December 16, 2013
Final Exam
CE/EE/TE 3302: Signals and Systems

NOTE: Please, complete the following table and keep record of your assignment number.

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Exercise 1. A system is described by the following differential equation
\[ \frac{d^2 y(t)}{dt^2} + 11 \frac{d y(t)}{dt} + 10 y(t) = \frac{d^2 x(t)}{dt^2} + 16 \frac{d x(t)}{dt} + 15 x(t) \]
where \( x(t) \) is the input signal, and \( y(t) \) is the output signal. Assume that the initial rest condition is satisfied.
A) Determine the frequency response of the system, i.e., \( H(j\omega) \) [pt. 10].
B) Determine the unit impulse response of the system, i.e., \( h(t) \) [pt. 10].
C) Determine the frequency response of the inverse system, i.e., \( G(j\omega) \) [pt. 5].
D) Determine the unit impulse response of the inverse system, i.e., \( g(t) \) [pt. 10].

Exercise 2. Consider the continuous-time signal
\[ x(t) = \frac{\sin(t - 5)}{t - 5} \]
Let \( y(t) = x^2(t) \). The following signals are sampled using a train of impulses with periodicity \( T \), \( \sum_{k=-\infty}^{+\infty} \delta(t - kT) \): signal \( x(t) \) is sampled to obtain \( x_s(t) \), and signal \( y(t) \) is sampled to obtain \( y_s(t) \).
A) Determine the range of values for \( T = T_x \) that allows complete recovery of \( x(t) \) from \( x_s(t) \) [pt. 10].
B) Determine the range of values for \( T = T_y \) that allows complete recovery of \( y(t) \) from \( y_s(t) \) [pt. 10].

Exercise 3. Consider the discrete-time sequence
\[ x[n] = n a^{n+2} u[n + 1] \]
where \( u[n] \) is the causal unit step function, and \( a \) is a constant value.
A) Compute the z-transform of \( x[n] \) [pt. 15].
B) Under what condition on \( a \) does the Fourier transform of \( x[n] \) converge? [pt. 10].

Exercise 4. The Fourier transform of a discrete-time signal \( x[n] \) is
\[ X(e^{j\omega}) = \frac{e^{-j\omega}}{1 + 4e^{-j2\omega}} \]
A) Derive the z-transform of \( x[n] \) [pt. 15]. (Hint: when deriving the RoC of the z-transform make sure that the condition for the existence of the Fourier transform is met.)
B) Derive the expression for $x[n]$ [pt. 15].

**Exercise 5.** Consider the discrete-time LTI system with the output signal defined as

$$g[n] = x[n] - x[n + 1]$$

where $x[n]$ is the signal at the system input. Let $h[n]$ be the unit impulse response of the LTI system. Let $g[n]$ be the unit impulse response of the inverse system of $h[n]$, that is $h[n] * g[n] = \delta[n]$, where $\delta[n]$ is the unit impulse.

A) Compute $h[n]$ [pt. 10].

B) Compute the z-transform of $h[n]$ [pt. 15].

C) Compute the z-transform of $g[n]$ [pt. 15]. (Hint: note that the sequence $g[n]$ is left-sided.)

D) Compute $g[n]$ [pt. 15]. (Hint: verify the correctness of your solution by showing that $h[n] * g[n] = \delta[n]$.)