LIFETIME MAXIMIZATION AND RESOURCE MANAGEMENT IN WIRELESS SENSOR NETWORKS

by

Frank (Farhad) Azadi Namin

APPROVED BY SUPERVISORY COMMITTEE:

Dr. Aria Nosratinia, Chair

Dr. Naofal Al-Dhahir

Dr. John P. Fonseka

To Behnam, Farzaneh, and Amir

LIFETIME MAXIMIZATION AND RESOURCE MANAGEMENT IN WIRELESS SENSOR NETWORKS

by

FRANK (FARHAD) AZADI NAMIN, B.S.E.E.

THESIS

Presented to the Faculty of

The University of Texas at Dallas

in Partial Fulfillment

of the Requirements

for the Degree of

MASTER OF SCIENCE IN ELECTRICAL ENGINEERING

THE UNIVERSITY OF TEXAS AT DALLAS

August, 2008

ACKNOWLEDGEMENTS

First and foremost I would like to give my deepest and most sincere gratitude to my parents and my dear brother, to whom this thesis is dedicated. Without their love, support, and encouragement this would not have been possible.

I would particularly like to thank my advisor, Professor Aria Nosratinia for his support and commitment. Over the past two years he was greatly instrumental in writing this thesis. I am very grateful for all his help. I would also like to thank my supervisory committee members, Professor Naofal Al-Dhahir and Professor John Fonseksa for their time and great input.

Last but not least, I would like to acknowledge my current and former colleagues, Ali Tajer, Negar Bazargani, Ramy Tannious, Mohammad AbolSoud, and Annie Le. I also have to thank my best friends Abteen Vaziri and Dariush Ferdows for their kindness and support.

July 2008

LIFETIME MAXIMIZATION AND RESOURCE MANAGEMENT IN WIRELESS SENSOR NETWORKS

Publication No. ____________________

Frank (Farhad) Azadi Namin, M.S.E.E.
The University of Texas at Dallas, 2008

Supervising Professor: Dr. Aria Nosratinia

A primary concern in the operation of cooperative wireless sensor networks is the issue of energy efficiency and lifetime maximization. This thesis addresses the problem of lifetime maximization under unequal and time-varying channel conditions and subject to a distortion constraints. The standard method for solving such dynamic stochastic problems is dynamic programming, which is a discrete method that must heavily quantize all quantities, is exponentially complex with respect to the number of states, and requires global information exchange among sensors at each iteration. Our goal in this thesis is to develop a practical, low-complexity solution which does not have the downfalls of the previous methods.

Three signalling schemes are considered in the context of joint estimation: orthogonal channels, beamforming, and shared channel with no phase information. In the final chapter the problem of binary detection in sensor networks is studied.

In the case of orthogonal channels, the distortion constraint is relaxed. We propose a simplified method via a decomposition approximation: The SNR requirement at the destination

is “divided” between sensors according to their battery powers and radio link statistics, and then each of the sensors’ operating power is carefully controlled over time to maximize the lifetime as well as maintain a certain required SNR at the receiver.

For the other two channel configurations, we consider the special case in which channels coefficients are independently, identically distributed. We argue that under these conditions a power scheduling scheme that minimizes the power consumption over any transmission period, maximizes the expected lifetime of the network. A closed from optimal solution is obtained for shared channels with phase information. In the case of shared channel with no phase information, the original problem is reduced to a standard linear programming problem and a closed form solution is obtained.

We also consider the problem of binary hypothesis detection in wireless sensor networks under power constraint. The objective is to solve the resource allocation problem for this distributed detection problem and find a suitable operating point for the network. We use the Neyman-Pearson (NP) criteria and design suitable transmission schemes and a fusion center detector, so that subject to transmission power constraints, detection probability is optimized. It is shown that the corresponding optimization problem is convex, optimality conditions are derived, and solutions for transmit powers as well as overall detector are obtained. Numerical simulations verify our results.

TABLE OF CONTENTS

LIST OF FIGURES

CHAPTER 1

INTRODUCTION

In this thesis we study problems related to the lifetime of wireless sensor networks. The first part of the thesis (Chapters 2, 3, and 4) discusses the lifetime maximization of sensor networks that jointly measure and estimate a physical parameter. In the latter part of the thesis, we discuss sensor networks whose task is to test a binary hypothesis (detection) based on a set of physical measurements.

Typically a wireless sensor network consists of a fusion center and a large number of inhomogeneously placed sensors. The goal of the sensors is to cooperatively monitor some physical or environmental quantity. Each sensor in the network takes a noise corrupted measurement of the unknown quantity; these measurements are then sent to the fusion center where an estimate is produced. A major challenge for these sensors is that they have limited and non-renewable energy sources. Thus energy efficiency is a major concern, since it allows us to maximize the lifetime of the network.

There are several definitions of the network lifetime. Lifetime can be defined as the time until the first sensor runs out of energy [1]. Others have used definitions that include fractions of surviving sensors [2]. We simply define the lifetime as the period in which the network can perform its desired task with acceptable quality. Chen and Zhao [3] derive a general formula for the lifetime of wireless sensor networks. They use the max-min approach to exploit the state information of the network to maximize the minimum residual energy across the network.

Most of the past work in this area concentrates on system-wide energy efficiency. Cui et al. [4] considers a static network and finds an optimal water-filling solution that minimizes the total energy expenditure of the network. Xiao et al. [5] considers the same problem except their observations are quantized into discrete messages and then transmitted to the fusion center. Optimal quantization levels and transmit power levels are determined. This class of solutions is generally known as minimum total energy (MTE). In MTE solutions, sensors with bad channel conditions will not transmit. Since each sensor has its own battery, this might not be a good solution. Chen et al. [6] considers a dynamic network and uses dynamic programming techniques to maximize the network lifetime. But in order to formulate a pragmatic dynamic programming approach, channel gains and battery powers must be heavily quantized, which introduces errors. Since channel gains and battery powers form part of the state vector, the complexity of dynamic programming is exponentially related to this quantization, as well as the number of sensors, which can be expressed as $\mathcal{O}(M^E Q^M)$, where Q is the number of quantized channel levels, M is the number of sensors, and E is the number of possible values for the residual energy of a sensor. Thus, for a fairly small network consisting of 20 sensors, with 50 possible energy levels and 5 quantized channel levels, the computation complexity is on the order 10^{79}. Another drawback of both the MTE and dynamic programming algorithms is that in each transmission block, each sensor has to know (in principle) all channel conditions, including those of other sensors'. This leads to a large communications overhead that is hard to implement in practice. Shu et al. [7] use fuzzy logic systems to analyze the lifetime problem. They show that a type-2 fuzzy membership function is the best model for a single node lifetime in wireless sensor networks.

