Infinite Length MMSE Decision Feedback Equalization (DFE)

\[ Y(D) = \|h\|Q(D)X(D) + \tilde{N}(D) \]

\[ B(D) = 1 + b_1 D + \ldots \]
Infinite-Length Decision Feedback Equalizer

\[ Z'(D) = Y(D)W(D) + (1 - B(D)) \hat{X}(D) \]

Assuming previous decisions are correct

\[ Z'(D) = X(D) + Y(D)W(D) - B(D)X(D) \]

Error Sequence

\[ E(D) = X(D) - Z'(D) = B(D)X(D) - Y(D)W(D) \]
MMSE-DFE

- Previous Decisions are assumed correct
- FFF shapes ISI into a causal part (post-cursor ISI) that can be cancelled by the strictly causal FBF

**Error Sequence Analysis**

\[ E(D) = X(D) - Z'(D) \]
\[ = X(D) - [W(D)Y(D) + (1 - B(D))X(D)] \]
\[ = X(D) - W(D)Y(D) - X(D) + B(D)X(D) \]
\[ = B(D)X(D) - W(D)Y(D) \]
Optimum Filter Coefficients

**Orthogonality Principle**

\[ E[E(D)Y^*(D^{-*})] = 0 \]

\[ \Rightarrow B(D)R_{XY}(D) = W(D)R_{YY}(D) \]

\[ \Rightarrow W(D) = \frac{B(D)R_{XY}(D)}{R_{YY}(D)} = B(D)W_{MMSE-LE}(D) \]

\[
W(D) = \frac{B(D)}{\|h\| \left( Q(D) + \frac{1}{SNR_{MFB}} \right)}
\]
Pole-Zero Interpretation of DFE

- Under ideal decisions assumption, transfer function of DFE feedback loop is \( 1/B(D) \)
- Hence, overall transfer function of DFE is \( W(D)/B(D) \). Therefore, in DFE, the 2 filters collaborate to synthesize a pole-zero approximation of the channel inverse which is more accurate (has more degrees of freedom) than a single-filter as in LE
Error Auto-Correlation

\[ R_{ee}(D) = E[E(D)E^*(D^{-*})] \]

where \( E(D) = B(D)X(D) - W(D)Y(D) \)
\[ = B(D)X(D) - B(D)W_{MMSE-LE}(D)Y(D) \]
\[ = B(D)E_{MMSE-LE}(D) \]

\[ \Rightarrow R_{ee,MMSE-DFE}(D) = B(D) \frac{\sigma_n^2}{\|h\|^2 \left( Q(D) + \frac{1}{SNR_{MFB}} \right)} B^*(D^{-*}) \]

Intuitively: we should choose the feedback filter to whiten the error sequence so that the input to the slicer is the desired information symbol + white noise
Spectral Factorization

- **Theorem**: An auto-correlation function $R(D)$ is factorizable if it can be written in the form

$$R(D) = \gamma_0^2 g(D)g^*(D^{-1})$$

where $g(D)$ is a causal, monic, and minimum phase function.

- A function $g(D)$ is called minimum phase if all its roots are outside the unit circle (note $D = z^{-1}$).

- The positive scalar $\gamma_0^2$ is given by

$$\gamma_0^2 = e^{\frac{T}{2\pi} \int_{-T}^{T} \log(R(e^{-j\omega T})) d\omega}$$

- **Fact**: For any auto-correlation function $R(D)$

$$e^{\frac{T}{2\pi} \int_{-T}^{T} \log(R(e^{-j\omega T})) d\omega} \leq \frac{T}{2\pi} \int_{-T}^{T} R(e^{-j\omega T}) d\omega$$

with equality iff $R(e^{-j\omega T})$ is a constant (arithmetic average versus geometric average).
DFE Optimization

**Spectral Factorization**

\[ Q(D) + \frac{1}{\text{SNR}_{MFB}} = \gamma_0^2 G(D) G^*(D^{-*}) \]

\( \gamma_0^2 \) is positive real number

\( G(D) \) is canonical (causal, monic, minimum phase)

\[ R_{ee, DFE-MMSE}(D) = \frac{B(D)}{G(D)} \frac{B^*(D^{-*})}{G^*(D^{-*})} \frac{\sigma_n^2}{\gamma_0^2 \|h\|^2} \]

