May 5, 2003

Final Exam

EE 3302: Signals and Systems

NOTE: **Please, complete the following table and keep record of your assignment number.**

First Name	
Last Name	
Student ID	
Assignment #	0

Exercise 1. A Linear and Time Invariant (LTI) system has the input and output signals related by the differential equation

$$\frac{d\ y(t)}{dt} + 3\ y(t) = \frac{d\ x(t)}{dt} + 4\ x(t)$$

A) Determine the frequency response of the system [pt. 10], the unit impulse response of the system [pt. 10], the frequency response of the inverse system [pt. 5], and the unit impulse response of the inverse system [pt. 10].

Exercise 2. Consider the two continuous-time signals

$$x(t) = \sin(\omega_0 t)$$

$$y(t) = x(t)\ \cdot\ \cos(\omega_1 t)$$

where ω_0 and ω_1 are positive finite values. The following signals are sampled using a train of impulses with periodicity T, $\sum_{k=-\infty}^{+\infty} \delta(t - kT)$: signal $x(t)$ is sampled to obtain $x_c(t)$, signal $y(t)$ is sampled to obtain $y_c(t)$, and signal $x(t) \cdot y(t)$ is sampled to obtain $p_c(t)$.

A) Determine the range of values for T that allow complete recovery of $x(t)$ from $x_c(t)$ [pt. 5].

B) Determine the range of values for T that allow complete recovery of $y(t)$ from $y_c(t)$ [pt. 10].

C) Determine the range of values for T that allow complete recovery of $x(t) \cdot y(t)$ from $p_c(t)$ [pt. 15].

Exercise 3. Consider the continuous-time signal

$$x(t) = \mathrm{e}^{-5t}.$$

The discrete-time sequence $x[n]$ is obtained by sampling $x(t)$ at a rate of one sample every $T = 0.2$ s, *starting* with the first sample at $t = 0$ and continuing until ∞.

A) Derive an expression for $x[n]$ [pt. 10].

B) Evaluate the z-transform of $x[n]$ [pt. 20].

Exercise 4. A digital filter consists of a series of two Linear and Time Invariant (LTI) sub-systems. The z-transform of the unit impulse response of the first sub-system is

$$H_1(z) = \frac{z}{z - 1/2} \qquad |z| > 1/2.$$

The z-transform of the unit impulse response of the second sub-system is

$$H_2(z) = \frac{z}{z - 1} \qquad |z| > 1.$$

Let $h[n]$ be the unit impulse response of the entire digital filter.

A) Derive the z-transform of $h[n]$ [pt. 10].

B) Derive $h[n]$ [pt. 20].

Exercise 5. A discrete-time signal $x[n]$ has the following z-transform

$$X(z) = \frac{1}{1+3z^{-1}} \cdot \frac{1}{1+5z^{-1}} \qquad |z| > 5.$$

A) Derive the z-transform of $y[n] = x[n+3]$ [pt. 20].