Exploiting Sparsity for Multiple Relay Selection with Relay Gain Control in Large AF Relay Networks

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***Abstract*—We propose a new approach for multiple relay selection in large amplify-and-forward (AF) relay networks based on exploiting the sparsity of the relay gain vector. We start by applying our design approach to formulate the problem of optimizing the selected relay index and gain under the mean squared error criterion and a sparsity constraint. Building on recent advances in sparse signal recovery theory, we propose a solution based on the orthogonal matching pursuit algorithm and investigate its performance-complexity tradeoffs. Then, we generalize our formulation to multiple-antenna large AF relay networks, which naturally leads to a joint multiple relay-antenna selection problem with sparse relay gain vector optimization. Simulation results demonstrate the performance gains of our proposed scheme over existing schemes for both single-antenna and multiple-antenna large AF relay networks. Alternatively, at the same performance level, our scheme selects fewer relays resulting in reduced implementation complexity.**

***Index Terms*—AF relay networks, antenna selection, MSE, OMP, relay gain, relay selection.**

I. Introduction

RELAY selection strategies for dual-hop amplify-and-forward (AF) multiple relay networks have drawn significant attention recently due to their performance gains and reduced implementation complexity [1]-[4]. In optimal single relay selection, the relay with the maximum end-to-end signal-to-noise ratio (SNR) is selected [2], [3]. Reference [3] was the first to generalize the idea of single relay selection to multiple relay selection and showed that it results in improved error rates. In [3], [4], each relay is simply assumed to either cooperate with full power or not cooperate, i.e., no relay gain optimization is performed.

Multiple relay selection in multi-input multi-output (MIMO) relay networks where all nodes are equipped with multiple antennas was recently investigated in [5]-[7]. In this case, both the selections of relays and antennas at each relay are jointly optimized to reduce the number of radio frequency (RF) chains at each relay due to practical implementation constraints [6], [7]. The authors in [6] proposed a greedy minimum mean squared error (MMSE) based antenna selection algorithm. Joint relay-antenna selection strategies based on end-to-end SNR were investigated in terms of the outage probability in [7]. However, the algorithms in [6], [7] select only one antenna pair at each selected relay which eventually limits the performance.

We propose a new approach for joint relay and antenna selection with relay gain control for single and multiple-antenna AF relay networks. Our focus is on **large** MIMO AF relay networks for three reasons: (i) the prohibitive computational complexity of exhaustive joint selection algorithms in large networks (ii) the significant reduction in implementation complexity achieved by relay and antenna selection in this case (iii) the sparse structure of the relay gain vector which can be exploited, as we show in this paper, to improve performance at practical complexity. Large relay networks are applicable in many practical scenarios including device-to-device (D2D) communication networks and wireless sensor networks which consist of a large number of cooperating nodes.

As a special case, when multiple relays each with a single antenna are deployed, the relay gain matrix is a diagonal matrix whose main diagonal contains only few non-zero elements (i.e. it is a **sparse** vector). We optimize this relay gain matrix to minimize the mean squared error (MSE) under the sparsity constraint. We build on recent advances in sparse signal recovery theory to solve the relay selection problem efficiently using the orthogonal matching pursuit (OMP) algorithm [8]. Then, we generalize our approach to large MIMO AF relay networks where our goal is to minimize the MSE by jointly selecting multiple relays and antennas while optimizing the relay gain matrix under a sparsity constraint.

Notation: We use the following standard notation in this paper. $|a|$ is the magnitude of a scalar; $||\mathbf{a}||_1$ and $||\mathbf{a}||_2$ are the $\mathcal{L}_1$ and $\mathcal{L}_2$ norms, respectively; and $||\mathbf{A}||_{\mathrm{F}}$ is the Frobenius norm. $\mathbf{a}^T(\mathbf{A}^T)$ is the transpose of a vector (a matrix); $\mathbf{a}^H(\mathbf{A}^H)$ is the complex-conjugate transpose of a vector (a matrix); and $\mathbf{A}^{-1}$ denotes the inverse of a square matrix. The $N \times N$ identity matrix is denoted by $\mathbf{I}_N$. The expectation of a random variable A is denoted by $\mathcal{E}[A]$; $\otimes$ denotes the Kronecker product; $\mathrm{vec}\{\mathbf{A}\}$ and $\mathrm{Tr}\{\mathbf{A}\}$ are the vectorization and the trace operator for a matrix $\mathbf{A}$; $\mathrm{diag}\{a_1(\mathbf{A}_1)\dots a_N(\mathbf{A}_N)\}$ denotes a diagonal matrix (a block diagonal matrix).