Variance of error \( \leftarrow r_{ee,0} = \frac{\|B\|^2}{\|G\|^2} \frac{\sigma_n^2}{\gamma_0^2 \|h\|^2} \geq \frac{\sigma_n^2}{\gamma_0^2 \|h\|^2} \)

Since \( B(D)/G(D) \) is also a monic polynomial
DFE Optimization

with equality iff \( B(D) = G(D) \Rightarrow \left\| \frac{B}{G} \right\|^2 = 1 \)

\[ \Rightarrow \sigma_{MMSE-DFE}^2 = \frac{\sigma_n^2}{\gamma_0^2 \|h\|^2} \]

\[ W(D) = \frac{B(D)}{\|h\left(Q(D) + \frac{1}{SNR_{MFB}}\right)} = \frac{1}{\|h\gamma_0^2 G^*(D^-)\|_{non-causal}} \]

Note the Feedforward filter coefficients are not the same as the LE coefficients
DFE Optimization

An Alternative formula for $\gamma_0^2$

$$\frac{T}{2\pi} \int_{-\pi/T}^{\pi/T} \ln \left( Q(e^{-j\omega T}) + \frac{1}{\text{SNR}_{\text{MFB}}} \right) d\omega = \ln(\gamma_0^2)$$

$$+ \frac{T}{2\pi} \int_{-\pi/T}^{\pi/T} \ln \left( G(e^{-j\omega T}) G^*(e^{-j\omega T}) \right) d\omega$$

second term $= \phi$ (proof sketch: write as ratio of products of 1st order pole-zero sections)

Details soon!
DFE Optimization

and use relation
\[
\frac{T}{2\pi} \int_{-\pi/T}^{\pi/T} \ln \left( Q(e^{-j\omega T}) + \frac{1}{SNR_{MFB}} \right) d\omega = \ln(\gamma_0^2)
\]

\[
\Rightarrow \gamma_0^2 = e
\]

\[
SNR_{MMSE-DFE} = \frac{E_x}{\sigma_{MMSE-DFE}^2} = \gamma_0^2 SNR_{MFB}
\]
Another formula for $\gamma_0^2$

Zero order term

\[
\left[ Q(D) + \frac{1}{SNR_{MFB}} \right]_0 = \gamma_0^2 \left[ G(D)G^* (D^{-*}) \right]_0
\]

\[
\Rightarrow 1 + \frac{1}{SNR_{MFB}} = \gamma_0^2 \| g \|^2 = \gamma_0^2 \left( 1 + \sum_{i=1}^{\infty} |g_i|^2 \right)
\]

\[
\Rightarrow \gamma_0^2 = \frac{1 + \frac{1}{SNR_{MFB}}}{1 + \sum_{i=1}^{\infty} |g_i|^2} \leq 1 + \frac{1}{SNR_{MFB}}
\]
**Special Case:**

For no ISI, \( Q(D) = 1 \Rightarrow G(D) = 1 \)

\[ \Rightarrow \gamma_0^2 = 1 + \frac{1}{SNR_{MFB}} \]

\[ \Rightarrow SNR_{MMSE-DFE} = \gamma_0^2 SNR_{MFB} = SNR_{MFB} + 1 \quad !! \text{bias} \]

\[ \Rightarrow SNR_{MMSE-DFE,U} = SNR_{MFB} \quad \left( \text{since channel is ISI-free} \right) \]
DFE Bias Analysis

\[ Z'(D) = X(D) - E(D) \]
\[ = X(D) - G(D)X(D) + \frac{1}{\|h\|\gamma_0^2G^*(D^{-*})} Y(D) \]
\[ = X(D) - G(D)X(D) + \frac{Q(D)}{\gamma_0^2G^*(D^{-*})} X(D) + \frac{\tilde{N}(D)}{\|h\|\gamma_0^2G^*(D^{-*})} \]
\[ = X(D) - G(D)X(D) + \frac{\gamma_0^2G(D)G^*(D^{-*})}{\gamma_0^2G^*(D^{-*})} \frac{1}{SNR_{MFB}} X(D) + noise \]
\[ = X(D) \left[ 1 - \frac{1/\text{SNR}_{MFB}}{\gamma_0^2G^*(D^{-*})} \right] + noise \]
DFE Bias Analysis

