Parallelizable, bidirectional methods for simulating optical-signal propagation

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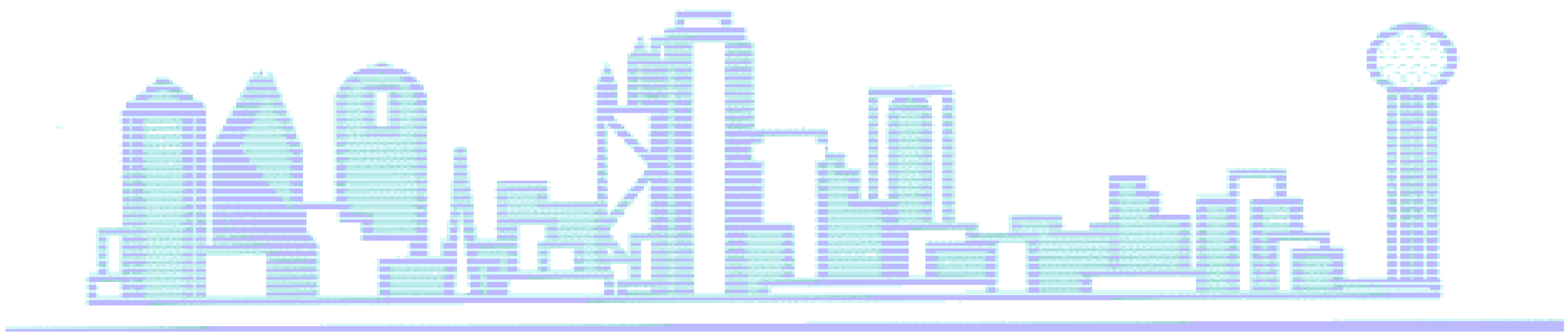


THANKS TO OUR SPONSORS

- WorldCom, Inc. (for support in 2001)
- Texas Telecommunications Education Consortium (for support in 1999 and 2000)

OUTLINE

- Sequential, unidirectional (SU) methods for simulating optical-signal propagation
- Parallelizable, bidirectional (PB) methods
  - ▷ FDTD
  - ▷ Split-step
- Verification
- Parallelizability measurements

SEQUENTIAL, UNIDIRECTIONAL (SU) METHODS (1)

- Propagation equation for the envelope $\mathcal{E}_R(z,t)$ of an optical signal that propagates towards $+z$:

$$\left(\frac{\partial}{\partial z} + \beta_{R,1}\frac{\partial}{\partial t}\right)\mathcal{E}_R = -\frac{\alpha_R \mathcal{E}_R}{2} + \mathsf{D}_t \mathcal{E}_R + \frac{2\pi\omega_R^2}{\beta_R c^2}\mathcal{P}_R$$

  ▷ Dispersion operator

$$\mathsf{D}_t \mathcal{E}_R(z,t) = \frac{i}{2\pi}\int_{-\infty}^{\infty}[\beta(\omega_R+\omega') - \beta(\omega_R) - \beta_1\omega']\tilde{\mathcal{E}}_R(z,\omega')e^{-i\omega' t}d\omega'$$

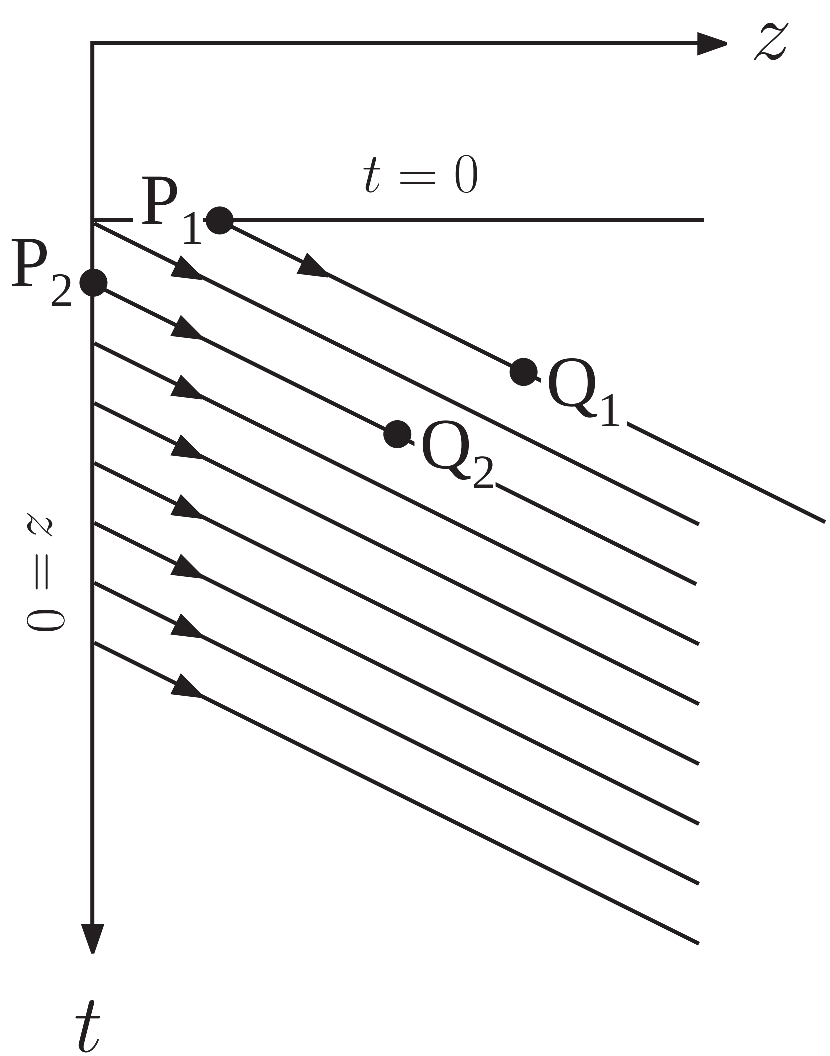
$$\approx -\frac{i}{2}\beta_{R,2}\frac{\partial^2 \mathcal{E}_R}{\partial t^2} + \frac{1}{6}\beta_{R,3}\frac{\partial^3 \mathcal{E}_R}{\partial t^3}$$

- Equation for a leftward-propagating signal:

$$\left(-\frac{\partial}{\partial z} + \beta_{L,1}\frac{\partial}{\partial t}\right)\mathcal{E}_L = -\frac{\alpha_L \mathcal{E}_L}{2} + \mathsf{D}_t \mathcal{E}_L + \frac{2\pi\omega_L^2}{\beta_L c^2}\mathcal{P}_L$$

- Characteristics are diagonal lines in the $z-t$ plane

CHARACTERISTICS FOR SIGNAL GOING TOWARD $+z$



SEQUENTIAL, UNIDIRECTIONAL (SU) METHODS (2)

- Equation for the envelope $\mathcal{E}_R(z', t')$ of an optical signal propagating towards $+z$, in co-moving coordinates:

$$\frac{\partial \mathcal{E}_R}{\partial z'}(z', t') = -\frac{\alpha_R}{2}\mathcal{E}_R(z', t') + \mathsf{D}_{t'}\mathcal{E}_R(z', t') + \frac{2\pi\omega_R^2}{\beta_R c^2}\mathcal{P}_R(z', t')$$

  - ▷ Co-moving coordinates for propagation towards $+z$ (don't work for propagation towards $-z$):

$$z' = z \qquad t' = t - \beta_{R,1} z$$

- Propagation equation can be solved numerically by marching in z', using
  - ▷ Finite differences in both z' and t', or
  - ▷ Beam propagation method
- Sequential computation in z'
- Parallelization is inhibited if the nonlinear polarization $\mathcal{P}_R(z', t')$ depends on the fields at times prior to t' (Example: Raman response function)

SEQUENTIAL, UNIDIRECTIONAL (SU) METHODS (3)

- Equation for the envelope $\mathcal{E}_L(z'', t'')$ of an optical signal propagating towards $-z$, in co-moving coordinates:

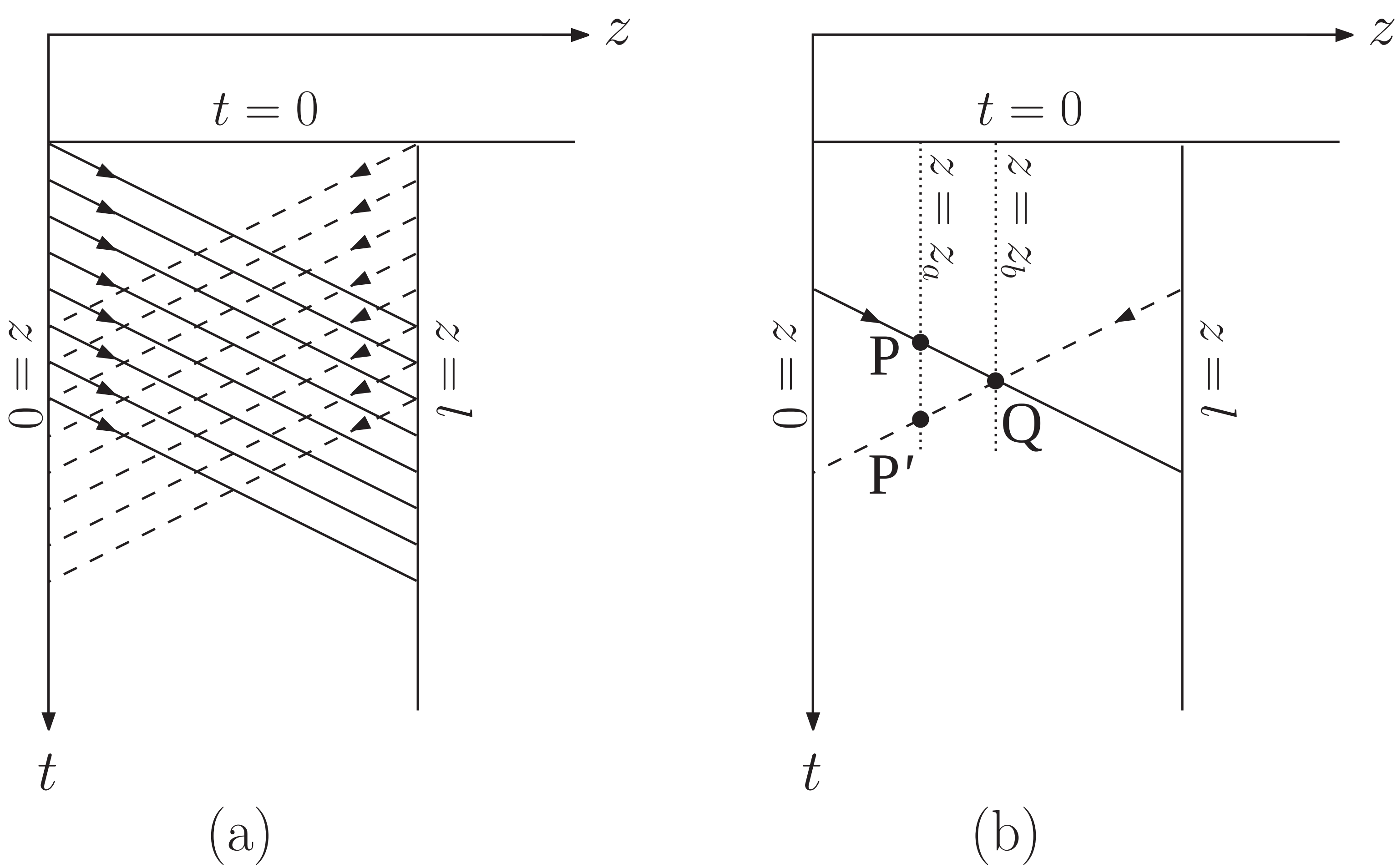
$$-\frac{\partial \mathcal{E}_L}{\partial z''}(z'', t'') = -\frac{\alpha_L}{2}\mathcal{E}_L(z'', t'') + \mathsf{D}_{t'}\mathcal{E}_L(z'', t'') + \frac{2\pi\omega_L^2}{\beta_L c^2}\mathcal{P}_L(z'', t'')$$

  - ▷ Co-moving coordinates for propagation towards $-z$ (don't work for propagation towards $+z$):

$$z'' = z \qquad t'' = t + \beta_{L,1} z$$

- Propagation equation for $\mathcal{E}_L$ can be solved numerically by marching from $z'' = \ell$ to $z'' = 0$ — but, if $\mathcal{E}_R$ is present, then:
  - ▷ $\mathcal{E}_R$ must be computed at $z = z_a$ before it is computed at $z = z_b$
  - ▷ $\mathcal{E}_L$ must be computed at $z = z_b$ before it is computed at $z = z_a$
  - ▷ Optical nonlinearities make $\mathcal{E}_R(P')$ depend on both $\mathcal{E}_R(Q)$ and $\mathcal{E}_L(Q)$, where Q corresponds to $z = z_b > z_a$
- **An iterative solution is required**

CHARACTERISTICS FOR SU METHODS APPLIED TO 2-WAY PROPAGATION



PARALLELIZABLE, BIDIRECTIONAL (PB) METHODS (1)

- March in t, not in z
  - Propagate “snapshots” of the optical field envelopes
  - Snapshot coordinates for signal propagating towards $+z$:

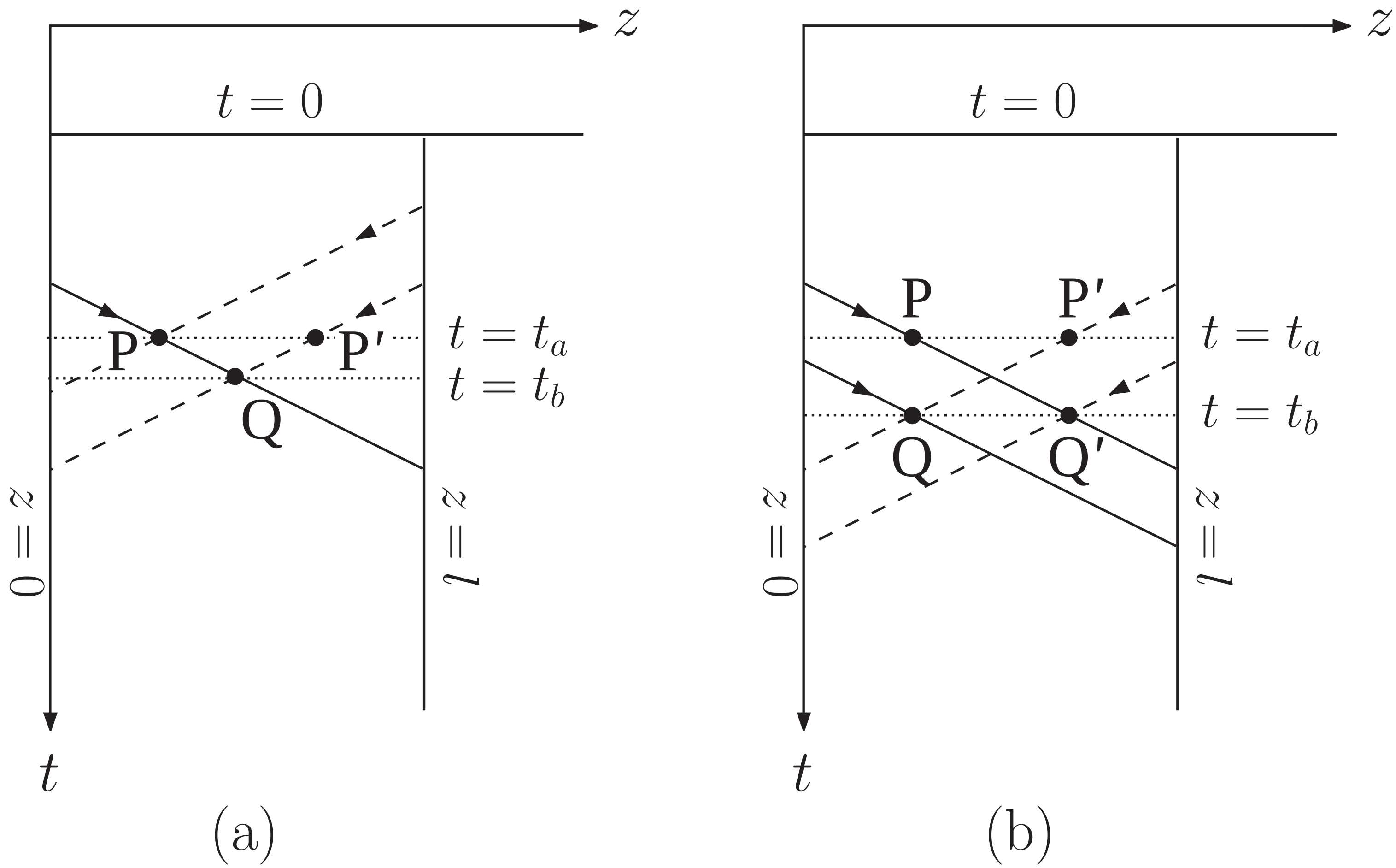
$$t_1 = t \qquad z_1 = z - \omega_{R,1} t \qquad \omega_{R,1} = \left.\frac{d\omega}{d\beta}\right|_{\omega=\omega_R}$$