There has also been a considerable amount of work on sensor network lifetime in the context of network routing. Many of these problems consider a multi-hop path between the sensors and the fusion sensor and try to find the optimum path to maximize the network lifetime. Zhu and Papavassiliou [8] present an analytical model to estimate and evaluate the node and network lifetime. Chang and Tassiulas [9] express the problem of the routing decision as a linear programming problem and show that MTE solutions are not necessarily optimal for routing problems. Ma and Aylor [10] consider a set of heterogenous nodes, in which sensors have different radio capacity, computation power, and some are mobile. They propose a protocol to build an optimal network topology. Madan and Lall [11] formulate the problem of optimal network routing as a linear programming problem. They propose a low complexity distributed subgradient algorithm to solve the problem. Gatzianas, and Georgiadis [12] consider a multi-hop network with a mobile sink. They propose a distributed algorithm based on the subgradient method to solve the problem. A fundamental feature of problems that attempt to maximize the lifetime in the network routing context is their definition of lifetime. In these problems once the first node runs out of power the network will partition and that is considered the end of lifetime.

We consider the problem of lifetime maximization for fusion center based wireless sensor networks under three different channel configurations. In each case, our goal is to maximize the number of transmissions while achieving an acceptable quality. The quality constraint is given as a minimum SNR requirement at the fusion center. We assume that our signal of interest θ is zero-mean Gaussian with unit variance. The channels experience quasi-static Rayleigh fading, and the noises are complex Gaussian distributed. Each sensor has an initial energy of $\mathcal{E}$. Let h_i denote the i-th channel coefficient between the sensor and the fusion center. We assume that $|h_i|$ is distributed according to:

$$f(|h_i|) = \frac{|h_i| e^{\frac{-|h_i|^2}{2\sigma_{hi}^2}}}{\sigma_{hi}^2} \tag{1.1}$$

where σ_{hi}^2 is known to us.

In orthogonal channels, we take advantage of the fact that the total SNR at the fusion center is the sum of per-sensor SNRs. To make the problem tractable, we relax our quality requirement from instantaneous SNR at receiver to an average SNR over time. Thus the network no longer can guarantee the quality of the calculated value at the fusion center in each transmission, but the average quality across time is guaranteed. This allows us to decompose the joint optimization problem into a series of independent optimization problems across sensors. Thus the optimization is simpler, but the quality of estimates is not as tightly guaranteed. This tradeoff may be necessary in sensor networks that are poor in computational ability. In many applications the average SNR criterion may be sufficient, including applications where the successive temporal estimates at the fusion center are averaged for better accuracy. If several estimates are averaged, then there is no sense in enforcing a strict quality constraint for each estimate, rather, one can enforce an average quality constraint.

Next we consider a shared channel where the fusion center has the phase information of all the incoming signals. In this case the sensors can transmit the signals to combine coherently at the receiver, i.e., beamforming. Unlike orthogonal channels, in this case the total SNR cannot be divided among sensors. We consider a special case in which all the channels are statistically identical. Having made this assumption, and also assuming that the lifetime of the network is large enough for the law of large numbers to take effect, we argue that a power scheduling scheme that minimizes the sum power at each transmission period maximizes the lifetime of the network. We then show that the problem can be converted into a convex optimization, for which obtain a closed form solution.

We then consider the case where the fusion center has no phase information for the incoming signals, thus they add incoherently. Similar to the previous case, we consider statistically identical channels. We show that the problem is equivalent to a convex optimization, and then using further manipulations show that it is a standard linear programming problem that can be solved efficiently. Although in general linear programming problems do not have closed form solutions, thankfully in this case a closed form solution has been obtained.

In the final chapter, we consider the problem of distributed detection under communication constraints in the context of sensor networks. A brief history of past work in this area is as follows. Chamberland and Veeravalli [13] consider decentralized detection with a capacity constraint over multiple access channels, showing that binary sensors are asymptotically optimal. In [14] they consider power constrained networks and provide asymptotic theoretical results, while we concentrate on practical operation of finite-size networks. Other works at first seem to hint at the same issues, e.g. [15] mentions optimal transmit powers, but only looks at network connectivity and does not address detection probability. To the best of our knowledge the problem of resource allocation to optimize distributed detection under power constraints, in the manner we describe, has to date remained open. We also mention that a significant body of work considers censoring sensors, where the constraint is on the number of transmitting sensors (degrees of freedom of the communication channel) [16], [17], and [18]. The overview paper of Chen et al. [19] is also noteworthy as a useful and recommended resource.

We consider a network of nodes whose observations are conditional on a certain hypothesis, which we desire to detect in a fusion center. The communication of sensors to the fusion center is hampered by noise, and is constrained by their sum-power. This constraint is motivated on the one hand by the low power sensors, and on the other hand by the interference footprint of the network. Each of the nodes calculates and transmits a signal to the base station to indicate its own likelihood function. We find individual transmission powers, with a knowledge of the channel gains and individual observation noise variances, so that subject to an overall power constraint the best detection probability is achieved. The objective of this chapter is therefore to provide a practical method to find the operating point of such a network for best performance.

CHAPTER 2

ORTHOGONAL CHANNELS

2.1 System Model

The received signal at the destination from the i-th sensor is:

$$y_i = h_i w_i(\theta + n_i) + n_{id} \quad i = 1, 2, \ldots, M$$

where w_i is the amplification gain for the i-th sensor which is related to transmit power by $p_i = w_i^2(1 + \sigma_i^2)$ and $n_{id} \sim \mathcal{CN}(0, \sigma_{id}^2)$ is the destination noise. Thus:

$$\mathbf{y} = \mathbf{H}\theta + \widetilde{\mathbf{n}}$$

where $\mathbf{y} = [y_1 \ldots y_M]^T$, $\mathbf{H} = [h_1 w_1 \ldots h_M w_M]^T$, and

$$\widetilde{\mathbf{n}} \sim \mathcal{N}\left(0, diag\left(|h_1|^2 w_1^2 \sigma_1^2 + \sigma_{1d}^2, \ldots, |h_M|^2 w_M^2 \sigma_M^2 + \sigma_{Md}^2\right)\right).$$