The bias term (factor that multiplies $X_k$) is

$$\alpha = \left[1 - \frac{1}{\gamma_0^2 \text{SNR}_M G^*(D^{-*})} \right]_0 = 1 - \frac{1}{\gamma_0^2 \text{SNR}_M} = 1 - \frac{1}{\text{SNR}_{MMSE-DFE}}$$

$$\alpha = \frac{\text{SNR}_{MMSE-DFE}}{\text{SNR}_{MMSE-DFE}} - 1 = \frac{\text{SNR}_{MMSE-DFE, U}}{\text{SNR}_{MMSE-DFE}} \leq 1$$

**Special Case**

ZF–DFE, set $\text{SNR} \to \infty$

$$\frac{W(D)}{B(D)} = \frac{1}{m_0^2 \|h\| P(D) P^*(D^{-*})} = \frac{1}{\|h\| Q(D)}$$

$$Q(D) = m_0^2 P(D) P^*(D^{-*})$$

$$B(D) = P(D); \quad W(D) = \frac{1}{m_0^2 \|h\| P^*(D^{-*})}$$

ZF-DFE transfer function
ZF-DFE

Output of Feedforward Filter:

\[ W(D) \| h \| Q(D) X(D) = \frac{m_0^2 P(D) P^* (D^{-*}) X(D)}{m_0^2 P^* (D^{-*})} = P(D) X(D) \]

\[ 1 = q_0 = m_0^2 \| P \|^2 \Rightarrow m_0^2 = \frac{1}{\| P \|^2} \]

FFF converts channel Q(D) into a canonical filter whose ISI is cancelled by the feedback filter!

\[ \sigma^2_{ZF-DFE} = \frac{\sigma_n^2}{\| h \|^2 m_0^2} = \frac{\sigma_n^2}{\| h \|^2} e^{-\frac{T}{2 \pi} \int_{-\pi/T}^{\pi/T} \ln(Q(e^{-j\omega T}))d\omega} \]

\[ SNR_{ZF-DFE} = m_0^2 SNR_{MFB} \]

ZF-DFE is not biased (\( \alpha = 1 \))
ZF-DFE Feedforward Filter Action

Channel Impulse Response

Feedforward Filter Impulse Response

Equivalent Channel at Feedforward Filter Output
MMSE-DFE Example

Note: we get the same results for $h(D)=1+0.9D$ since we assume analog matched filter so $Q(D)$ would be the same!

$h(D) = 1 + 0.9D^{-1}$

\[
\frac{1}{SNR_{MFB}} + Q(D) = \frac{0.9D + 1.991 + 0.9D^{-1}}{1.81}
\]

\[
= 0.785(1 + 0.633D)(1 + 0.633D^{-1})
\]

\[
W(D) = \frac{1}{\gamma_0^2 \|h\|G^*(D^{-*})} = \frac{1}{\sqrt{1.81(0.785)(1 + 0.633D^{-1})}}
\]

\[
= \frac{0.9469}{1 + 0.633D^{-1}}
\]

non-causal realized with delay

\[
w_k = (0.9469)(-0.633)^{-k}u(-k) : k \leq 0
\]

Feedforward Filter
Impulse Response
DFE Example (Cont’d)

\[ B(D) = G(D) = 1 + 0.633D \quad (Canonical) \]

1 feedback tap given by \( 1 - B(D) = -0.633D \)

\[ \sigma_{MMSE-DFE}^2 = \frac{\sigma_n^2}{\gamma_0^2 \| h \|^2} \quad \Rightarrow \quad \text{SNR}_{MMSE-DFE} = \gamma_0^2 \frac{10}{\text{SNR}_{MFB}} \]

\[ = (0.785)(10) = 7.85 \]

\[ \Rightarrow \text{SNR}_{MMSE-DFE,U} = 7.85 - 1 = 6.85 = 8.4dB \]

\[ \Rightarrow \text{Loss from SNR}_{MFB} = 10 - 8.4 = 1.6dB \]

\[
\begin{cases}
\text{c.f. 4.3dB} \\
\text{for MMSE - LE}
\end{cases}
\]
ZF-DFE Example

\[ Q(D) = \frac{1}{1.81} (1 + 0.9D)(1 + 0.9D^{-1}) = m_0^2 P(D) P^*(D^{-*}) \]

\[ m_0^2 = \frac{1}{1.81} = 0.5525 \]

\[ W(D) = \frac{1}{(0.5525)(\sqrt{1.81})(1 + 0.9D^{-1})} \]

\[ SNR_{ZF-DFE} = 5.525 = 7.4\text{dB} \]