  - Snapshot coordinates for signal propagating towards $-z$:

$$t_2 = t \qquad z_2 = z + \omega_{R,1} t$$

  - Use data only from $z \pm \omega_{R,1}\Delta t$ at $t - \Delta t$ in computing $\mathcal{E}_R(z,t)$ and $\mathcal{E}_L(z,t)$
  - Parallelizable, because the fields at different z's are computed independently
  - Lower memory usage than when the SU method is applied to 2-way propagation

CHARACTERISTICS FOR PARALLELIZABLE BIDIRECTIONAL (PB) METHODS



VALIDATION OF DISPERSION COMPUTATION

- Validate formulation of dispersion in terms of z-derivatives:
  - ▷ Dispersion operator in terms of spatial derivatives:

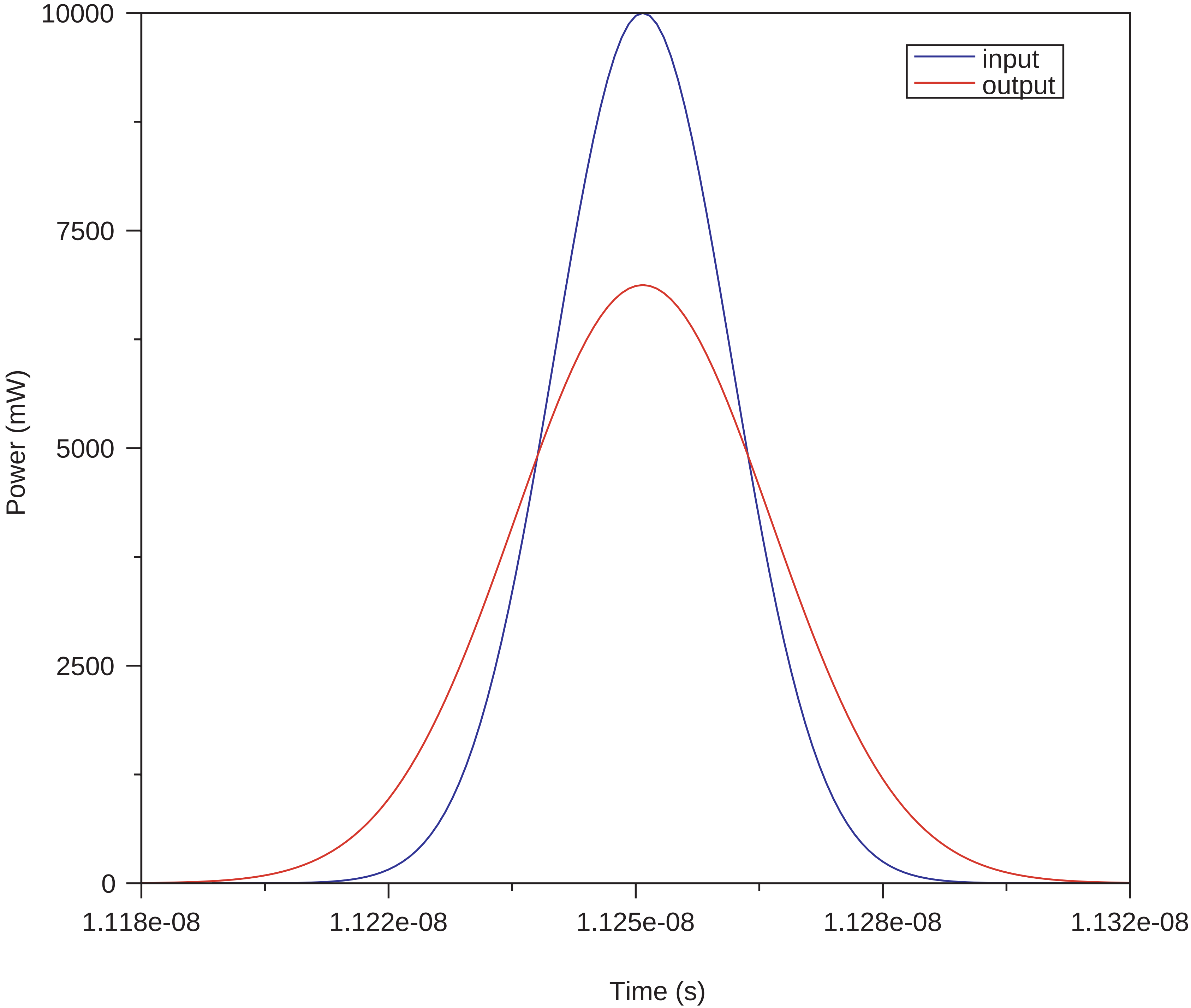
$$\mathsf{D}_{z_2}\overline{\mathcal{E}}_L = \frac{i}{2}\omega_{L,2}\frac{\partial^2\overline{\mathcal{E}}_L}{\partial z_2^2} + \frac{1}{6}\omega_{L,3}\frac{\partial^3\overline{\mathcal{E}}_L}{\partial z_2^3}$$

  - ▷ Derivatives of ω with respect to β:

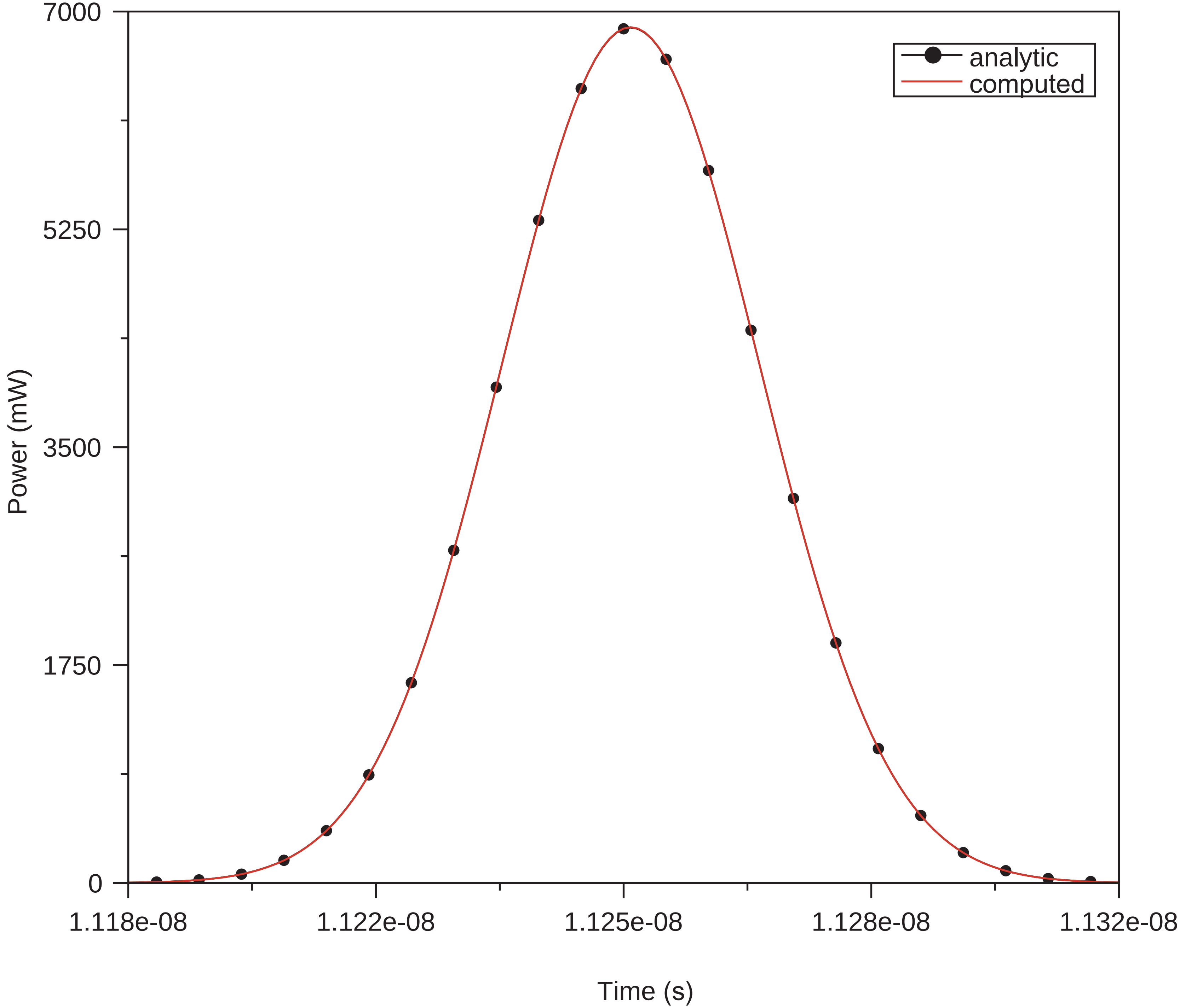
$$\omega_{R,1} = \frac{1}{\beta_{R,1}} = v_{g,R} \approx \frac{c}{n_R}$$

