

April 26, 2017

Final Exam

# EE/TE 4367: Telecommunications Networks

NOTE: Please, complete the following table and keep record of your assignment number.

First Name	
Last Name	
Student ID	
Assignment #	0

**Exercise 1.** Representing all the relevant intermediate steps, find a minimum weight spanning tree on the

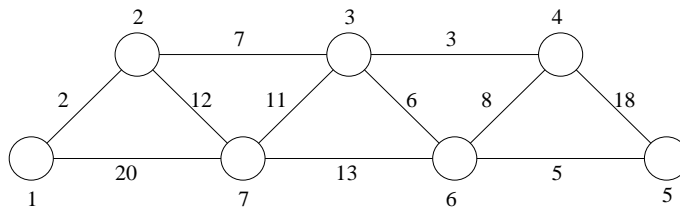


Figure 1: Undirected graph with 7 nodes and 8 edges.

graph shown in Fig. 1 using, respectively,

- A) the Prim-Dijkstra algorithm choosing node 3 as the root vertex [pt. 10],
- B) the Kruskal algorithm [pt. 10].

**Exercise 2.** Consider the graph shown in Fig. 2. Note that every arc in the figure represents two distinct

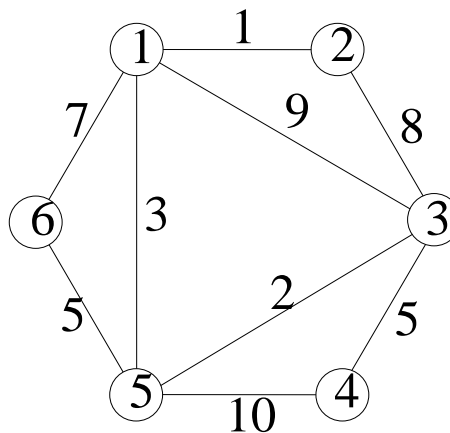


Figure 2: Undirected graph with 6 nodes and 9 edges.

directed arcs with opposite directions and same weight (shown next to the arc). Using a graphical or matrix based representation of each intermediate iteration, find the shortest path from every node to node 1 as indicated below.

- A) Run the first two iterations of the Dijkstra algorithm and show the path costs at the end of the second iteration [pt. 15].
- B) Identify the paths found at the end of the second iteration [pt. 10].

- C) Continue to run iterations until a stop condition is reached, reporting the paths found and their respective costs [pt. 15].

**Exercise 3.** Consider the open network of two queues  $Q1$  and  $Q2$ . Each queue has a single server. The service times at the queues are independent and exponentially distributed with mean  $1/\mu_1$  and  $1/\mu_2$ , respectively. Customers (or jobs) entering the network form a Poisson arrival process with rate  $\lambda$ . Upon entering the network of queues, a job chooses to enter  $Q1$  with probability  $p$ , or  $Q2$  with probability  $1 - p$ . Jobs leaving  $Q1$  will choose to either re-enter  $Q1$  (with probability  $q_1$ ) or depart from the network for good (with probability  $1 - q_1$ ). Jobs leaving  $Q2$  will choose to either re-enter  $Q2$  (with probability  $q_2$ ) or depart from the network for good (with probability  $1 - q_2$ ).

- A) Find the stability conditions of the network of queues [pt. 10].
- B) Find  $N$ , defined as the average number of jobs in the entire network of queues at steady state [pt. 15].
- C) Find  $T$ , defined as the average time spent in the network of queues by a generic job [pt. 15].
- D) Define  $\hat{T}_1$  as the average total time spent in the network of queues by a job choosing to go through  $Q1$ . Define  $\hat{T}_2$  as the average total time spent in the network of queues by a job choosing to go through  $Q2$ . Find  $p_T$ , defined as the value of  $p$  which guarantees a fair system, i.e.,  $\hat{T}_1 = \hat{T}_2$ . Note that  $p_T$  must be a probability and there may be additional constraints on  $\lambda$  (beside those required for stability) for the solution to exist. Report any such additional constraint(s) on  $\lambda$ . [pt. 15].
- E) Assuming now that  $p = p_T$  and  $q_1 = q_2 = q$ , derive the stability conditions for the network of queues (answer must contain only  $q$ ,  $\lambda$ ,  $\mu_1$  and  $\mu_2$ ). Report any additional constraint(s) on  $\lambda$  for the solution to exist [hint: recall that  $p_T$  must be a probability] [pt. 15].