Loss of 2.6dB from SNR_{MFB}

One feedback tap of -0.9

**Detailed Proof for Salz Formula**

*(Thanks to former EE6353 student Shahab Sanayei)*

*(not a recommended bed-time reading!)*:

\[ Q(D) + \frac{1}{SNR_{MFB}} = \gamma_0^2 g(D) g^*(D^{-*}) \]

\[ \Rightarrow \frac{T}{2\pi} \int_{-\pi/T}^{\pi/T} \log(Q(e^{-j\omega T}) + \frac{1}{SNR_{MFB}}) d\omega = \log \gamma_0^2 + \frac{T}{2\pi} \int_{-\pi/T}^{\pi/T} \log(g(e^{-j\omega T}) g^*(e^{j\omega T})) d\omega \]
Salz Formula

\[ R.H.S = \log \gamma_0^2 - \frac{1}{2\pi j} \int_{-\pi}^{\pi} \log(g(e^{-j\omega T})g^*(e^{j\omega T})) \frac{de^{-j\omega T}}{e^{-j\omega T}} \]

Contour Integrals!

\[ = \log \gamma_0^2 - \frac{1}{2\pi j} \oint_{\mathbb{D}} \log(g(D)g^*(D^{-1})) \frac{dD}{D} \quad \mathbb{D} \text{ is the unit disc} \]

\[ = \log \gamma_0^2 - \frac{1}{2\pi j} \oint_{\mathbb{D}} \log(g(D)) \frac{dD}{D} - \frac{1}{2\pi j} \oint_{\mathbb{D}} \log(g^*(D^{-1})) \frac{dD}{D} \]

\[ \rightarrow Z = D^{-1} \Rightarrow \frac{dD}{D} = -\frac{dZ}{Z} \quad \text{and} \quad D^{-1} \xrightarrow{Z\rightarrow Z^{-1}} \mathbb{D} \]
Hence: R.H.S = \( \log \gamma_0^2 - \frac{1}{2\pi j} \int_D \log (g(D)) \frac{dD}{D} + \frac{1}{2\pi j} \int_z \log (g^*(z)) \frac{dz}{z} \)

Since \( g(D) \) and \( g^*(z) \) are both min phase hence they are both analytic

\[ \Rightarrow R.H.S = \log \gamma_0^2 - \frac{1}{2\pi j} \log(g(0)) + \frac{1}{2\pi j} \log(g^*(0)) \]

but \( g(0) = g^*(0) = 1 \); hence \( \log(\gamma_0^2) = \frac{T}{2\pi} \int_{-\pi/T}^{\pi/T} \log(Q(e^{j\omega T}) + \frac{1}{SNR_{MFB}})d\omega \)
Minimum-Phase Channels

• Consider 2-ray multipath channel \( h(D) = c_1 + c_2 D^m \)
  where \( m \) is differential path delay (in symbol periods).

• The \( m \) zeros of \( h(D) \) satisfy the relation \( |D| = |\frac{c_1}{c_2}|^{1/m} \geq 1 \)
  \( h(D) \) **minimum phase** \( \iff |c_1| \geq |c_2| \)
  (shorter delay path has larger magnitude)

• Effects of fading, shadowing, and reflections cause channel to alternate between minimum and non-minimum phase

• **Minimum-phase channels are easier to equalize!**
Zero-Forcing DFE

- Special case of MMSE-DFE by letting $SNR \rightarrow \infty$
- Spectral factorization \(Q(D) = \overline{h}(D)\overline{h}^*(D^{-*}) = m_0^2 P(D)P^*(D^{-*})\)
- Feedback filter is $1-R(D)$ and feedforward filter is $1/(m_0^2 \|h\|P^*(D^{-*}))$
- Infinite-length ZF-DFE is unbiased

- For **minimum-phase** channels:
  1) $m_0^2 = |\overline{h}_0|^2 = \frac{|h_0|^2}{\|h\|^2}$
  2) Feedback filter is $1-P(D) = 1 - \frac{\overline{h}(D)}{h_0} = 1 - \frac{h(D)}{h_0} = -\sum_{i=1}^v \frac{h_i}{h_0} D^i$
  3) Combined matched filter and feedforward filter is a scalar gain!

