May 8, 2002

Final Exam

EE 6340: Introduction to Telecommunications Networks

NOTE: **Please, complete the following table and keep record of your assignment number.**

First Name	
Last Name	
Student ID	
Assignment #	0

Exercise 1. Consider a dual-processor router in which two processors, processor 1 and processor 2, are used to process the incoming packets. Each processor has its own queue, hypothetically with ∞ capacity. A packet dispenser at the router input, randomly determines which processor will process the incoming packet. With probability p, processor 1 will be chosen. With probability $1-p$, processor 2 will be chosen. Assume that packets arrive at a Poisson rate of λ, and the processing time of the processors is exponentially distributed with rate, respectively, μ_1 and μ_2. In general, $\mu_1 \neq \mu_2$.

A) Choose the appropriate queue (or network of queues) to model the system and determine stability conditions [pt. 5].

B) Compute the expected time spent in the system (including service time) by a generic packet as a function of p, i.e., $T(p)$ [pt. 10].

C) Assume that $\mu_1 + \mu_2 = \mu$, where μ is a constant value. Find the value of p that will maximize λ_s, where λ_s is the maximum arrival rate that does not create instability [pt. 5]. Let p_m be such value. Compare $T(p_m)$ with the expected time spent by a packet in a M/M/1 with service rate μ [pt. 5].

Exercise 2. Consider a M/G/1/1 queue with Poisson arrival rate λ, and expected service time $\bar{X}$.

A) Determine the stability conditions of the queue [pt. 5].

B) Derive the blocking probability, P_b [pt. 10]. (A packet is blocked upon arrival when it cannot be stored in the queue due to lack of space.)

C) Derive the probability that the server is not busy [pt. 10].

D) Compute the expected time spent by a customer in the system (including service time) [pt. 5].

Exercise 3. Consider a M/G/1 queue that is used to model a transmission system in which packets are transmitted only after a reservation is made. Packets arrive at the queue according to a Poisson process with rate λ. Time is divided to form cycles. A cycle consists of a *reservation interval* followed by a *transmission interval*. The reservation interval lasts V seconds, where V is a random variable with average $\bar{V}$ and second moment $\bar{V^2}$. During the transmission interval packets whose reservation was made during the preceding reservation interval are transmitted. Transmission time of each packet is a random variable, X, with average $\bar{X}$ and second moment $\bar{X^2}$. When no packets are there to transmit, the transmission time in that cycle is zero, and a new reservation interval will start immediately.

The following rule is used to determine which packets are scheduled for transmission in each cycle: only packets that are already in the queue at the time the cycle begins will be reserved to be transmitted during that cycle. Packets that arrive during the cycle will be transmitted during the next cycle.

Notes: 1) the residual time, R, should include both the residual service time and the residual reservation time; 2) in the described system every packet has to wait in queue at least for a full reservation interval (on the contrary, in the M/G/1 with vacation model, only packets arriving while the server is on vacation are forced to wait till the server comes back from vacation).

A) Derive the expected waiting time for the generic packet, W [pt. 15].

B) Compute the expected cycle length, e.g., reservation time plus transmission time [pt. 10].

C) Derive the expected waiting time for the generic packet when the reservation time is deterministic and equal to A [pt. 10].

D) Compute the value of $\rho = \lambda\bar{X}$, such that $W = 3\bar{X}$, assuming that both reservation time and transmission time are deterministic and with same value , i.e., $X = A$ [pt. 5].