$$\omega_{R,2} = -\frac{\beta_{R,2}}{\beta_{R,1}^3} = \frac{\lambda_R^2 v_{g,R}^3}{2\pi c} D_R$$

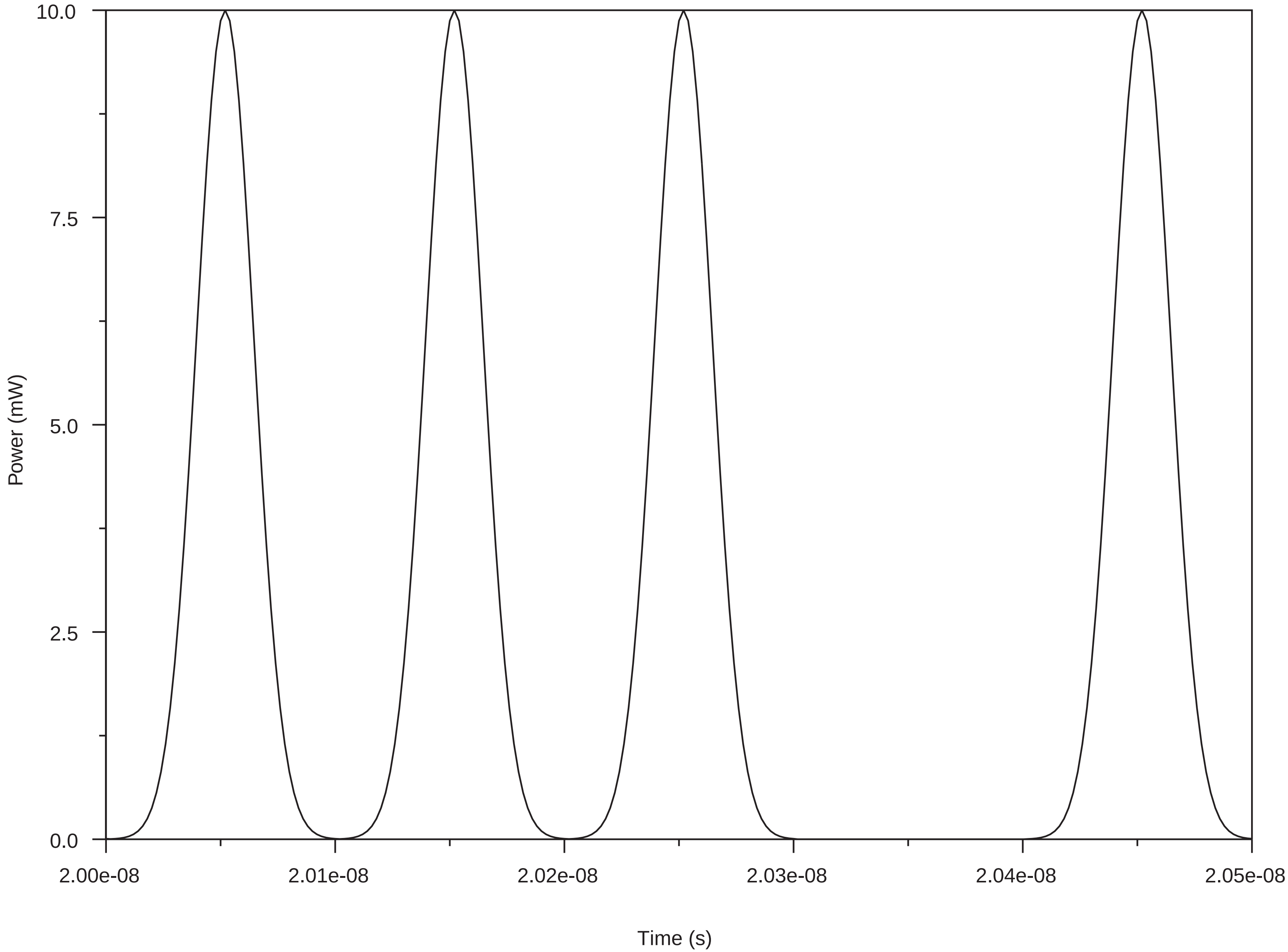
  - ▷ Compare computation with analytic formula for a dispersing Gaussian pulse



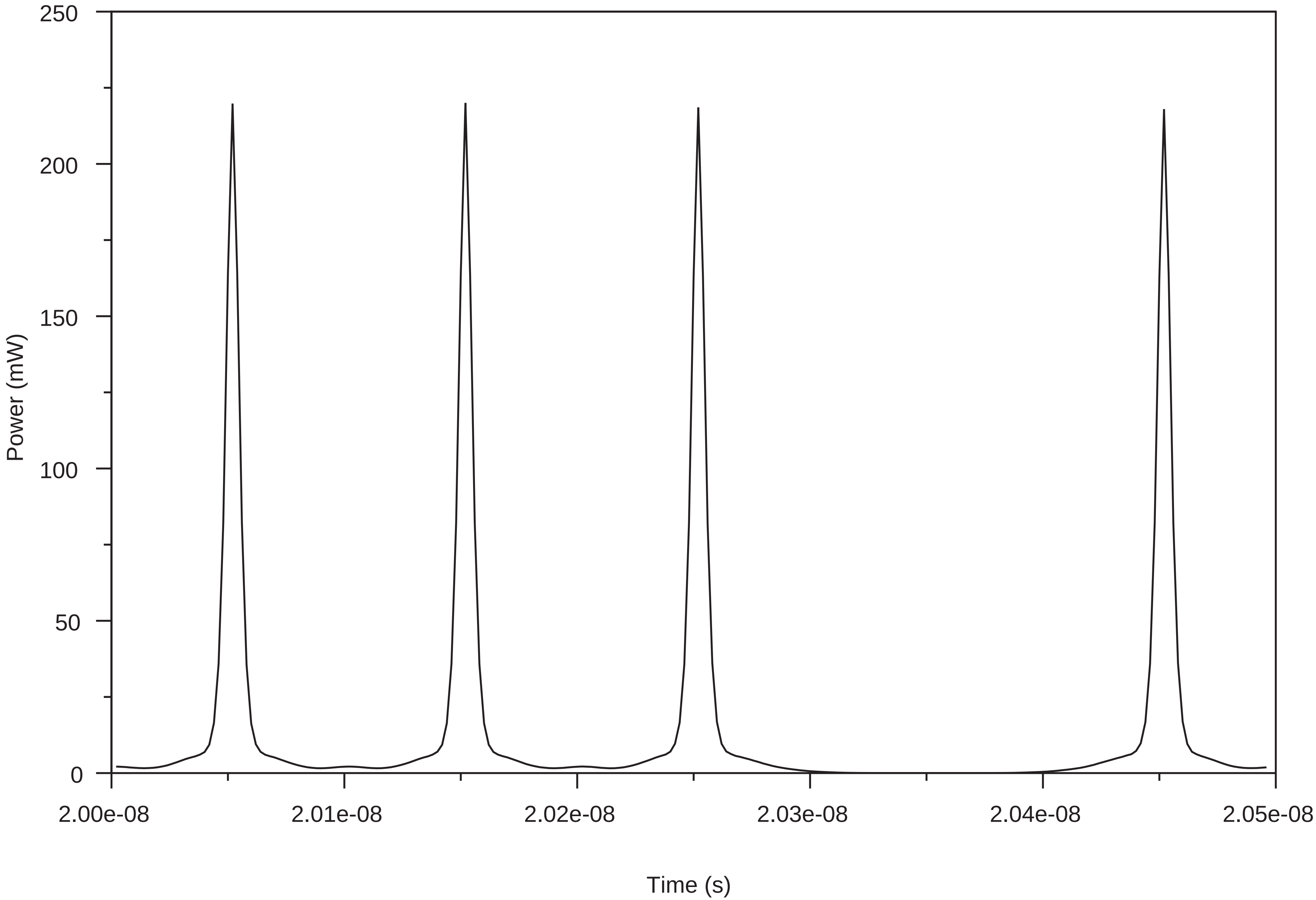
10000
7500
5000
2500
0
Power (mW)
1.118e-08
1.122e-08
1.125e-08
1.128e-08
1.132e-08
Time (s)
input
output