II. System Model

We consider two-hop AF relay networks where there is one source (S), one destination (D), and *N* relays (R), as shown in Fig. 1. The direct link between the source and the destination is ignored due to its assumed large path loss. We assume a half-duplex signaling mode where in the first phase the source broadcasts the signal to all relays. Then, in the second phase, each relay transmits the received signal multiplied by a gain factor. In addition, we assume that the destination knows all the channel information as assumed also in [3] and [6].

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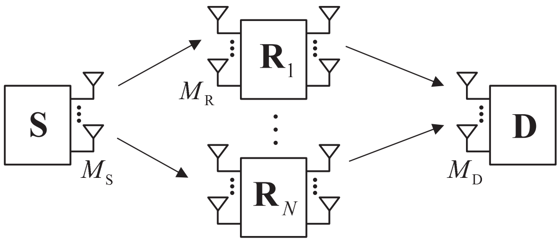

Fig. 1. Multiple AF relay network.

We assume $M_{\rm S}$ antennas at the source, $M_{\rm D}$ antennas at the destination, and $M_{\rm R}$ antennas at each relay. The $M_{\rm S}\times M_{\rm R}$ S-R channel matrix from the source to the *n*-th relay is denoted by $\mathbf{H}_{\rm SR}^n$ and the $M_{\rm D}\times M_{\rm R}$ R-D channel matrix from the *n*-th relay to the destination is denoted by $\mathbf{H}_{\rm RD}^n$. We assume that those two channel matrices are constant during the two transmission phases. Moreover, the elements of $\mathbf{H}_{\rm SR}^n$ and $\mathbf{H}_{\rm RD}^n$ are independent circularly-symmetric complex Gaussian random variables with zero mean and unit variance. The $NM_{\rm R}\times 1$ noise vector $\mathbf{v}$ at all relays and the $M_{\rm D}\times 1$ noise vector at the destination are mutually independent circularly-symmetric complex additive white Gaussian noise (AWGN) vectors modeled as $\mathbf{R}_{\mathbf{vv}}=\mathcal{E}[\mathbf{vv}^H]=\sigma_v^2\mathbf{I}_{NM_{\rm R}}$ and $\mathbf{R}_{\mathbf{ww}}=\mathcal{E}[\mathbf{ww}^H]=\sigma_w^2\mathbf{I}_{M_{\rm D}}$, respectively. During the first transmission phase, the received signal vector $\mathbf{y}_{\rm R}$ through the S-R channels $\mathbf{H}_{\rm SR}=[\mathbf{H}_{\rm SR}^1\ldots\mathbf{H}_{\rm SR}^n\ldots\mathbf{H}_{\rm SR}^N]^T$ at the multiple relays is given by

$$\mathbf{y}_{\rm R}=\mathbf{H}_{\rm SR}\mathbf{x}+\mathbf{v}, \tag{1}$$

where the $M_{\rm S}$-dimensional vector $\mathbf{x}$ is the transmitted signal from the source with transmission power $\mathrm{Tr}\{\mathbf{R}_{\mathbf{xx}}\}=\sigma_x^2$ where $\mathbf{R}_{\mathbf{xx}}=\mathcal{E}[\mathbf{xx}^H]$. In the second transmission phase, the relays amplify via an AF relays gain matrix $\mathbf{G}$ and forward the received signal $\mathbf{y}_R$ to the destination. The received signal at the destination is

$$\mathbf{y}_{\rm D}=\mathbf{H}_{\rm RD}\mathbf{G}\mathbf{H}_{\rm SR}\mathbf{x}+\mathbf{H}_{\rm RD}\mathbf{G}\mathbf{v}+\mathbf{w}, \tag{2}$$

where $\mathbf{H}_{\rm RD}=[\mathbf{H}_{\rm RD}^1\ldots\mathbf{H}_{\rm RD}^n\ldots\mathbf{H}_{\rm RD}^N]$ is the compound R-D channel matrix.