Now the linear minimum mean square estimator (LMMSE) of θ can be written as [20]:

$$\hat{\theta} = \mathbf{H}^T \left(\mathbf{H}\mathbf{H}^T + diag\left(|h_1|^2 w_1^2 \sigma_1^2 + \sigma_{1d}^2, \ldots, |h_M|^2 w_M^2 \sigma_M^2 + \sigma_{Md}^2\right)\right)^{-1} \mathbf{y} \tag{2.1}$$

The mean square error of the LMMSE is given by [20]:

$$\begin{aligned} \bar{D} &= \left(1 + \mathbf{H}^T \mathbf{C}_{\tilde{n}}^{-1} \mathbf{H}\right)^{-1} \\ &= \left(1 + \sum_{i=1}^{M} \frac{w_i^2 |h_i|^2}{w_i^2 \sigma_i^2 |h_i|^2 + \sigma_{id}^2}\right)^{-1} \\ &= \left(1 + \sum_{i=1}^{M} \left(\sigma_i^2 + \frac{(1+\sigma_i^2)\sigma_{id}^2}{p_i |h_i|^2}\right)^{-1}\right)^{-1} \\ &= \left(1 + \sum_{i=1}^{M} \left(\frac{p_i |h_i|^2}{\sigma_i^2 p_i |h_i|^2 + (1+\sigma_i^2)\sigma_{id}^2}\right)\right)^{-1} \end{aligned} \tag{2.2}$$

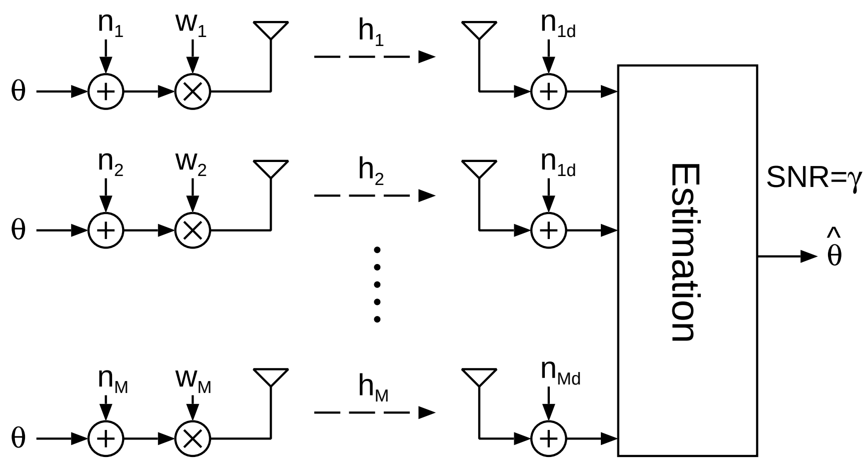

Figure 2.1. Network model

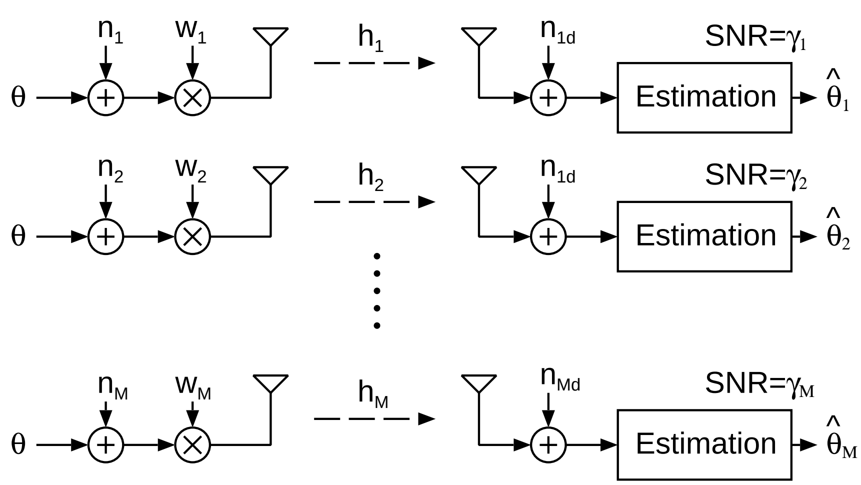

Figure 2.2. Network decomposition for the lifetime problem.

Our goal is to operate the network so that the mean error is no worse than a given value D. So

$$\left(1+\sum_{i=1}^{M}\left(\frac{p_i|h_i|^2}{\sigma_i^2 p_i|h_i|^2+(1+\sigma_i^2)\sigma_{id}^2}\right)\right)^{-1} \le D$$

$$\sum_{i=1}^{M}\left(\frac{p_i|h_i|^2}{\sigma_i^2 p_i|h_i|^2+(1+\sigma_i^2)\sigma_{id}^2}\right) \ge D^{-1}-1 \tag{2.3}$$

Let $D^{-1}-1 \equiv \gamma$. The i-th term in the summation of equation (2.3) corresponds to the SNR at the fusion center due to i-th sensor. We shall use this structure of the SNR equation to decompose and simplify the optimization problem.

2.2 Power Scheduling

Suppose we measure the channel conditions every T seconds. We assume that N is the number of transmissions before the network runs out of energy. So network lifetime is NT seconds. For simplicity we set $T=1$. So we can write the problem as :

$$\max \; E[N] \tag{2.4}$$

$$\text{s.t.} \quad \left[\sum_{i=1}^{M}\frac{p_{ij}|h_{ij}|^2}{\sigma_i^2 p_{ij}|h_{ij}|^2+(1+\sigma_i^2)\sigma_{id}^2}\right] \ge \gamma \qquad j=1,\ldots,N$$

$$p_{ij} \ge 0 \qquad \forall i,j$$

$$\sum_{j=1}^{N} p_{ij} \le \mathcal{E} \qquad i=1,\ldots,M$$

where $|h_{ij}|$ is the channel coefficient for the i-th channel during the j-th transmission period and p_{ij} is the corresponding transmission power. Now we slightly modify the problem by changing the first constraint from a hard constraint to an expected value. We also know that at optimality the fourth constraint is active. We can approximate the SNR constraint by twice using the weak law of large numbers [21].

$$\sum_{j=1}^{N}\sum_{i=1}^{M}\frac{p_{ij}|h_{ij}|^2}{\sigma_i^2 p_{ij}|h_{ij}|^2+(1+\sigma_i^2)\sigma_{id}^2} \ge \gamma N$$

$$\sum_{i=1}^{M} NE\left[\frac{p_i|h_i|^2}{\sigma_i^2 p_i|h_i|^2+(1+\sigma_i^2)\sigma_{id}^2}\right] \geq \gamma N$$

where the expected value is the average SNR due to the i-th channel at the fusion center over the lifetime of the network. Define

$$\gamma_i \triangleq E\left[\frac{p_i|h_i|^2}{\sigma_i^2 p_i|h_i|^2+(1+\sigma_i^2)\sigma_{id}^2}\right] \tag{2.5}$$