7000
5250
3500
1750
0
Power (mW)
1.118e-08
1.122e-08
1.125e-08
1.128e-08
1.132e-08
Time (s)
analytic
computed



10.0
7.5
5.0
2.5
0.0
Power (mW)
2.00e-08
2.01e-08
2.02e-08
2.03e-08
2.04e-08
2.05e-08
Time (s)



250
200
150
100
50
0
Power (mW)
2.00e-08
2.01e-08
2.02e-08
2.03e-08
2.04e-08
2.05e-08
Time (s)

PARALLEL EXECUTION (1)

- Amdahl's law applied to parallel execution:
  - ▷ Let f be the fraction of a program that is parallelizable, and let P be the number of processors
  - ▷ The speedup in wall-clock time due to execution with P processors instead of 1 processor is

$$S(P) = \frac{1}{(1-f) + \frac{f}{P}}$$

  - ▷ This version of Amdahl's law ignores the dependence of f on P due to the dependence of I/O and memory/CPU latency on P
  - ▷ The actual speedup is always less than the asymptotic speedup,

$$S_{\text{asymp}} = \lim_{P\to\infty} S(P) = \frac{1}{1-f}$$

PARALLEL EXECUTION (2)

- Number of processors (P_F) needed to achieve a fraction F of the asymptotic speedup:

$$S(P_F) = \frac{1}{(1-f) + \frac{f}{P_F}} = F S_{\text{asymp}} = \frac{F}{1-f}$$