III. Multiple Relay Selection with Gain Control For SISO Relay Networks

In this section, we propose a multiple relay selection scheme for single-input single-output (SISO) relay networks where all nodes are equipped with a single antenna. First, we formulate the problem of multiple relay selection with relay gain control for MSE minimization in SISO relay networks. In the case of SISO relay networks, the gain matrix $\mathbf{G}$ becomes a *diagonal matrix* $\mathbf{G}_{\rm S}=\mathrm{diag}\{g_{\rm S}^1\ldots g_{\rm S}^n\ldots g_{\rm S}^N\}$ whose diagonal element $g_{\rm S}^n$ is the gain associated with the *n*-th selected relay. Exploiting the diagonal structure of $\mathbf{G}_{\rm S}$, Equation (2) can be written in the following equivalent form

$$y_{\rm D}=\mathbf{g}_{\rm S}^H\mathbf{h}x+\mathbf{g}_{\rm S}^H\tilde{\mathbf{v}}+w \tag{3}$$

where $\mathbf{g}_{\rm S}=\mathrm{vec}\{\mathbf{G}_{\rm S}\}=[g_{\rm S}^1\ldots g_{\rm S}^N]^T$ is the gain vector which is the diagonal of $\mathbf{G}_{\rm S}$. The $N\times 1$ vectors $\mathbf{h}$ and $\tilde{\mathbf{v}}$ have the form $[h_{\rm RD}^1h_{\rm SR}^1\ldots h_{\rm RD}^Nh_{\rm SR}^N]^T$ and $[h_{\rm RD}^1v_1\ldots h_{\rm RD}^Nv_N]^T$, respectively, where $h_{\rm SR}^n$ and $h_{\rm RD}^n$ are the S-R and R-D channel elements associated with the *n*-th relay, respectively. The noise components at the *n*-th relay and destination are denoted by v_n and w, respectively. Defining a relay selection vector $\mathbf{g}_{\rm S}$, the error signal is defined as follows

$$e=x-\left\{\mathbf{g}_{\rm S}^H\left(\mathbf{h}x+\tilde{\mathbf{v}}\right)+w\right\}. \tag{4}$$

Hence, the MSE at the destination can be written as

$$\begin{aligned}\mathcal{E}[|e|^2]&=\sigma_x^2-\mathbf{g}_{\rm S}^H\underbrace{\mathbf{h}\sigma_x^2}_{\triangleq\tilde{\mathbf{h}}}-\underbrace{\mathbf{h}^H\sigma_x^2}_{\triangleq\tilde{\mathbf{h}}^H}\mathbf{g}_{\rm S}\\&\quad+\mathbf{g}_{\rm S}^H\underbrace{\left(\sigma_x^2\mathbf{hh}^H+\mathbf{R}_{\tilde{v}\tilde{v}}\right)}_{\triangleq\mathbf{R}}\mathbf{g}_{\rm S}+\sigma_w^2\\&=\sigma_x^2-\mathbf{g}_{\rm S}^H\tilde{\mathbf{h}}-\tilde{\mathbf{h}}^H\mathbf{g}_{\rm S}+\mathbf{g}_{\rm S}^H\mathbf{R}\mathbf{g}_{\rm S}+\sigma_w^2,\end{aligned} \tag{5}$$

where the covariance matrix of the relay noise vector is $\mathbf{R}_{\tilde{v}\tilde{v}}=\mathcal{E}[\tilde{\mathbf{v}}\tilde{\mathbf{v}}^H]=\mathrm{diag}\{\sigma_{\tilde{v}_1}^2\ldots\sigma_{\tilde{v}_n}^2\ldots\sigma_{\tilde{v}_N}^2\}$ whose *n*-th element is $\sigma_{\tilde{v}_n}^2=|h_{\rm RD}^n|^2\sigma_v^2$. Considering the Cholesky factorization of the positive-definite matrix $\mathbf{R}=\mathbf{LL}^H$ where $\mathbf{L}$ is an $N\times N$ lower-triangular matrix, we can rewrite (5) as follows [9]

$$\mathrm{MSE}=\sigma_x^2-\mathbf{g}_{\rm S}^H\mathbf{LL}^{-1}\tilde{\mathbf{h}}-\tilde{\mathbf{h}}^H\mathbf{L}^{-H}\mathbf{L}^H\mathbf{g}_{\rm S}+\mathbf{g}_{\rm S}^H\mathbf{LL}^H\mathbf{g}_{\rm S}+\sigma_w^2. \tag{6}$$

