1. **(100 points)**

Five telemetry signals, $m_i(t)$, i = 1,2,…,5, each of bandwidth 1 kHz, 2 kHz, 2 kHz, 5 kHz, and 10 kHz, respectively, are to be transmitted simultaneously by binary PCM. The maximum tolerable error in the sample amplitude of the signals are to be 0.2%, 0.1%, 0.1%, 0.5%, and 0.5% respectively. The first signal is to be sampled at the Nyquist rate, the second at 25% over the Nyquist rate and the rest at 50% over the Nyquist rate. Framing and synchronization of the data requires an additional 1% extra bits.

a. Determine the minimum possible data rate (bits/second) that must be transmitted.

b. Determine the maximum bandwidth required to transmit the composite signal.

c. If 8-ary signaling is used, determine the new maximum bandwidth required to transmit the composite signal. Repeat for 16-ary.

d. What is SNR_Q of the first signal given uniform quantization with m_p equaling 10 times the rms value of $m_1(t)$? Repeat for a μ-law quantizer with μ = 255 and the same number of bits. Which is better?

1) Determine sampling rates

$$F_s \geq 2B$$

$$F_{sm_1} = 2(1kHz) = 2kHz$$

$$F_{sm_2} = 2(2kHz) \cdot 1.25 = 5kHz$$

$$F_{sm_3} = 2(2kHz) \cdot 1.50 = 6kHz$$

$$F_{sm_4} = 2(5kHz) \cdot 1.50 = 15kHz$$

$$F_{sm_5} = 2(10kHz) \cdot 1.50 = 30kHz$$

F_s is the sampling frequency and B is the bandwidth of the signal.

2) Determine number of quantization levels

$$Error_{\max} = \frac{\Delta V}{2 \cdot m_p} \cdot 100\ \%$$

$$L = \frac{2 m_p}{\Delta V} \Rightarrow\Rightarrow \quad Error_{\max} = \frac{1}{L} \cdot 100\ \%$$

$$L_{m_1} = \frac{1}{0.002} = 500$$

$$L_{m_2} = L_{m_3} = \frac{1}{0.001} = 1000$$

$$L_{m_4} = L_{m_5} \frac{1}{0.005} = 200$$

3) Determine number of bits required per sample

$$Bn = \log_2 L$$

$$Bn_{m_1} = \log_2 500 \approx \log_2 512 = 9$$

$$Bn_{m_2} = \log_2 1000 \approx \log_2 1024 = 10$$

$$Bn_{m_3} = 10$$

$$Bn_{m_4} = Bn_{m_5} = 8$$

4) Determine data rate for pure data

$$Datarate = \frac{samples}{\sec} \cdot \frac{bits}{sample} \Rightarrow bits/\sec$$

$$dr_{m_1} = 2.0kHz \cdot 9bits = 19kbps$$

$$dr_{m_2} = 5.0kHz \cdot 10bits = 50kbps$$

$$dr_{m_3} = 6.0kHz \cdot 10bits = 60kbps$$

$$dr_{m_4} = 15kHz \cdot 8bits = 120kbps$$

$$dr_{m_5} = 30kHz \cdot 8bits = 240kbps$$

$$dr_{data} = 489kbps$$

5) Adjust for framing and synchronization

$$dr_{total} = dr_{data} \cdot 1.01 = 489kbps \cdot 1.01 = 493.89kbps \approx 494kbps$$

a. Minimum data rate = 494kbps

In binary transmission, 2B bits can be sent for B Hz of transmission bandwidth.

b. Minimum transmission bandwidth = 494kbps / 2 = 247 kHz

In M-ary transmission, 2B x $\log_2$ M bits can be sent for B Hz of transmission bandwidth.

c. Minimum transmission bandwidth = 494kbps / 2$\log_2$ 8 = 82.33 kHz
Minimum transmission bandwidth = 494kbps / 2$\log_2$ 16 = 61.75 kHz

6) Calculate SNR's

$$uniform \Rightarrow SNR_Q = 6.02Bn + 4.77 - 10\log_{10}\left(\frac{m_p^{\ 2}}{\tilde{m}_t^{\ 2}}\right)$$

$$SNR_Q = 6.02 \cdot 9 + 4.77 - 10\log_{10}\left(\frac{(10 \cdot \tilde{m}_t)^2}{\tilde{m}_t^{\ 2}}\right) = 58.95 - 10\log_{10}(100) = 38.95dB$$

$$m - law \Rightarrow SNR_Q = 6.02Bn + 4.77 - 20\log_{10}\left(\ln(1+m)\right)$$

$$SNR_Q = 6.02 \cdot 9 + 4.77 - 20\log_{10}\left(\ln(256)\right) = 58.95 - 14.878 = 44.07dB$$

d. μ-law is better