\[
\frac{\overline{h}^*(D^{-*})}{\|h_0\|^2 P^*(D^{-*})} = \frac{h^*(D^{-*})/\|h\|}{\|h\|^2} \frac{h^*_0}{h_0} = 1
\]
MMSE-DFE vs. ZF-DFE

- MMSE-DFE is superior to ZF-DFE at low SNR
- Both structures become identical at high SNR
- At very low SNR where noise dominates ISI, MMSE-DFE converges to a matched filter
DFE Error Propagation

• Most analyses assume correct past decisions for tractability (accurate at high SNR)

• A single decision error in DFE results in incorrect estimate of post-cursor ISI possibly causing future decision errors

• Long feedback filter exacerbates error propagation

• On channels w/ spectral nulls, the performance advantage of DFE over LE far outweighs effects of error propagation

• Precoding technique is used to eliminate error propagation when channel is known at transmitter
Precoding

**Idea:** Move DFE feedback section to transmitter where no decision errors occur

**Caution:** simple moving of feedback loop $1/B(D)$ to transmitter increases transmitted power (degrades DFE)

**Solution:** Use *Modulo arithmetic* to limit the increase in transmitted power (proposed by Tomlinson & Harashima)
Precoding

• Used to eliminate error propagation in DFE by moving feedback filter to transmitter
• Requires channel knowledge at transmitter
• Consider the ZF-DFE where

\[ W(D) = \frac{1}{m_0^2 \|h\| P^*(D^{-*})} \text{ and } B(D) = P(D) \]

where \( Q(D) = m_0^2 P(D) P^*(D^{-*}) \)

\[ \Rightarrow Y(D) = \|h\| Q(D) \tilde{X}(D) + \tilde{N}(D) \]
Precoding Analysis

Feedforward Filter Output

\[ \Rightarrow Z(D) = W(D)Y(D) \]

\[ = \|h\|W(D)Q(D)\tilde{X}(D) + W(D)\tilde{N}(D) \]

\[ = \|h\| \frac{m_0^2 P(D)P^*(D^{-*})}{m_0^2 \|h\|P^*(D^{-*})} \frac{X(D)}{P(D)} + \tilde{N}(D) \]

\[ \tilde{X}(D) = \frac{X(D)}{P(D)} \]
Precoding Analysis

\[ = X(D) + \tilde{N}(D) \]

where

\[ R_{\tilde{N}}(D) = W(D)\sigma_n^2 Q(D)W^*(D^{--}) \]

\[ = \frac{\sigma_n^2 \cdot m_0^2 P(D)P^*(D^{--})}{(m_0^2)^2\|h\|^2 P(D)P^*(D^{--})} = \frac{\sigma_n^2}{m_0^2\|h\|^2} \rightarrow \text{white!} \]
**Problem:** Transmit Power is boosted

**Solution:** Modulo Operator

\[ \Gamma_M(t) = t - Md \left\lfloor \frac{t + \frac{Md}{2}}{Md} \right\rfloor \]

\( t \) is a real number while \( M \) and \( d \) are integers

uniformly-distributed between \(-M\) and \(M\)

\[ \text{Ex: } d = 2, \ M = 4 \]

\[ \Gamma_4(t) = t - 8 \left\lfloor \frac{t + 4}{8} \right\rfloor \]

\( \left\lfloor \cdot \right\rfloor \) denotes largest integer less than or equal to

---

Sawtooth function
Properties of $\Gamma_M(x)$

\[
\Gamma_M(X \pm Y) = \Gamma_M(X) \oplus_M \Gamma_M(Y)
\]

where \(X \oplus_M Y \triangleq \Gamma_M(X \pm Y)\)

Input to modulo operator

\[
\Psi(D) = X(D) + (1 - B(D))\tilde{X}(D)
\]

\[
\Psi_k = X_k - \sum_{i=1}^{\infty} b_i \tilde{X}_{k-i}
\]

Output of modulo operator

where \(\tilde{X}_k = \Gamma_M(\Psi_k) = \Gamma_M\left(X_k - \sum_{i=1}^{\infty} b_i \tilde{X}_{k-i}\right)\)
Precoded ZF-DFE