By completing the square in (6), we get

$$\mathrm{MSE}=\underbrace{\sigma_x^2-\tilde{\mathbf{h}}^H\mathbf{L}^{-H}\mathbf{L}^{-1}\tilde{\mathbf{h}}+\sigma_w^2}_{\triangleq\mathrm{MSE}_{\min}}+\underbrace{\left\|\mathbf{L}^H\mathbf{g}_{\rm S}-\mathbf{L}^{-1}\tilde{\mathbf{h}}\right\|_2^2}_{\triangleq\mathrm{MSE}_{\rm excess}}. \tag{7}$$

Since $\mathrm{MSE}_{\min}$ does not depend on $\mathbf{g}_{\rm S}$, the MSE is minimized by minimizing the term $\mathrm{MSE}_{\rm excess}$ which can be controlled through the relay gain vector $\mathbf{g}_{\rm S}$. To select multiple relays which minimize MSE from (7), there are two classes of algorithms: convex optimization and greedy algorithms with the latter being more suitable to the joint relay-antenna selection problem due to its low complexity. We use the OMP algorithm in [8] since it is widely recognized as a canonical greedy algorithm for sparse signal recovery and most other greedy-type algorithms, such as the more recent algorithm in [10], are modifications of it.

Here, we only provide a sketch of the OMP algorithm due to space limitations. We denote the OMP algorithm by the function OMP(sensing matrix, measurement vector, stopping criterion) which is well documented in the sparse signal recovery literature; see e.g. [8]. In our formulation of SISO relay networks, the OMP algorithm proceeds by finding, in each iteration, one column of the matrix $\mathbf{L}^H$ which is the most correlated with the residual error vector obtained by subtracting the contributions of the selected relays in the previous iteration from the vector $\mathbf{L}^{-1}\tilde{\mathbf{h}}$. We consider the OMP algorithm with two different stopping criteria. The first criterion finds the sparse vector $\mathbf{g}_{\rm S}$ which determines the indices and gains of the selected relays to guarantee a tolerable MSE increase ε as given by

$$\mathbf{g}_{\rm S}=\mathrm{OMP}\left(\mathbf{L}^H,\mathbf{L}^{-1}\tilde{\mathbf{h}},\ \left\|\mathbf{L}^H\mathbf{g}_{\rm S}-\mathbf{L}^{-1}\tilde{\mathbf{h}}\right\|_2^2\leq\varepsilon\right). \tag{8}$$

The design parameter ε is chosen according to the tolerable performance loss $\mathrm{MSE}_{\rm excess}$. The second stopping criterion is the desired number of selected relays K_{R} as given by

$$\mathbf{g}_{\mathrm{S}} = \mathrm{OMP}\left(\mathbf{L}^{H}, \mathbf{L}^{-1}\tilde{\mathbf{h}},\, K_{\mathrm{R}}\,\text{iterations}\right). \tag{9}$$

Since one relay is selected at each iteration of the OMP algorithm, setting the value of K_{R} yields an index set of K_{R} selected multiple relays. The OMP algorithm in (9) has a computational complexity of $O(K_{\mathrm{R}}^2 N)$.

IV. Joint Relay and Antenna Selection with Gain Control for MIMO Relay Networks

In this section, we extend our approach to MIMO AF relay networks to perform joint selection of relays and their antennas with relay antenna gain optimization. For the case of MIMO relay networks, the gain matrix $\mathbf{G}$ becomes a *block diagonal matrix* $\mathbf{G}_{\mathrm{M}} = \mathrm{diag}\{\mathbf{G}_{\mathrm{M}}^1, \ldots, \mathbf{G}_{\mathrm{M}}^n, \ldots, \mathbf{G}_{\mathrm{M}}^N\}$ instead of a diagonal matrix $\mathbf{G}_{\mathrm{S}}$ as in SISO relays networks. Note that sub-matrix $\mathbf{G}_{\mathrm{M}}^n$ is associated with the *n*-th relay. We assume that $M_{\mathrm{S}} = M_{\mathrm{D}}$, and at the destination the error vector before equalization is defined as follows

$$\mathbf{e} = \mathbf{x} - \left(\mathbf{H}_{\mathrm{RD}}\mathbf{G}_{\mathrm{M}}\mathbf{y}_{\mathrm{R}} + \mathbf{w}\right). \tag{10}$$