The problem can now be expressed as M separate and independent optimizations. Our goal is to maximize the lifetime of each sensor such that over its lifetime it provides an average SNR of γ_m with $\sum_{m=1}^{M}\gamma_m = \gamma$. Assuming an initial energy of $\mathcal{E}$, since each sensor has its own energy supply, maximizing the lifetime is equivalent to minimizing the power consumption. So we have M convex optimization problems.

$$\begin{aligned} &\min \sum_{j=1}^{N} p_{ij} \\ &s.t.\ E\left[\frac{p_{ij}|h_{ij}|^2}{\sigma_i^2 p_{ij}|h_{ij}|^2+(1+\sigma_i^2)\sigma_{id}^2}\right] \geq \gamma_i \qquad j=1,\dots,N \\ &p_{ij} \geq 0 \qquad \forall j \end{aligned}$$

The last constraint was eliminated since it has no bearing on the optimal solution. However it will be used later to determine the expected lifetime of the network. Using the weak law of large numbers [21] we can rewrite the problem.

$$\begin{aligned} &\min \sum_{j=1}^{N} p_{ij} \\ &s.t.\ \sum_{j=1}^{N}\left[\frac{p_{ij}|h_{ij}|^2}{\sigma_i^2 p_{ij}|h_{ij}|^2+(1+\sigma_i^2)\sigma_{id}^2}\right] \geq \gamma_i N \\ &p_{ij} \geq 0 \qquad \forall j \end{aligned} \tag{2.6}$$

The Lagrangian $\mathcal{L}$ of (2.6) can be written as:

$$\mathcal{L}(\mathbf{p},\lambda,\nu) = \sum_{j=1}^{N} p_{ij} - \sum_{j=1}^{N}\lambda_j p_{ij} + \nu_i\left(\gamma_i N - \sum_{j=1}^{N}\left(\frac{p_{ij}|h_{ij}|^2}{\sigma_i^2 p_{ij}|h_{ij}|^2+(1+\sigma_i^2)\sigma_{id}^2}\right)\right)$$

The Karush-Kuhn-Tucker (KKT) conditions [22] for the problem are given by:

1. Primary

$$\sum_{j=1}^{N}\left[\frac{p_{ij}|h_{ij}|^2}{\sigma_i^2 p_{ij}|h_{ij}|^2+(1+\sigma_i^2)\sigma_{id}^2}\right]\geq \gamma_i N$$

$$p_{ij}\geq 0 \qquad \forall j$$

2. Dual

$$\nu_i \geq 0$$

$$\lambda_j \geq 0 \qquad \forall j$$

3. Complementary Slackness

$$\nu_i\left(\gamma_i N-\sum_{j=1}^{N}\left(\frac{p_{ij}|h_{ij}|^2}{\sigma_i^2 p_{ij}|h_{ij}|^2+(1+\sigma_i^2)\sigma_{id}^2}\right)\right)=0$$

$$\sum_{j=1}^{N}\lambda_j p_{ij}=0$$

4. Gradient

$$\frac{\partial\mathcal{L}}{\partial p_{ij}}=1-\lambda_j-\nu_i\frac{|h_{ij}|^2(1+\sigma_i^2)\sigma_{id}^2}{(\sigma_i^2 p_{ij}^2|h_{ij}|^2+(1+\sigma_i^2)\sigma_{id}^2)^2}=0$$

If $\nu_i=0$, the Gradient condition, implies $\lambda_j=1\ \forall j$ then complementary slackness implies that $p_{ij}=0\ \forall j$. This result is not acceptable, therefore we must have $\nu_i>0$. Since $\nu_i>0$ again due to complementary slackness, the first constraint in (2.6) is active at the optimal point. The optimal solution is obtained by solving the KKT conditions:

$$\begin{cases}\lambda_j=1-\nu_i\frac{|h_{ij}|^2(1+\sigma_i^2)\sigma_{id}^2}{\left(\sigma_i^2 p_{ij}^2|h_{ij}|^2+\left(1+\sigma_i^2\right)\sigma_{id}^2\right)^2} & \\ \nu_i\leq\frac{\left(\sigma_i^2 p_{ij}^2|h_{ij}|^2+\left(1+\sigma_i^2\right)\sigma_{id}^2\right)^2}{|h_{ij}|^2(1+\sigma_i^2)\sigma_{id}^2} & \\ p_{ij}>0\Rightarrow\lambda_j=0 & \text{if }\ \nu_i>\frac{((1+\sigma_i^2)\sigma_{id}^2)^2}{|h_{ij}|^2(1+\sigma_i^2)\sigma_{id}^2} \\ \lambda_j>0\Rightarrow p_{ij}=0 & \text{if }\ \nu_i<\frac{((1+\sigma_i^2)\sigma_{id}^2)^2}{|h_{ij}|^2(1+\sigma_i^2)\sigma_{id}^2}\end{cases}$$

Solving for $p_{ij} > 0$ and $\lambda_j = 0$ from KKT conditions, we get the following water-filling solution.

$$p_{ij} = \begin{cases} \frac{|h_{ij}|\sqrt{\nu_i}\sqrt{(1+\sigma_i^2)\sigma_{id}^2} - (1+\sigma_i^2)\sigma_{id}^2}{\sigma_i^2 |h_{ij}|^2} & |h_{ij}|^2 > \frac{(1+\sigma_i^2)\sigma_{id}^2}{\nu_i} \\ 0 & |h_{ij}|^2 < \frac{(1+\sigma_o^2)\sigma_{id}^2}{\nu_i} \end{cases} \tag{2.7}$$

The main challenge of the problem now lies in finding the value of ν_i since we do not know the future values for $|h_{ij}|$s. We only know they are i.i.d distributed according to (1.1). In order to find the value of ν_i, first express the SNR (S) in terms of our optimal power transmission solution.