Exercise 4. Find a minimum weight spanning tree of the graph in Fig. 1 (representing all necessary

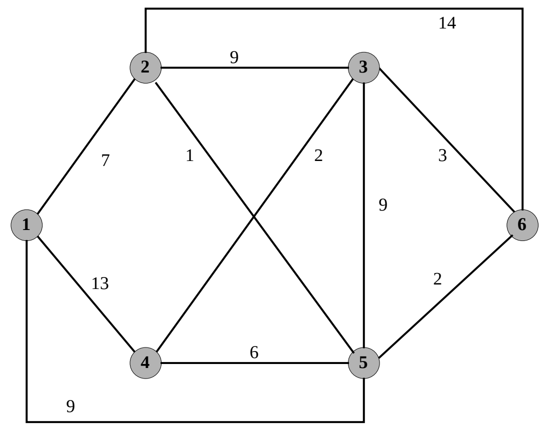

Figure 1: Undirected Graph with six vertices and eleven undirected edges.

intermediate steps) using, respectively,

A) the Prim-Dijkstra algorithm using vertex 1 as the root vertex [pt. 10],

B) the Kruskal algorithm [pt. 10].

Exercise 5. Consider the graph shown in Fig. 2. Using a graphical or matrix based representation of each

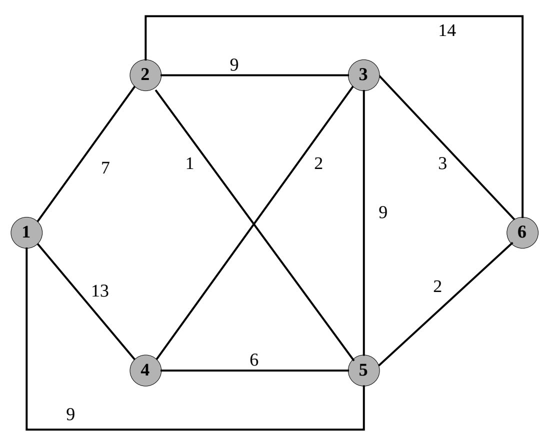

Figure 2: Undirected Graph with six vertices and eleven undirected edges.

intermediate iteration, find the shortest path from any node to node 1 by applying:

A) the Bellman-Ford algorithm [pt. 15],

B) the Dijkstra algorithm [pt. 15].

Exercise 6. Apply the Floyd-Warshall algorithm to the network shown in Fig. 3. Let $D^k_{i,j}$ be the matrix of the (temporary) distances (path lengths) from node i to node j at step k of the algorithm. Let $\Pi^k_{i,j}$ be the predecessor matrix for the path from node i to node j at step k of the algorithm.

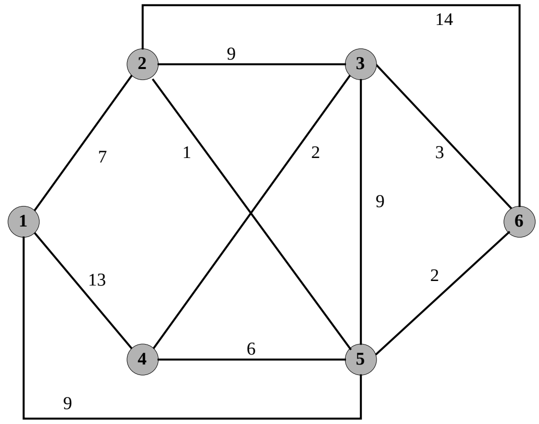

Figure 3: Undirected Graph with six vertices and eleven undirected edges.

A) Show the values of matrix $D^k_{i,j}$ at each iteration k of the algorithm for $k = 0, 1, 2, 3$ [pt. 15].

B) Show the values of matrix $\Pi^k_{i,j}$ at each iteration k of the algorithm for $k = 0, 1, 2, 3$ [pt. 10].

C) Print the paths found at iteration $k = 3$ from node 1 to every other node, with their respective length values [pt. 5]. (Hint: to print the paths apply the following algorithm to $\Pi^3_{i,j}$:

PRINT-ALL-PAIRS-SHORTEST-PATH ($\Pi_{i,j}, i, j$)
  if $i = j$
    then print i
    else if $\pi_{i,j}$ =NIL
      then print "no path from i to j"
      else PRINT-ALL-PAIRS-SHORTEST-PATH ($\Pi_{i,j}, i, \pi_{i,j}$)
        print j

where $\pi_{i,j}$ is the element in position (i, j) of the matrix $\Pi_{i,j}$.)