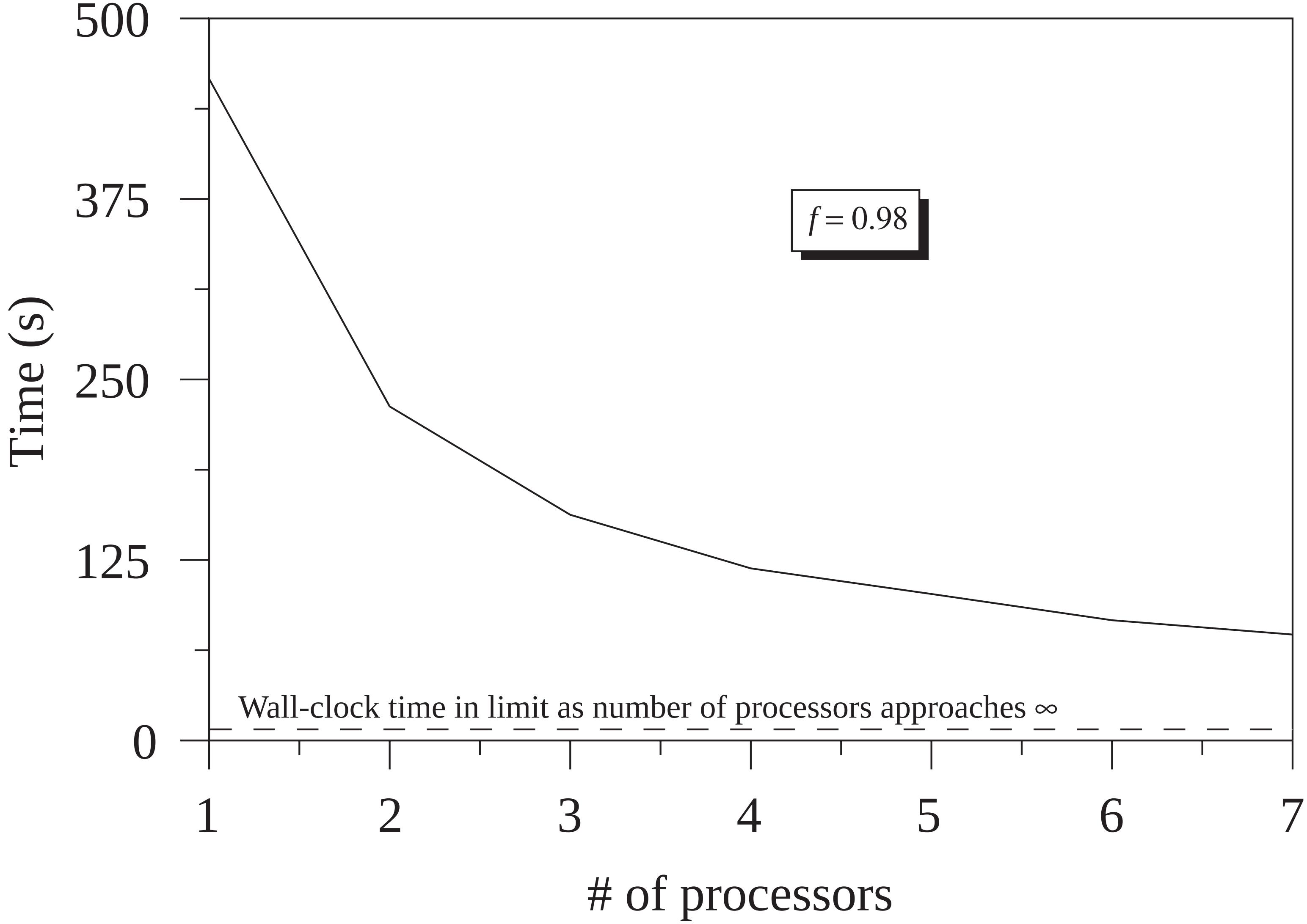
Then

$$P_F = \left(\frac{F}{1-F}\right)\left(\frac{f}{1-f}\right)$$

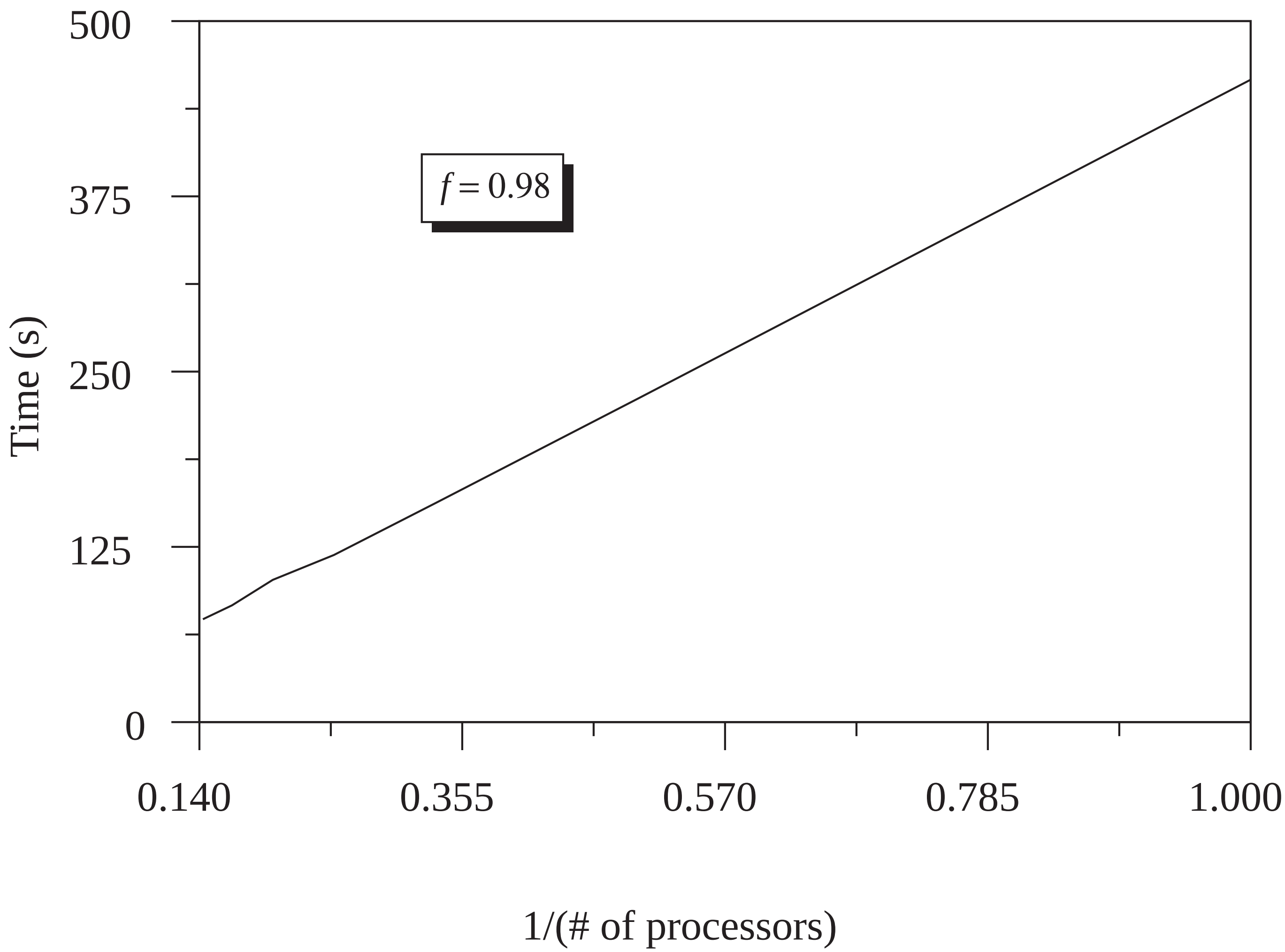
  - ▷ Estimate of the point of diminishing returns that can be obtained by using additional processors to reduce a computation's wall-clock time:

$$P_{1/2} = \frac{f}{1-f}$$

TIME vs. NUMBER OF PROCESSORS (PB METHOD)



TIME vs. (NUMBER OF PROCESSORS)$^{-1}$ (PB METHOD)



PARALLEL EXECUTION (3)

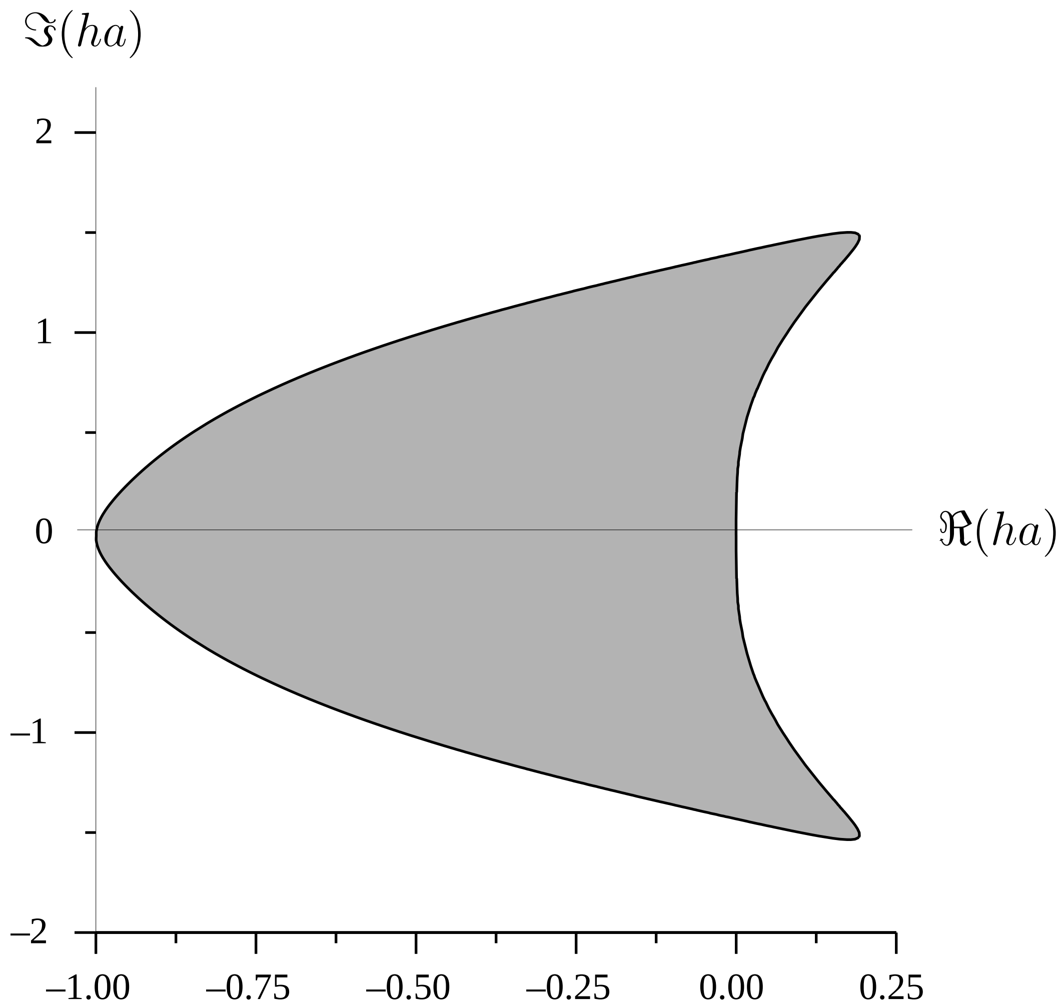
- Examples of asymptotic speedup as a result of parallel execution (if $P_{1/2} < 1$, then more than half the asymptotic speed is achieved with one processor):