$$S_{ij} = \begin{cases} \frac{1}{\sigma_i^2} - \frac{\alpha_i}{\sigma_i^2}\frac{1}{|h_{ij}|} & |h_{ij}| > \alpha_i \\ 0 & |h_{ij}| < \alpha_i \end{cases} \tag{2.8}$$

where S_{ij} is the received SNR at the fusion center from the i-th sensor at the j-th transmission period and $\alpha_i \equiv \sqrt{\frac{(1+\sigma_i^2)\sigma_{id}^2}{\nu_i}}$. Then find the expected value of the the SNR due to the i-th sensor at the fusion center over the lifetime of the network, which we denote by $E[S_i]$. Assuming that $|h_i|$ has pdf defined in(1.1), we can find the $E[S_i]$. In general if a random variable Y is defined in terms of another random variable X, in the following manner:

$$Y = \begin{cases} C + g(X) & X > b \\ 0 & X < b \end{cases}$$

then

$$E[Y] = C\int_b^\infty f(x)dx + \int_b^\infty f(x)g(x)dx \tag{2.9}$$

where $f(x)$ is the pdf of X. Using (2.9) and assuming that $|h_i|$ has the pdf defined in(1.1), then

$$E[S_i] = \frac{1}{\sigma_i^2}\exp\left(\frac{-\alpha_i^2}{2\sigma_{hi}^2}\right) - \frac{\alpha_i}{\sigma_i^2}\int_{\alpha_i}^\infty \frac{1}{\sigma_{hi}^2}\exp\left(\frac{-x^2}{2\sigma_{hi}^2}\right)dx$$

$$E[S_i] = \frac{1}{\sigma_i^2}\exp\left(\frac{-\alpha_i^2}{2\sigma_{hi}^2}\right) - \frac{\alpha_i}{\sigma_i^2}\left(\frac{\sqrt{2\pi}}{2\sigma_{hi}}\left(1 - \operatorname{erf}\left(\frac{\alpha_i}{\sqrt{2}\sigma_{hi}}\right)\right)\right) \tag{2.10}$$

where $\operatorname{erf}(\cdot)$ is the one-sided error function. From (2.4), we know that $E[S_i] = \gamma_i$, so to find the value of ν_i, we solve:

$$\gamma_i = \frac{1}{\sigma_i^2}\exp\left(\frac{-\alpha_i^2}{2\sigma_{hi}^2}\right) - \frac{\alpha_i}{\sigma_i^2}\left(\frac{\sqrt{2\pi}}{2\sigma_{hi}}\left(1 - \operatorname{erf}\left(\frac{\alpha_i}{\sqrt{2}\sigma_{hi}}\right)\right)\right) \tag{2.11}$$

There is no closed form solution of the above equation but it can be solved numerically. Having found ν_i for $i = 1, \ldots, M$ then

$$p_{ij} = \frac{\sqrt{(1+\sigma_i^2)\,\sigma_{id}^2}}{\sigma_i^2 |h_{ij}|^2} \left(\sqrt{\nu_i}|h_{ij}| - \sqrt{(1+\sigma_i^2)\,\sigma_{id}^2} \right)^+$$

where we define $(x)^+ = \max\{x, 0\}$. We can also find $E[p_i]$:

$$E[p_i] = \frac{\sqrt{\pi \nu_i (1+\sigma_i^2)\sigma_{id}^2}}{\sqrt{2}\sigma_{hi}\sigma_i^2} \left[1 - \text{erf}\left(\frac{\alpha_i}{\sqrt{2}\sigma_{hi}} \right) \right] - \frac{(1+\sigma_i^2)\sigma_{id}^2}{2\sigma_i^2 \sigma_{hi}^2} E_1 \left(\frac{\alpha_i^2}{2\sigma_{hi}^2} \right)$$

where $E_n(x) = \int_1^\infty \frac{e^{-xt}}{t^n} dt, (x > 0, n = 0, 1, ...)$ is the exponential integral function.

Once we have found $E[p_i]$, the expected lifetime of the node can also be determined. Assume that the i-th node has initial energy of $\mathcal{E}$ $\mathcal{E} = \sum_{j=1}^N p_{ij}$. Assuming $N \gg 1$ it follows from the Law of Large Numbers [21] that $\sum_{j=1}^N p_{ij} = NE[p_i]$, so $N = \frac{\mathcal{E}}{E[p_i]}$. Thus

$$N = \frac{\mathcal{E}}{\left\{ \frac{\sqrt{\pi \nu_i (1+\sigma_i^2)\sigma_{id}^2}}{\sqrt{2}\sigma_{hi}\sigma_i^2} \left[1 - \text{erf}\left(\frac{\alpha_i}{\sqrt{2}\sigma_{hi}} \right) \right] - \frac{(1+\sigma_i^2)\sigma_{id}^2}{2\sigma_i^2 \sigma_{hi}^2} E_1 \left(\frac{\alpha_i^2}{2\sigma_{hi}^2} \right) \right\}} \tag{2.12}$$

We can also find the pdf of S_i of the sensor at the fusion center. Finding the pdf of S_i allows us to do outage analysis for the sensor. The pdf of S_i, which we denote by $f_{S_i}(s)$, is defined by the following expression:

$$f_{S_i}(s) = \left[1 - \exp\left(\frac{-\alpha_i^2}{2\sigma_{hi}^2} \right) \right] \delta(s) + \frac{\sigma_i^2 \alpha_i^2}{\sigma_{hi}^2 (1 - s\sigma_i^2)^3} \exp\left(\frac{-\alpha_i^2}{2\sigma_{hi}^2 (1 - s\sigma_i^2)^2} \right) \tag{2.13}$$

where $\delta(s)$ is Dirac's delta function. It can be seen from the pdf that S_i is a mixed random variable and its support region is $[0, \frac{1}{\sigma_i^2})$.

2.3 Network Power Scheduling

In the previous section we found the optimal solution that will maximize the lifetime of a sensor such that the expected value of the received SNR at the fusion center will be a given value. Now we consider the question of how to assign an expected value for each sensor (γ_i) such that $\sum_{i=1}^M \gamma_i = \gamma$ and all the nodes have the same expected lifetime (N). We assume that our network consists of M sensors. We use the algorithm in Fig. 2.3 to find N and $\gamma_1, \gamma_2, ..., \gamma_M$.

$N = N_0$
Use (2.12) to find ν_i for $i = 1, 2, ..., M$
Use (2.10)to find $E[s_i]$ for $i = 1, 2, ..., M$
while $\left|\sum_{i=1}^{M} E\left[S_i\right] - \gamma\right| > \epsilon$
$N \leftarrow N \frac{\gamma}{\sum_{i=1}^{M} E[S_i]}$
Use (2.12) to find ν_i for $i = 1, 2, ..., M$
Use (2.10)to find $E[s_i]$ for $i = 1, 2, ..., M$
end
$\gamma_i = E[S_i]$

Figure 2.3. Algorithm for allocation of partial SNR's

2.4 Numerical Results

We perform two experiments and in each case we compare the performance of our decomposition lifetime maximizing (DLM) method against the minimum-total-energy (MTE) [4] and equal-power (EP) strategies. For the EP strategy, we assign power to each sensor according to the residual energy left in the sensor.

