May 1, 2006

Final Exam

EE 3302: Signals and Systems

NOTE: **Please, complete the following table and keep record of your assignment number.**

First Name	
Last Name	
Student ID	
Assignment #	0

Exercise 1. A system is described by the following differential equation

$$\frac{d^2\ y(t)}{dt^2} + 10\ \frac{d\ y(t)}{dt} + 21\ y(t) = \frac{d^2\ x(t)}{dt^2} + 8\ \frac{d\ x(t)}{dt} + 15\ x(t)$$

where $x(t)$ is the input signal, and $y(t)$ is the output signal. Assume that the initial rest condition is satisfied.

A) Determine the frequency response of the system [pt. 10].

B) Determine the unit impulse response of the system [pt. 10].

C) Determine the frequency response of the inverse system [pt. 5].

D) Determine the unit impulse response of the inverse system [pt. 10].

Exercise 2. Consider the continuous-time signal

$$x(t) = \frac{\sin(\omega_1 t)}{t}$$

where ω_1 is a positive finite value. Let $y(t) = x(t) \cdot \mathrm{e}^{-j\omega_0 t}$. where ω_0 is a positive finite value. The following signals are sampled using a train of impulses with periodicity T, $\sum_{k=-\infty}^{+\infty} \delta(t - kT)$: signal $x(t)$ is sampled to obtain $x_c(t)$, and signal $y(t)$ is sampled to obtain $y_c(t)$.

A) Determine the range of values for T that allows complete recovery of $x(t)$ from $x_c(t)$ [pt. 5].

B) Determine the range of values for T that allows complete recovery of $y(t)$ from $y_c(t)$ [pt. 10].

Exercise 3. Consider the two discrete-time sequences

$$x_1[n] = \alpha^{2n}\ u[n]$$

and

$$x_2[n] = -\beta^n\ u[-n-1],$$

where $u[n]$ is the causal unit step function, α and β are two positive real constants. A third signal is obtained using the convolution sum, e.g., $x[n] = x_1[n] * x_2[n]$.

A) Compute the z-transform of $x_1[n]$ [pt. 5].

B) Compute the z-transform of $x_2[n]$ [pt. 5].

C) Compute the z-transform of $x[n]$ [pt. 10].

D) Determine under what condition on α and β the z-transform of $x[n]$ exists [pt. 10].

E) Determine under what condition on α and β the discrete-time Fourier transform of $x[n]$ exists [pt. 5].

Exercise 4. Consider a discrete-time LTI system with unit impulse response $h[n]$. Let $x[n]$ and y[n] be the input and output signal, respectively. Let the z-transform of $h[n]$ be

$$H(z) = \frac{z}{z-1} \qquad |z| > 1.$$

Let the z-transform of $x[n]$ be

$$X(z) = 1 + z^{-1} \qquad |z| \neq 0.$$

A) Derive the z-transform of $y[n]$ [pt. 5].

B) Derive, sketch and label carefully $y[n]$ [pt. 15].

Exercise 5. A discrete-time signal $x[n]$ has the following z-transform

$$X(z) = \frac{1}{1+z^{-1}} \qquad |z| > 1.$$

A) Derive the z-transform of $y[n] = x^2[n]$ [pt. 15].