The error correlation matrix, $\mathbf{R}_{\mathbf{ee}} = \mathcal{E}[\mathbf{e}\mathbf{e}^H]$ is given by

$$\begin{aligned}\mathbf{R}_{\mathbf{ee}} &= \mathbf{R}_{\mathbf{xx}} - \mathbf{R}_{\mathbf{x}\mathbf{y}_{\mathrm{R}}}(\mathbf{H}_{\mathrm{RD}}\mathbf{G}_{\mathrm{M}})^H - \mathbf{H}_{\mathrm{RD}}\mathbf{G}_{\mathrm{M}}\mathbf{R}_{\mathbf{y}_{\mathrm{R}}\mathbf{x}} \\ &\quad + \mathbf{H}_{\mathrm{RD}}\mathbf{G}_{\mathrm{M}}\mathbf{R}_{\mathbf{y}_{\mathrm{R}}\mathbf{y}_{\mathrm{R}}}(\mathbf{H}_{\mathrm{RD}}\mathbf{G}_{\mathrm{M}})^H + \mathbf{R}_{\mathbf{ww}} \\ &= \underbrace{\left(\mathbf{R}_{\mathbf{xx}} - \mathbf{R}_{\mathbf{x}\mathbf{y}_{\mathrm{R}}}\mathbf{R}_{\mathbf{y}_{\mathrm{R}}\mathbf{y}_{\mathrm{R}}}^{-1}\mathbf{R}_{\mathbf{y}_{\mathrm{R}}\mathbf{x}}\right) + \mathbf{R}_{\mathbf{ww}}}_{\triangleq \mathbf{A}} \\ &\quad + \underbrace{\left(\mathbf{H}_{\mathrm{RD}}\mathbf{G}_{\mathrm{M}} - \mathbf{R}_{\mathbf{x}\mathbf{y}_{\mathrm{R}}}\mathbf{R}_{\mathbf{y}_{\mathrm{R}}\mathbf{y}_{\mathrm{R}}}^{-1}\right)}_{\triangleq \mathbf{B}}\mathbf{R}_{\mathbf{y}_{\mathrm{R}}\mathbf{y}_{\mathrm{R}}} \\ &\quad \cdot \underbrace{\left(\mathbf{H}_{\mathrm{RD}}\mathbf{G}_{\mathrm{M}} - \mathbf{R}_{\mathbf{x}\mathbf{y}_{\mathrm{R}}}\mathbf{R}_{\mathbf{y}_{\mathrm{R}}\mathbf{y}_{\mathrm{R}}}^{-1}\right)^H}_{\triangleq \mathbf{B}^H},\end{aligned} \tag{11}$$

where $\mathbf{R}_{\mathbf{x}\mathbf{y}_{\mathrm{R}}} = \mathcal{E}[\mathbf{x}\mathbf{y}_{\mathrm{R}}^H] = \mathbf{R}_{\mathbf{xx}}\mathbf{H}_{\mathrm{SR}}^H$,
$\mathbf{R}_{\mathbf{y}_{\mathrm{R}}\mathbf{y}_{\mathrm{R}}} = \mathcal{E}[\mathbf{y}_{\mathrm{R}}\mathbf{y}_{\mathrm{R}}^H] = \mathbf{H}_{\mathrm{SR}}\mathbf{R}_{\mathbf{xx}}\mathbf{H}_{\mathrm{SR}}^H + \mathbf{R}_{\mathbf{vv}}$.