In the first experiment, we show the effect of increasing number of sensors M, under equal statistics, where the required SNR at destination is normalized to the number of sensors, i.e., $M\gamma_0$. For our method this means that each sensor on average will provide an SNR of γ_0 over its lifetime. This normalization removes the effect of additional energy injected into the network via additional sensors. The results show the consistence of the performance of our method across different sizes. From the numerical results one suspects that our method may provide an upper bound for the MTE algorithm as the number of sensors approaches infinity, but we have no theoretical results at the present to support this conjecture. (See Fig. 2.4).

For the second experiment we generate σ_i^2, $\mathcal{E}_i$, and σ_{hi} randomly such that, $\sigma_i^2 \in \mathcal{U}[0.05, 0.1]$, $\mathcal{E}_i \in \mathcal{U}[250, 500]$, and $\sigma_{hi} \in \mathcal{U}[0.1, 0.2]$. Where $\mathcal{U}[a, b]$ denotes uniform distribution between a and b. For simplicity we take all the destination noise variances to be 0.08

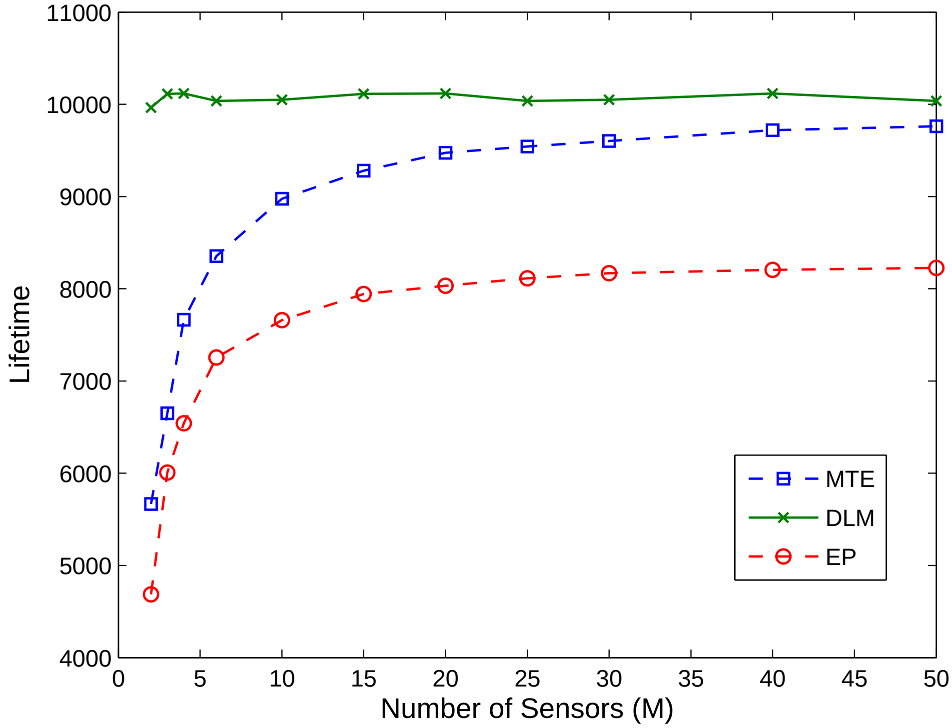

Figure 2.4. Lifetime comparisons for identical sensors

and we require an average SNR of $10dB$ at the receiver. (See Fig. 2.5).

Please note that no dynamic programming simulations are presented due to their immense computational complexity that is beyond most existing computational facilities. We note that the main point of this work is not to claim any improvement over dynamic programming, but rather to provide a pragmatic method with fewer requirements.

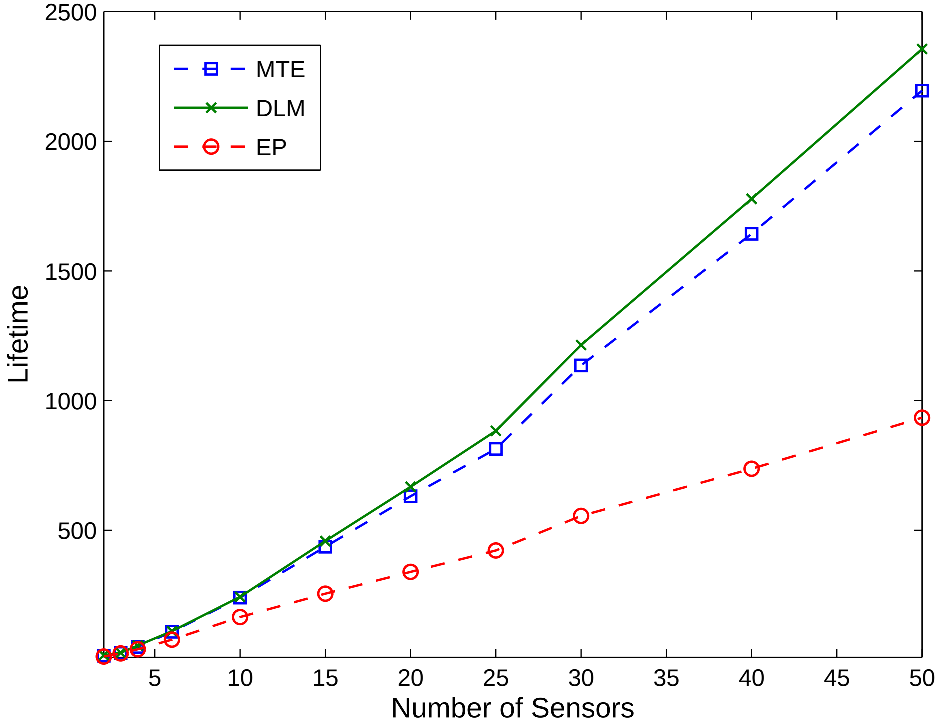

Figure 2.5. Lifetime Comparisons for randomly chosen sensors

CHAPTER 3

SHARED CHANNEL WITH PHASE INFORMATION

3.1 System Model

For this section we assume that all sensors know the phase information of channel coefficients of all the other sensors. Thus sensors can adjust their phases so that they will add coherently at the fusion center, i.e., beamforming. The received signal at the destination is:

$$y = \sum_{i=1}^{M} h_i w_i(\theta + n_i) + n_d \tag{3.1}$$

where $n_d \sim \mathcal{CN}(0, \sigma_d^2)$ and $n_i \sim \mathcal{CN}(0, \sigma_i^2)$.