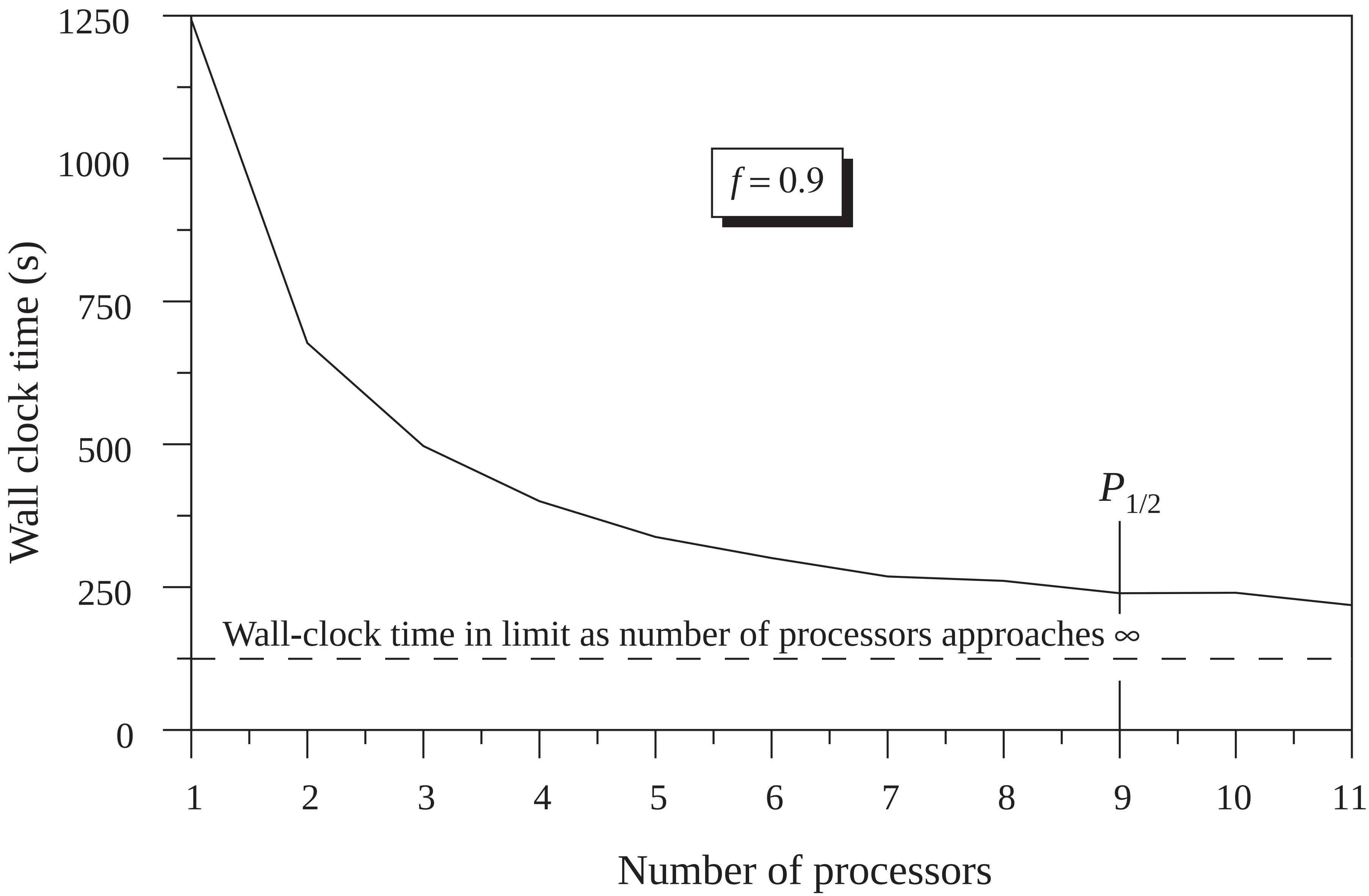
f	S_{asymp}	$P_{1/2}$
0.10	1.11	0.11
0.20	1.25	0.25
0.30	1.43	0.43
0.40	1.67	0.67
0.50	2.00	1
0.75	4.00	3
0.80	5.00	4
0.90	10.0	9
0.95	20.0	19
0.98	50.0	49
0.99	100	99

SUMMARY

- Family of numerical methods for efficiently simulating counter-propagating, strongly interacting waves
  - ▷ Highly parallelizable ⇒ high computational speed
    - ∘ Short threads ⇒ not suitable for clusters
  - ▷ Can handle dispersion, all nonlinearities, and very long bit streams
    - ∘ Can handle arbitrary dispersion and Raman gain profiles
- Comparison with previous work where marching in time is used:
  - ▷ Computational electromagnetics methods solve for the full **E** and **H** fields without introducing a slowly varying envelope
    - ∘ Time step is orders of magnitude shorter than ours ⇒ much greater computational effort
  - ▷ Gaeta, Boyd, Milloni, and Ackerhalt (*Phys. Rev. Lett.* **58**, 2432–2435 (1987))
    - ∘ No treatment of group-velocity dispersion
    - ∘ No exploitation of parallelism

MIDPOINT-TRAPEZOIDAL STABILITY REGION





1250
1000
750
500
250
0
Wall clock time (s)
f = 0.9
P1/2
Wall-clock time in limit as number of processors approaches ∞
1
2
3
4
5
6
7
8
9
10
11
Number of processors

PARALLELIZABLE, BIDIRECTIONAL (PB) METHODS (2)

- Propagation equations for field envelopes in snapshot coordinates:

$$\frac{\partial \overline{\mathcal{E}}_R}{\partial t_1}(z_1, t_1) = \overline{\mathcal{R}}(z_1, t_1) = -\frac{\omega_{R,1}\alpha_R \overline{\mathcal{E}}_R}{2} + \mathsf{D}_{z_1}\overline{\mathcal{E}}_R + 2\pi\omega_R \overline{\mathcal{P}}_R$$

$$\frac{\partial \overline{\mathcal{E}}_L}{\partial t_2}(z_2, t_2) = \overline{\mathcal{L}}(z_2, t_2) = (\omega_{R,1} - \omega_{L,1})\frac{\partial \overline{\mathcal{E}}_L}{\partial z_2} - \frac{\omega_{L,1}\alpha_L \overline{\mathcal{E}}_L}{2} + \mathsf{D}_{z_2}\overline{\mathcal{E}}_L + 2\pi\omega_L \overline{\mathcal{P}}_R$$

  ▷ Dispersion operator in terms of spatial derivatives:

$$\mathsf{D}_{z_2}\overline{\mathcal{E}}_L = \frac{i}{2}\omega_{L,2}\frac{\partial^2 \overline{\mathcal{E}}_L}{\partial z_2^2} + \frac{1}{6}\omega_{L,3}\frac{\partial^3 \overline{\mathcal{E}}_L}{\partial z_2^3}$$

  ▷ Derivatives of ω with respect to β:

$$\omega_{R,1} = \frac{1}{\beta_{R,1}} = v_{g,R} \approx \frac{c}{n_R}$$

$$\omega_{R,2} = -\frac{\beta_{R,2}}{\beta_{R,1}^3} = \frac{\lambda_R^2 v_{g,R}^3}{2\pi c} D_R$$

MIDPOINT PREDICTOR FOR PB METHOD

- Discretization:

$$\mathcal{E}_{R;m,n} = \mathcal{E}_R(z = mh, t = nk) \quad \text{and} \quad \overline{\mathcal{E}}_{R;m,n} = \overline{\mathcal{E}}_R(z_1 = mh, t_1 = nk)$$

  ▷ These are not evaluated at the same space-time point, because

$$z_1 = z - \omega_{R,1} t$$

  ▷ Magic time step: $h = \Delta z = \omega_{R,1}\Delta t = \omega_{R,1} k$

- Midpoint predictor in snapshot coordinates:

$$\overline{\mathcal{E}}^{\text{pred}}_{R;m-n-1,n+1} = \overline{\mathcal{E}}_{R;m-n-1,n-1} + 2k\overline{\mathcal{R}}_{m-n-1,n}$$

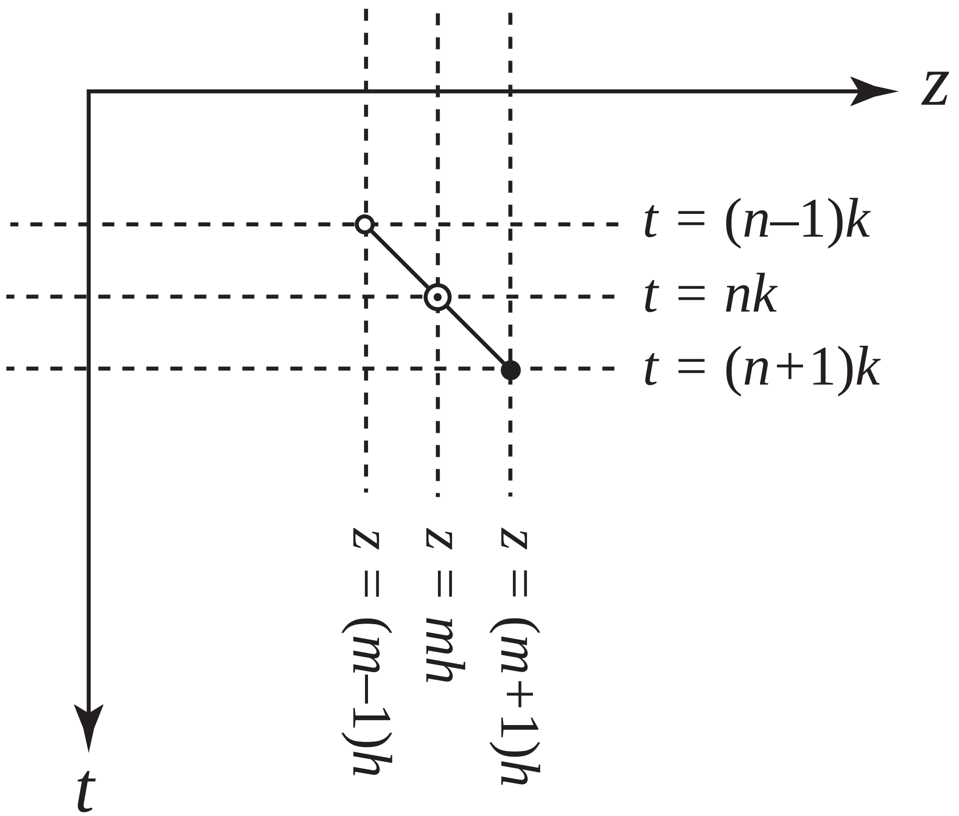
$$\overline{\mathcal{E}}^{\text{pred}}_{L;m+n+1,n+1} = \overline{\mathcal{E}}_{L;m+n+1,n-1} + 2k\overline{\mathcal{L}}_{m+n+1,n}$$