Channel Output with Precoded Input

\[ Y(D) = \|h\|Q(D)\tilde{X}(D) + \tilde{N}(D) \]

Precoded ZF-DFE Feedforward Filter Output

\[ Z(D) = W(D)Y(D) = \frac{\|h\|m_0^2P(D)P^*(D^{-*})}{m_0^2\|h\|P^*(D^{-*})} \tilde{X}(D) + \tilde{N}(D) \]

\[ = P(D)\tilde{X}(D) + \tilde{N}(D) \]

\[ \Rightarrow Z_k = \tilde{X}_k + \sum_{i=1}^{\infty} b_i \tilde{X}_{k-i} + \tilde{n}_k \]
After modulo operation at FFF output

\[ \Gamma_M(Z_k) = \Gamma_M \left[ \tilde{X}_k + \sum_{i=1}^{\infty} b_i \tilde{X}_{k-i} + \tilde{n}_k \right] \]

\[ = \Gamma_M \left[ \Gamma_M \left( X_k - \sum_{i=1}^{\infty} b_i \tilde{X}_{k-i} \right) + \sum_{i=1}^{\infty} b_i \tilde{X}_{k-i} + \tilde{n}_k \right] \]

\[ = \Gamma_M \left[ \Gamma_M \left( X_k - \sum_{i=1}^{\infty} b_i \tilde{X}_{k-i} \right) + \sum_{i=1}^{\infty} b_i \tilde{X}_{k-i} + \tilde{n}_k \right] \]

\[ \text{(a)} \quad = \Gamma_M \left[ X_k - \sum_{i=1}^{\infty} b_i \tilde{X}_{k-i} + \sum_{i=1}^{\infty} b_i \tilde{X}_{k-i} + \tilde{n}_k \right] \]

\[ = \Gamma_M \left[ X_k + \tilde{n}_k \right] \]

\[ \text{(a)} \quad = \Gamma_M \left[ X_k + \Gamma_M (\tilde{n}_k) \right] \]
\[ X_k \oplus_M \Gamma_M(\tilde{n}_k) \iff \text{signal plus noise} \]

\[ = X_k + n'_k \iff \text{what is the pdf of } n'_k \]

where (a) follows from

\[ \Gamma_M[\Gamma_M(a) + b] = \Gamma_M(\Gamma_M(a)) \oplus_M \Gamma_M(b) \]

\[ = \Gamma_M(a) \oplus_M \Gamma_M(b) \]

\[ = \Gamma_M(a + b) \]

Transmit Power: \( X_k \) Original PAM signal: \( \varepsilon_X = \frac{M^2 - 1}{12} d^2 \)

\( \tilde{X}_k \) uniform in \( \left[ \frac{-Md}{2}, \frac{Md}{2} \right] \iff \varepsilon_X = \frac{M^2d^2}{12} \)

\( \Rightarrow \text{Power increase is } \frac{M^2}{M^2 - 1} \Rightarrow 1 \text{ as } M \to \infty \)
Precoding Summary

Transmitter

\[ x_k \xrightarrow{\text{1-B(D)}} \tilde{x}_k \xrightarrow{\text{Modulo}} x'_k \]

1. Modulo Operator \( \Gamma_M(x) \)

\[ \Gamma_M(x) = x - Md \left\lfloor \frac{x + \frac{Md}{2}}{Md} \right\rfloor \]

For M-ary constellation with distance \( d \). \( \left\lfloor x \right\rfloor \) denotes largest integer less than or equal to \( x \).

2. For iid input, precoder output is iid and \textit{uniformly distributed} over \( \left[ -\frac{Md}{2}, \frac{Md}{2} \right) \) resulting in slight power increase.

Receiver

\[ y_k \xrightarrow{W(D)} x_k \]

\[ \begin{align*}
\frac{SNR_{MMSE-DFE}}{SNR_{MMSE-DFE,U}}
\end{align*} \]
Precoding Summary

• Eliminates DFE error propagation
• Allows us to combine DFE with coding schemes (unlike conventional DFE which requires instantaneous reliable decisions which are only available after decoder delay!)
• Requires perfect knowledge of feedback filter impulse response (possible with time-invariant or slowly time varying channels). Estimated at receiver and sent back via reverse channel at expense of rate loss
• Precoding slightly increases transmitted power (by factor of $M/(M-1)$ for QAM)