Hence, the MSE is given by

$$\mathrm{MSE} = \mathrm{Tr}\left\{\mathbf{A}\right\} + \mathrm{Tr}\left\{\mathbf{B}\mathbf{R}_{\mathbf{y}_{\mathrm{R}}\mathbf{y}_{\mathrm{R}}}\mathbf{B}^H\right\}. \tag{12}$$

Since the first term does not depend on $\mathbf{G}_{\mathrm{M}}$, the MSE is minimized by minimizing the second term of (12). Considering the Cholesky factorization of the positive-definite $\mathbf{R}_{\mathbf{y}_{\mathrm{R}}\mathbf{y}_{\mathrm{R}}} = \mathbf{L}_{\mathbf{y}_{\mathrm{R}}}\mathbf{L}_{\mathbf{y}_{\mathrm{R}}}^H$, the second term of (12) can be rewritten as

$$\begin{aligned}\mathrm{Tr}\left\{\mathbf{B}\mathbf{L}_{\mathbf{y}_{\mathrm{R}}}\mathbf{L}_{\mathbf{y}_{\mathrm{R}}}^H\mathbf{B}^H\right\} &= \left\|\mathbf{B}\mathbf{L}_{\mathbf{y}_{\mathrm{R}}}\right\|_{\mathrm{F}}^2 \\ &\overset{(a)}{\leq} \left\|\mathbf{B}\right\|_{\mathrm{F}}^2 \left\|\mathbf{L}_{\mathbf{y}_{\mathrm{R}}}\right\|_{\mathrm{F}}^2,\end{aligned} \tag{13}$$

where the inequality (a) follows from the Cauchy-Schwartz inequality. By minimizing $\mathrm{MSE}_{\mathrm{excess}} \triangleq ||\mathbf{B}||_{\mathrm{F}}^2 = ||\mathbf{H}_{\mathrm{RD}}\mathbf{G}_{\mathrm{M}} - \mathbf{R}_{\mathbf{x}\mathbf{y}_{\mathrm{R}}}\mathbf{R}_{\mathbf{y}_{\mathrm{R}}\mathbf{y}_{\mathrm{R}}}^{-1}||_{\mathrm{F}}^2$, we also minimize MSE. This is done by a reformulation to get a vector form of $\mathbf{G}_{\mathrm{M}}$ as in the case of SISO relay networks as follows,

$$\begin{aligned}\mathrm{MSE}_{\mathrm{excess}} = ||\, &\underbrace{(\mathbf{I}_{NM_{\mathrm{R}}} \otimes \mathbf{H}_{\mathrm{RD}})}_{\triangleq \mathbf{H}} \cdot \underbrace{\mathrm{vec}\left\{\mathbf{G}_{\mathrm{M}}\right\}}_{\triangleq \mathbf{g}_{\mathrm{M}}} \\ &- \underbrace{\mathrm{vec}\left\{\mathbf{R}_{\mathbf{x}\mathbf{y}_{\mathrm{R}}}\mathbf{R}_{\mathbf{y}_{\mathrm{R}}\mathbf{y}_{\mathrm{R}}}^{-1}\right\}}_{\triangleq \mathbf{c}}\,||_2^2.\end{aligned} \tag{14}$$

To reduce the algorithm's complexity, we perform dimension reduction by exploiting the block diagonal structure of $\mathbf{G}_{\mathrm{M}}$. The first term $\mathbf{H}\mathbf{g}_{\mathrm{M}}$ of the $\mathrm{MSE}_{\mathrm{excess}}$ in (14) is equal to

$$\begin{aligned}\mathbf{H}\mathbf{g}_{\mathrm{M}} = &\underbrace{\mathrm{diag}\left\{\mathbf{I}_{M_{\mathrm{R}}} \otimes \mathbf{H}_{\mathrm{RD}}^1, \ldots, \mathbf{I}_{M_{\mathrm{R}}} \otimes \mathbf{H}_{\mathrm{RD}}^N\right\}}_{\triangleq \tilde{\mathbf{H}}} \\ &\cdot \underbrace{\left[\tilde{\mathbf{g}}_{\mathrm{M}}^1, \ldots, \tilde{\mathbf{g}}_{\mathrm{M}}^N\right]^T}_{\triangleq \tilde{\mathbf{g}}_{\mathrm{M}}},\end{aligned} \tag{15}$$

