April 26, 2006
Final Exam
EE 6340: Introduction to Telecommunications Networks

NOTE: Please, complete the following table and keep record of your assignment number.

| First Name |  |
| Last Name |  |
| Student ID |  |
| Assignment # | 0 |

Exercise 1. Representing all the relevant intermediate steps, find a minimum weight spanning tree on the graph shown in Fig. 1 using, respectively,

A) the Prim-Dijkstra algorithm choosing node 1 as the root vertex [pt. 10],

B) the Kruskal algorithm [pt. 10].

C) Identify manually the minimum weight tree connecting nodes 1, 2, 4, and 5 [pt. 10].

D) Design a polynomial algorithm that finds the tree connecting nodes 1, 2, 4, and 5, with the lowest possible weight (possibly, the same weight found in C)] [pt. 10].

Exercise 2. Consider the flow network shown in Fig. 2. The label on the link indicates the capacity of the link.

A) Using a graphical based representation of each intermediate iteration, find the maximum flow in the flow network from node 1 to 8. Use a shortest path approach when finding the augmenting path [pt. 20].

B) Determine the maximum capacity of the flow [pt. 5].

C) Determine all the minimum cuts between node 1 and 8 [pt. 10].
Exercise 3. Consider a two-server system with exponential service time, in which server 1 has service rate \( \mu_1 \), and server 2 has service rate \( \mu_2 \). The arrival process is Poisson with rate \( \lambda \). An arriving job has two options: with probability \( p_1 \) it chooses server 1, and with probability \( p_2 = 1 - p_1 \) it chooses server 2. If the chosen server is free, service begins immediately. If the chosen server is busy, the arrival is blocked and discarded.

A) Build the Markov chain of the queue and determine the stability condition [pt. 10].
B) Derive the steady state probabilities of the Markov chain [pt. 10].
C) Derive \( P_b \), defined as the blocking probability of a generic arrival [pt. 10].
D) Derive \( N \), defined as the expected number of jobs in the system [pt. 10].
E) Derive \( T \), defined as the expected sojourn time in the system [pt. 10].

Exercise 4. \( K \) transmitters share the same radio channel using time division multiple access. Time is divided to form time frames of equal duration. Each frame contains \( K \) time slots. The time slot duration is \( d \). Node \( i \) can use slot \( i \) of each frame to transmit one packet. Assume that at node \( i \), packets have constant length to fit the slot transmission time and are generated according to a Poisson arrival process with rate \( \lambda \). Assume that the transmission buffer is unlimited.

A) Determine the queueing model for the described system, and determine the stability condition [pt. 10].
B) Derive \( W \), defined as the waiting time of a packet in the queue till its transmission begins [pt. 20].
C) Derive \( T \), defined as the time interval between the moment the packet is generated and the moment the last bit of that packet is transmitted [pt. 10].
D) Derive \( N \), defined as the average number of packets at the transmitter, including the one (if any) in transmission [pt. 10].