November 30, 2007

Final Exam

EE 3302: Signals and Systems

NOTE: **Please, complete the following table and keep record of your assignment number.**

First Name	
Last Name	
Student ID	
Assignment #	0

Exercise 1. A system is described by the following differential equation

$$\frac{d^2\ y(t)}{dt^2} + 8\ \frac{d\ y(t)}{dt} + 15\ y(t) = \frac{d^2\ x(t)}{dt^2} + 7\ \frac{d\ x(t)}{dt} + 10\ x(t)$$

where $x(t)$ is the input signal, and $y(t)$ is the output signal. Assume that the initial rest condition is satisfied.

A) Determine the frequency response of the system [pt. 10].

B) Determine the unit impulse response of the system [pt. 10].

C) Determine the frequency response of the inverse system [pt. 5].

D) Determine the unit impulse response of the inverse system [pt. 10].

Exercise 2. Consider the continuous-time signal

$$x(t) = \frac{\sin(\alpha t)}{t}$$

where α is a positive finite value. Let $y(t) = x^2(t)$. The following signals are sampled using a train of impulses with periodicity T, $\sum_{k=-\infty}^{+\infty} \delta(t - kT)$: signal $x(t)$ is sampled to obtain $x_c(t)$, and signal $y(t)$ is sampled to obtain $y_c(t)$.

A) Determine the range of values for T that allows complete recovery of $x(t)$ from $x_c(t)$ [pt. 10].

B) Determine the range of values for T that allows complete recovery of $y(t)$ from $y_c(t)$ [pt. 10].

Exercise 3. Consider the two discrete-time sequences

$$x_1[n] = 2^n u[n+1] \qquad \text{and} \qquad x_2[n] = \left(\frac{1}{3}\right)^n u[-n]$$

where $u[n]$ is the causal unit step function. A third signal is obtained using the convolution sum, e.g., $x[n] = x_1[n] * x_2[n]$.

A) Compute the z-transform of $x_1[n]$ [pt. 10].

B) Compute the z-transform of $x_2[n]$ [pt. 10].

C) Compute the z-transform of $x[n]$ [pt. 10].

D) Does the Fourier transform of $x[n]$ converge? [pt. 5].

Exercise 4. The algebraic part of the z-transform of a discrete-time signal $x[n]$ is

$$X(z) = \frac{1}{(1+2jz^{-1})^2}$$

The region of convergence (RoC) is not explicitly given, but it is known that point $z_1 = -5$ belongs to the RoC.

A) Determine the RoC of $X(z)$ from the information available [pt. 10].

B) Derive $x[n]$ using the RoC found in **A)** [pt. 15].

C) Does the Fourier transform of $x[n]$ converge? [pt. 5].

Exercise 5. Consider a discrete-time LTI system with unit impulse response $h[n] = \delta[n] - \delta[n-1]$, where $\delta[n]$ is the unit impulse function. Let $x[n]$ and $y[n]$ be the input and output signal, respectively. Let the z-transform of $x[n]$ be

$$X(z) = \frac{1}{z(z^2 - \frac{2}{3}z - \frac{1}{3})} \qquad |z| > 1$$

A) Derive the z-transform of $y[n]$ [pt. 15].

B) Derive, sketch and label carefully $y[n]$ [pt. 15].