A Comparison of Time Horizon Models to Forecast Enrollment

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FORECAST PROCESS with DIFFERENT TIME HORIZONS

LONG-RANGE STRATEGIC FORECAST

INTERMEDIATE-RANGE FORECAST

SHORT-RANGE TACTICAL FORECAST

Long-Range Strategic Planning

Annual / Biennium Planning and Budgeting

Operations and Execution Planning

Forecasting Supports Planning
FORECAST PROCESS with DIFFERENT TIME HORIZONS

LONG-RANGE STRATEGIC FORECAST (10-15 YEARS OUT)

Enrollment, SCH, Revenues, Expenditures, FTE Faculty, Federal R&D Expenditures, Endowment, Physical Plant / Campus Master Plan, Degree Programs

INTERMEDIATE-RANGE FORECAST (1-2 YEARS OUT)

Enrollment, SCH, Revenues, Expenditures, Faculty and Staff Hiring, Student Financial Aid, Tuition / Fees, Administrative Computer Systems, Course Inventory

SHORT-RANGE TACTICAL FORECAST (1-3 SEMESTERS OUT)

Enrollment, SCH, Revenue, Expenditures, Classroom Scheduling

Forecasts with Interlocking Time Horizons
ENROLLMENT DRIVES EVERYTHING

Enrollment = f (X1, X2, … , Xn)

SCH = g(Enrollment)

Revenue ($) = h(SCH)

These models are recursive; i.e., the output of one model becomes the input of the next model.
HUB AND SPOKES

Goal: Accuracy & Timing

BOX-JENKINS MODEL

REGRESSION MODEL

MATRICULATION MODEL

MARKOV CHAIN MODEL

Short-Range Tactical Forecast System Structure
REGRESSION MODEL

• Good for both aggregated and disaggregated data
• Handles unusual events using dummy variables
• Prediction variables must themselves be forecasted for future semesters (this is usually done using compound growth formula, Box-Jenkins, business judgment / scenario forecasting)
• Sample size can vary from a few observations to many
• Can establish “causal” relationships among variables
• Appropriate for capturing turning points in data
• Generally requires that random error term be distributed as bell curve (hypothesis testing)
• Useful for goal-seeking (run model in reverse to assess inputs that yield targeted output)

SOFTWARE TOOLS: SAS, Stata, RATS, Eviews, S+, SPSS
BOX-JENKINS MODEL

• Requires a sample of 40-60 observations

• Extremely simple model with no prediction variables, only time-series history

• Requires intervention of analyst to identify model structure using graphs of sample autocorrelation and partial autocorrelation functions; thus, can be time-consuming to construct many models

• Usually works best on more aggregated data with discernable trend and seasonal patterns

• Seasonal models much more difficult to formulate

• Requires stationary data; i.e., de-trended (no linear or curvilinear trend) and constant variance across sample

• SOFTWARE TOOLS: SAS, Stata, RATS, Eviews, S+
MARKOV CHAIN MODEL

• Sample size of three semester-to-semester changes is adequate to estimate state transition probabilities, if stable
• Closely replicates the physical student pipeline
• Transition probabilities are required to be time invariant (constant over time)
• New students (first time and transfers-in) need to be forecasted outside the MC system using traditional methods
• Lends itself to finer granularity of data; e.g., baccalaureate degree model has classification states (freshmen, sophomores, juniors, seniors)
• No goodness of fit statistics to assess model adequacy a priori
• Gain in-depth of understanding of underlying data generating process (DGP)
• Transaction-oriented approach to modeling time-series
• SOFTWARE TOOLS: Customized programming in SAS, Excel
TREND-SEASONAL REGRESSION MODEL

Use stepwise selection of key prediction variables from Master Menu of 33 Time-Sensitive Trend, Seasonal, and Event History variables conjectured to affect enrollment by semester:

\[ \text{Enrollment}(t) = B_0 \text{ (Intercept)} + B_1 \text{ Time} + B_2 \text{ Spring Intercept Dummy} + B_3 \text{ Spring Slope Dummy} + B_4 \text{ Summer Intercept Dummy} + B_5 \text{ Summer Slope Dummy} + \text{(AY Dummies)} + \text{random error}(t) \]

Correct for all violations of underlying statistical assumptions to sharpen forecasts.
UTD ENROLLMENT FORECASTING - REGRESSION MODEL
TWO TECHNICALLY EQUIVALENT BOX-JENKINS MODELS

Notation:  
Y(t) = Fall Enrollment at time t
Z(t) = Y(t) - Y(t-1) = First Difference to de-trend the data

E(t) = random error (bell curve)

Example:  
Enrollment Fall 2001 = 12,455 = Y(t-1)
Enrollment Fall 2002 = 13,229 = Y(t)
First Difference = 13,229 - 12,455 = 774 = Z(t)

MA(1) Model:  
Z(t) = B0 + e(t) - [ B1 * e(t-1) ]

AR(1) Model:  
Z(t) = A0 + e(t) + [ A1 * Z(t-1) ]
MARKOV CHAIN TRANSITION GRAPH (JUNIORS)

Econometric Models

Enrollment
Input Streams

International students

Student Pipeline
( Goal – Reduce Friction)

Forecast of Total First-time Students

Forecast of Total Transfers - In

MI (missing) has two components:
(1) Inflows = stop-outs + drop-outs + transfers-out
(2) Outflows = sum of all stop-outs prior to t+1 with enrollment in t+1

MI Inflow at t+1

Stop-out re-enrollment t+2 to t+4

Re-enroll after t+1

Inflows = stop-outs + drop-outs + transfers-out

MI (missing) has two components:

MI Inflow at t+1

Stop-out re-enrollment t+2 to t+4

Re-enroll after t+1

Enrollment and/or
Time Leakages

Forecast of Total Transfers - In

Source Data: Spring 2001 (t) to Fall 2001 (t + 1)
MC TRANSITION PROBABILITIES (%)


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Fall 2003 Enrollment = Spring 2003 Enrollment - Graduates - Missing + First-Time Fall 2003 + Transfers-In Fall 2003 + Stop-Outs Returning Fall 2003

= 7,570 - 851 - 1,339 + 947 + 1,325 + 578

= 8,230 undergraduate students
ENROLLMENT FORECASTS FOR FALL 2003

• Trend-Seasonal Regression Model = 13,656
• Box-Jenkins MA(1) Model = 13,570
• Box-Jenkins AR(1) Model = 13,803
• Average of two Box-Jenkins Models = 13,687
• Markov Chain Model = 13,717
• Average of three models = 13,687

For MC model, baccalaureate enrollment forecast was inflated using 60% - 40% split between undergraduate and graduate students)

THECB Certified Fall 2003 Headcount = 13,718
NEXT STEPS IN RESEARCH AND DEVELOPMENT

• Finish development of Undergraduate Markov Chain Model before tackling Masters and Doctoral models

• Refine estimation of missing (MI) or “out-of-system” state

• Model time-varying state transition probabilities using logistic regression, as required

• Develop robust econometric models for enrollment input streams (first-time students and transfers-in)

• Implement a working software system that includes simulation and what-if capabilities

• Disaggregate data by student classification and academic school

• Generate SCH forecasts driven by enrollment forecasts using distributions of semester credit hours for part-time and full-time students

• Repeat above for Revenue ($) using distribution of tuition and fees for in-state and out-of-state tuition status