where the $1 \times M_{\mathrm{R}}^2$ vector $\tilde{\mathbf{g}}_{\mathrm{M}}^n$ is the vector form of the sub- matrix $\mathbf{G}_{\mathrm{M}}^n$, i.e. $\tilde{\mathbf{g}}_{\mathrm{M}}^n = \mathrm{vec}\{\mathbf{G}_{\mathrm{M}}^n\}^T$. Hence, minimizing $\mathrm{MSE}_{\mathrm{excess}}$ by jointly selecting relays and antennas with antenna gain control is achieved by either one of the following two OMP algorithm formulations

$$\tilde{\mathbf{g}}_{\mathrm{M}} = \mathrm{OMP}\left(\tilde{\mathbf{H}}, \mathbf{c}, ||\tilde{\mathbf{H}}\tilde{\mathbf{g}}_{\mathrm{M}} - \mathbf{c}||_2^2 \leq \varepsilon\right), \tag{16}$$

$$\tilde{\mathbf{g}}_{\mathrm{M}} = \mathrm{OMP}\left(\tilde{\mathbf{H}}, \mathbf{c},\, K_{\mathrm{A}}\,\text{iterations}\right), \tag{17}$$

where K_{A} denotes the desired number of selected antenna pairs. By using K_{A} (the number of iterations) as the stopping criterion in (17) rather than K_{R} as in SISO relay networks, we can easily control the number of selected antenna pairs while ensuring a small number of selected relays in the MIMO relay network. The OMP algorithm of (17) has a computational complexity of $O(K_{\mathrm{A}}^2 N M_{\mathrm{R}}^2 + K_{\mathrm{A}}^2 N M_{\mathrm{R}} M_{\mathrm{D}})$. Our proposed scheme is different from the scheme of [6] in that our sub-matrix $\mathbf{G}_{\mathrm{M}}^n$ contains several non-zero elements; hence, multiple antenna pairs are selected at each selected relay resulting in improved performance.

V. Simulation Results

In this section, we compare the bit error rate (BER) of our proposed relay selection scheme and the schemes in [3] and [6] for SISO and MIMO multiple relay networks, respectively. In all simulations, we assume BPSK transmission. For a fair comparison, the same total transmit power constraint is assumed. The total power used at all relays is denoted by $\mathrm{Tr}\{\mathcal{E}[\mathbf{G}\mathbf{y}_{\mathrm{R}} \cdot (\mathbf{G}\mathbf{y}_{\mathrm{R}})^H]\} = \sigma_x^2$. We use the OMP formulation in (9) and (17) due to its flexibility in setting the number of selected relays.

Fig. 2 shows the BER comparison in both SISO and MIMO relay networks with *N*=50 relays. In the SISO case, our proposed multiple relay selection with gain control minimizing end-to-end MSE is compared with the multiple relay selection maximizing end-to-end SNR in [3]. For our simulation parameters, the scheme in [3] selects around 24 relays. Hence, in our OMP-based scheme, a fixed number of relays is used as the stopping criterion in (9); namely K_{R} =24 relays are selected and their gains are optimized. As expected, the best relay selection scheme exhibits the worst performance resulting from selecting a single relay. Our proposed OMP-based multiple relay selection scheme outperforms [3] due to relay gain optimization and also due to the different selected relay indices. For the MIMO case, Fig. 2 compares the BER of the greedy MSE minimization (GMM) antenna selection scheme in [6] with our OMP approach in (17) for joint selection of

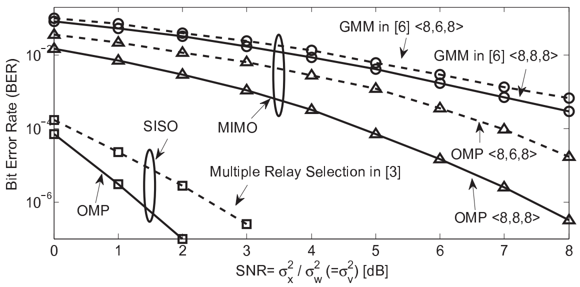

Fig. 2. BER comparison at different SNR levels (dB).