- Midpoint predictor in laboratory coordinates:

$$\mathcal{E}^{\text{pred}}_{R;m,n+1} = \mathcal{E}_{R;m-2,n-1} + 2k\mathcal{R}_{m-1,n}$$

$$\mathcal{E}^{\text{pred}}_{L;m,n+1} = \mathcal{E}_{L;m+2,n-1} + 2k\mathcal{L}_{m+1,n}$$

COMPUTATIONAL "MOLECULE" FOR MIDPOINT PREDICTOR



TRAPEZOIDAL CORRECTOR FOR PB METHOD

- In snapshot coordinates:

$$\overline{\mathcal{E}}_{R;m-n-1,n+1} = \overline{\mathcal{E}}_{R;m-n-1,n} + \frac{1}{2}k\left[\overline{\mathcal{R}}^{\text{pred}}_{m-n-1,n+1} + \overline{\mathcal{R}}_{m-n-1,n}\right]$$

$$\overline{\mathcal{E}}_{L;m+n+1,n+1} = \overline{\mathcal{E}}_{L;m+n+1,n} + \frac{1}{2}k\left[\overline{\mathcal{L}}^{\text{pred}}_{m+n+1,n+1} + \overline{\mathcal{L}}_{m+n+1,n}\right]$$

- In laboratory coordinates:

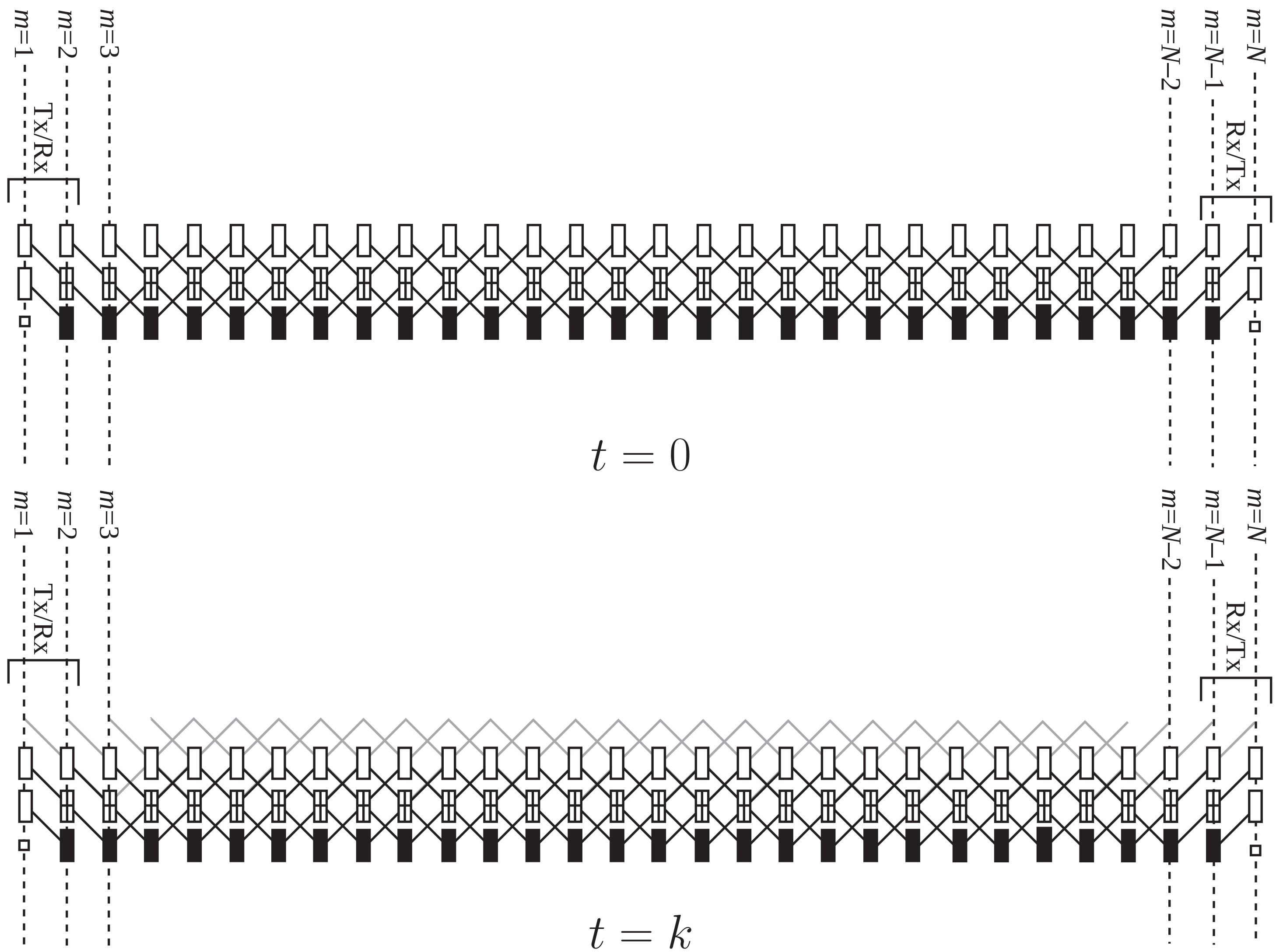
$$\mathcal{E}_{R;m,n+1} = \mathcal{E}_{R;m-1,n} + \frac{1}{2}k\left[\mathcal{R}^{\text{pred}}_{m,n+1} + \mathcal{R}_{m-1,n}\right]$$

$$\mathcal{E}_{L;m,n+1} = \mathcal{E}_{L;m+1,n} + \frac{1}{2}k\left[\mathcal{L}^{\text{pred}}_{m,n+1} + \mathcal{L}_{m+1,n}\right].$$

PB/SPLIT-STEP METHOD

- Propagate arrays of field values for a range of times, not individual field values
  - ▷ Discretization:
  $$\mathcal{E}_{R;m,n}(p\tau) = \mathcal{E}_R(mh, nk + p\tau)$$
  where $p \in (0 : N_\tau - 1)$
  $$\tilde{\mathcal{E}}_{R;m,n}(\omega_\ell) = \sum_{p=0}^{N_\tau - 1} e^{-i\omega_\ell p\tau} \mathcal{E}_{R;m,n}(p\tau)$$
  where $\omega_\ell = \dfrac{2\pi\ell}{k}$ and $\ell \in \left(-\dfrac{N_\tau}{2} : \dfrac{N_\tau}{2} - 1\right)$
  - ▷ Take the step from $t = nk$ to $t = (n+1)k$ in signal space $(\forall p)$ using the source terms that are "diagonal" in signal space
  - ▷ Take the step from $t = nk$ to $t = (n+1)k$ again in transform space $(\forall \ell)$, using the source terms that are "diagonal" in transform space

COMPUTATIONAL LATTICE FOR PB METHOD (1)



COMPUTATIONAL LATTICE FOR PB METHOD (2)