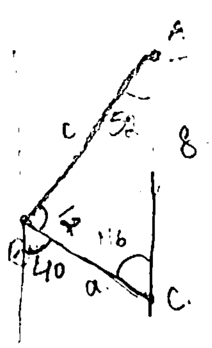
$\Rightarrow \sin b = \frac{2.15 \times 0.261}{4.21} \Rightarrow b = 7.65°$

$\sin a = \frac{6.25 \times 0.261}{4.21} \Rightarrow a = 22.79°$

11. A ship travels 60 miles due east, then adjusts its course northward. After traveling 80 miles in that direction, the ship is 139 miles from its point of departure. Describe this adjustment:



12. The course for a race starts at point A and proceeds in the direction S 52° W to point B, then in the direction S 40°E to point C, and finally back to A. Point C lies 8 kilometers directly south of point A. Approximate the total distance of the race course:



$\frac{\sin 116}{c} = \frac{\sin 12}{8} = \frac{\sin 52}{a}$

$\frac{\sin 116}{c} = 0.025 \Rightarrow c = 34.58$

$\frac{\sin 52}{a} = 0.025 \Rightarrow a = 30.32$

Total race course $= 72.9$ Kms.