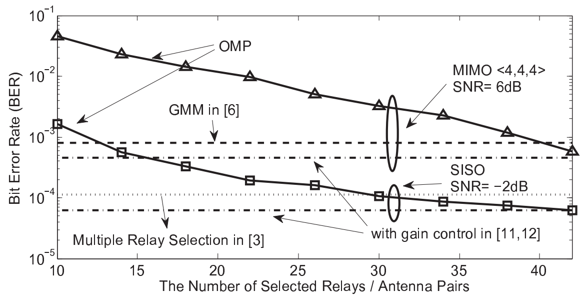

Fig. 3. BER comparison as a function of the number of selected relays (in the SISO case) and antennas (in the MIMO case).

relays and their antennas with gain control minimizing end-to-end MSE. We set $M_{\rm S} = M_{\rm D} = 8$ and $M_{\rm R} = 6$ or 8, and denote this tuple choice by $\langle M_{\rm S}, M_{\rm R}, M_{\rm D}\rangle$ in the Figure. At the destination, the MMSE equalizer matrix $\mathbf{K}$ is

$$\mathbf{K} = \mathbf{R_{xx}}\mathbf{H}^H(\mathbf{R_{ww}} + \mathbf{H}_{\rm RD}\mathbf{G}_{\rm M}\mathbf{R_{vv}} \cdot\mathbf{G}_{\rm M}^H\mathbf{H}_{\rm RD}^H + \mathbf{H}\mathbf{R_{xx}}\mathbf{H}^H)^{-1}$$

where $\mathbf{H} = \mathbf{H}_{\rm RD}\mathbf{G}_{\rm M}\mathbf{H}_{\rm SR}$. For our simulation parameters, the scheme of [6] selects 44 and 46 relays for $M_{\rm R}$ =6 and 8, respectively. For our OMP-based scheme in (17), $K_{\rm A} = 110$ and 150 for $M_{\rm R}$ =6 and 8, respectively, which results in the same average number of selected relays as GMM. It is clear that our scheme outperforms the GMM scheme in [6] because the latter selects only one antenna pair at each selected relay. Specifically, if the n-th relay is selected, the resultant gain matrix $\mathbf{G}_{\rm M}^n$ of the GMM algorithm contains only one non-zero element whose index corresponds to an antenna pair in the n-th relay. Hence, the maximum number of antenna pairs is limited by N out of the available $NM_{\rm R}^2$ antenna pairs in the GMM algorithm. In addition, all non-zero elements of $\mathbf{G}_{\rm M}$ are equal since no antenna gain optimization is performed under the overall power constraint. However, in our scheme, the resultant gain matrices contain multiple non-zero elements and their values are optimized under the overall power constraint. We emphasize that the relay indices selected by our scheme are different from those selected by the GMM algorithm.

Fig. 3 compares the BER of our OMP approach with the relay selection algorithms in [3], [6] and with those whic perform gain control in [11], [12] as a function of the number of selected relays or antenna pairs for N=100 SISO or N=50 MIMO relay networks, respectively. As explained earlier, adding a relay or antenna pair in OMP is achieved by adjusting the number of iterations in (9) or in (17), respectively. For our SISO simulation parameters, the scheme in [3] selects around 46 relays and its BER performance with the gain control scheme developed in [11] is achieved by OMP with only 42 selected relays. In the MIMO case, our OMP-based approach with only 42 antenna pairs at 29 relays achieves similar BER performance as the GMM algorithm with the gain control scheme developed in [12] which selects around 44 relays. Hence, by exploiting sparsity of the relay gain vector, our OMP-based approach achieves the same performance as state-of-the-art selection algorithms, enhanced with gain control optimization, while reducing the number of selected relays. This, in turn, reduces implementation complexity, AF protocol signaling overhead, and power consumption in the RF front-ends at the relays. Note that AF relays must buffer the received signal block in the first time slot until it is transmitted (after amplification) in the second time slot which is essential for synchronization. This buffering operation cannot be efficiently done in the analog domain. Instead, it is done digitally at baseband; hence the need for power-consuming down/up conversion operations even at AF relays.

VI. Conclusion

We proposed a new computationally-efficient approach, based on exploiting the sparsity of the relay antennas gain vector, for joint MMSE selection of relays and antennas with relay gain matrix optimization in large dual-hop MIMO AF relay networks. Simulation results show that our proposed scheme outperforms existing schemes under the same overall transmit power constraint. Alternatively, at the same BER, our scheme reduces the number of selected relays resulting in reduced implementation complexity.

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