The MMSE estimator of θ and its average SNR are given by [20]:

$$\hat{\theta} = \frac{\sum_{i=1}^{M} |h_i| w_i}{\left(\sum_{i=1}^{M} |h_i| w_i\right)^2 + \sum_{i=1}^{M} w_i^2 |h_i|^2 \sigma_i^2 + \sigma_d^2} y$$

$$\frac{\left(\sum_{i=1}^{M} |h_i| w_i\right)^2}{\sum_{i=1}^{M} w_i^2 |h_i|^2 \sigma_i^2 + \sigma_d^2}$$

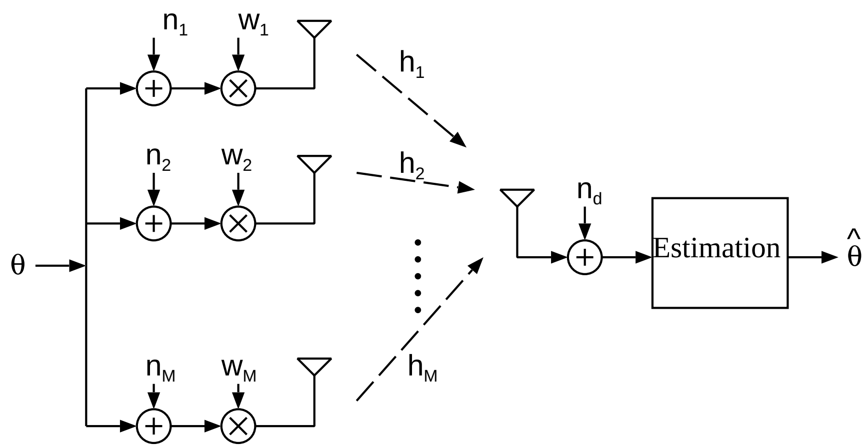

Figure 3.1. Network model for shared channel

3.2 Power Scheduling

Let N denote the number of transmissions before the network dies. Our goal is to maximize the expected lifetime of our network subject to a required SNR constraint (γ).

$$\begin{aligned}
&\max \ E[N] && && (3.2)\\
\text{s.t.}\ &\frac{\left(\sum_{i=1}^{M}|h_{ij}|w_{ij}\right)^2}{\sum_{i=1}^{M} w_{ij}^2|h_{ij}|^2\sigma_i^2+\sigma_d^2}\geq\gamma && j=1,\ldots,N\\
&p_{ij}\geq 0 && \forall i,j\\
&\sum_{j=1}^{N}p_{ij}\leq\mathcal{E} && i=1,\ldots,M
\end{aligned}$$

where $|w_{ij}|$ is the amplification gain for the i-th sensor during the j-th transmission period and $p_{ij}=w_{ij}^2(1+\sigma_i^2)$.

We consider a special case of the above problem in which all the channel coefficients are i.i.d distributed. So all the channel coefficients are Rayleigh distributed with the same variance. Under these assumptions and also assuming that $N \gg 1$, due to the law of large numbers [21] over the lifetime of the network all the channels behave approximately the same. So if we find a power-scheduling method that minimizes the power consumption over each transmission period, it will maximize the expected lifetime of the network and all the nodes will run out of energy at approximately the same time.

Therefore, at each transmission period we solve the following problem. For simplicity we disregard the time index and only show the sensor index.

$$
\begin{aligned}
&\min \sum_{i=1}^{M} w_i^2(1+\sigma_i^2) \\
&\text{s.t.} \frac{\left(\sum_{i=1}^{M} |h_i| w_i\right)^2}{\sum_{i=1}^{M} w_i^2 |h_i|^2 \sigma_i^2 + \sigma_d^2} \geq \gamma \\
&- w_i \leq 0 \quad i = 1, 2, \ldots, M
\end{aligned}
\tag{3.3}
$$

At the first glance, the problem does not look convex due to the first constraint. We can rewrite the first constraint as:

$$\gamma \sum_{i=1}^{M} w_i^2 |h_i|^2 \sigma_i^2 + \gamma \sigma_d^2 \leq \left(\sum_{i=1}^{M} |h_i| w_i\right)^2$$

We can disregard $\gamma\sigma_d^2$, and γ since they are constants and will have no affect on convexity or non-convexity of the constraint.

$$
\begin{aligned}
\sum_{i=1}^{M} w_i^2 |h_i|^2 \sigma_i^2 &\leq \left(\sum_{i=1}^{M} |h_i| w_i\right)^2 \\
\left(\sum_{i=1}^{M} w_i^2 |h_i|^2 \sigma_i^2\right)^{1/2} &\leq \left(\sum_{i=1}^{M} |h_i| w_i\right)
\end{aligned}
\tag{3.4}
$$

We can rewrite (3.4) as:

$$\|\mathbf{G}\mathbf{w}\|_2 \leq \mathbf{h}^T \mathbf{w} \tag{3.5}$$

where $\mathbf{G} = diag\,(\sigma_1^2|h_1|^2, \sigma_2^2|h_2|^2, \ldots, \sigma_M^2|h_M|^2)$, $\mathbf{w} = [w_1 w_2 \ldots w_M]^T$, and $\mathbf{h} = [|h_1||h_2| \ldots |h_M|]^T$. Equation (3.5) defines a second order cone [22] in $\mathbb{R}^{M+1}$ and it is a convex set. So our problem is in fact a second order cone programming problem (SOCP) which is equivalent to a convex quadratically constrained quadratic program (QCQP) [22]. There are various efficient numerical methods available to solve SOCP problems. However in our case we can also find the closed form solution. The Lagrangian $\mathcal{L}$ of (3.3) is given by:

$$\mathcal{L}(\mathbf{w},\lambda,\mathbf{u}) = \sum_{i=1}^{M} w_i^2(1+\sigma_i^2) + \lambda\left(\gamma\sum_{i=1}^{M} w_i^2|h_i|^2\sigma_i^2 + \gamma\sigma_d^2 - \left(\sum_{i=1}^{M} w_i|h_i|\right)^2\right) - \sum_{i=1}^{M} u_i w_i$$

We start by writing the KKT optimality conditions:

1. Primary

$$\gamma\sum_{i=1}^{M} w_i^2|h_i|^2\sigma_i^2 + \gamma\sigma_d^2 - \left(\sum_{i=1}^{M} |h_i|w_i\right)^2 \leq 0$$

$$w_i \geq 0 \qquad i = 1,2,\ldots,M$$

2. Dual

$$\lambda \geq 0, \mathbf{u} \geq 0$$

3. Complimentary Slackness

$$\lambda\left(\gamma\sum_{i=1}^{M} w_i^2|h_i|^2\sigma_i^2 + \gamma\sigma_d^2 - \left(\sum_{i=1}^{M} w_i|h_i|\right)^2\right) = 0$$

$$\lambda > 0 \implies \left(\gamma\sum_{i=1}^{M} w_i^2|h_i|^2\sigma_i^2 + \gamma\sigma_d^2 - \left(\sum_{i=1}^{M} w_i|h_i|\right)^2\right) = 0$$

$$u_i w_i = 0 \qquad i = 1,2,\ldots,M$$

4. Gradient

$$\frac{\partial\mathcal{L}}{\partial w_j} = 0 \qquad j = 1,2,\ldots,M \tag{3.6}$$

$$2w_j(1+\sigma_j^2) + 2\lambda\gamma w_j|h_j|^2\sigma_j^2 - 2\lambda|h_j|\left(\sum_{i=1}^{M} w_i|h_i|\right) - u_j = 0 \qquad j = 1,2,\ldots,M$$

Using the complimentary slackness conditions we can make the following conclusions regarding the activity of constraints at optimality:

$$\begin{aligned}
\lambda > 0 \Longrightarrow \left(\gamma \sum_{i=1}^{M} w_i^2 |h_i|^2 \sigma_i^2 + \gamma\sigma_d^2 - \left(\sum_{i=1}^{M} w_i |h_i|\right)^2\right) = 0 \\
\left(\gamma \sum_{i=1}^{M} w_i^2 |h_i|^2 \sigma_i^2 + \gamma\sigma_d^2 - \left(\sum_{i=1}^{M} w_i |h_i|\right)^2\right) > 0 \Longrightarrow \lambda = 0 \\
u_i > 0 \Longrightarrow w_i = 0 \qquad i = 1, 2, \ldots, M \\
w_i > 0 \Longrightarrow u_i = 0 \qquad i = 1, 2, \ldots, M
\end{aligned} \tag{3.7}$$

If $\lambda = 0$, then using the gradient condition (3.6), we will have $u_j = 2w_j(1 + \sigma_j^2)$. Multiplying both sides by w_j and using the complimentary slackness condition $u_i w_i = 0$, we will have $w_j = 0 \quad \forall j$ which is meaningless. So λ is strictly greater than zero. Using the first relationship in (3.7), this would imply that at optimality, the SNR requirement is active. So at the optimal operating point:

$$\left(\gamma \sum_{i=1}^{M} w_i^2 |h_i|^2 \sigma_i^2 + \gamma\sigma_d^2 - \left(\sum_{i=1}^{M} w_i |h_i|\right)^2\right) = 0$$

If $u_j > 0$, using the third relationship in (3.7) implies $w_j = 0$. Substituting $w_j = 0$ in (3.6), we get $u_j = 0$, which is a contradiction. So $u_j = 0 \quad \forall j$. In order to find λ, we rewrite the gradient equations (3.6) in the following form:

$$w_j = \frac{\lambda |h_j| \sum_{i=1}^{M} w_i |h_i|}{(1 + \sigma_j^2) + \lambda\gamma |h_j|^2 \sigma_j^2} \qquad j = 1, 2, \ldots, M \tag{3.8}$$

Multiplying the j-th equation in (3.8) by $|h_j|$ and then adding all the resulting equations:

$$\sum_{j=1}^{M} w_j |h_j| = \left(\sum_{j=1}^{M} w_j |h_j|\right) \sum_{j=1}^{M} \frac{\lambda |h_j|^2}{(1 + \sigma_j^2) + \lambda\gamma |h_j|^2 \sigma_j^2}$$

Dividing both sides by $\sum_{j=1}^{M} w_j |h_j|$:

$$\sum_{j=1}^{M} \frac{\lambda |h_j|^2}{(1 + \sigma_j^2) + \lambda\gamma |h_j|^2 \sigma_j^2} = 1 \tag{3.9}$$

$\lambda = \lambda_0$
while

$$\left|1 - \sum_{j=1}^{M} \frac{\lambda |h_j|^2}{(1+\sigma_j^2) + \lambda\gamma |h_j|^2 \sigma_j^2}\right| > \epsilon$$

$$\lambda = \frac{\lambda}{\sum_{j=1}^{M} \frac{\lambda |h_j|^2}{(1+\sigma_j^2)+\lambda\gamma|h_j|^2\sigma_j^2}}$$

end

Figure 3.2. Algorithm for numerical calculation of λ

All the variables in (3.9) are known except λ and it can be solved numerically via the simple algorithm in Fig. 3.2.

We also need to find the value for $\sum_{j=1}^{M} w_j|h_j|$. In order to find the value of $\sum_{j=1}^{M} w_j|h_j|$ we consider the first constraint in (3.3) which we already proved is going to be active at optimality.

$$\left(\sum_{j=1}^{M} w_j|h_j|\right)^2 = \gamma \sum_{j=1}^{M} w_j^2 |h_j|^2 \sigma_j^2 + \gamma\sigma_d^2 \tag{3.10}$$

For simplicity let $c \triangleq \left(\sum_{j=1}^{M} w_j|h_j|\right)^2$. From (3.8) we substitute w_j into (3.10). We have:

$$c = \gamma \sum_{j=1}^{M} \frac{\lambda^2 |h_j|^4 \sigma_j^2 c}{\left[(1+\sigma_j^2) + \lambda\gamma|h_j|^2\sigma_j^2\right]^2} + \gamma\sigma_d^2 \tag{3.11}$$

$$c = \gamma\sigma_d^2 \left(1 - \gamma \sum_{j=1}^{M} \frac{\lambda^2 |h_j|^4 \sigma_j^2}{\left[(1+\sigma_j^2) + \lambda\gamma|h_j|^2\sigma_j^2\right]^2}\right)^{-1}$$

Having found $\left(\sum_{j=1}^{M} w_j|h_j|\right)$ and λ we can use (3.8) to find the optimal amplification gains and hence the transmission power levels.

3.3 Numerical Results

In the first simulation we consider the lifetime of a network with statistically identical channels. We compare the performance of our optimal (Opt) method against a purely

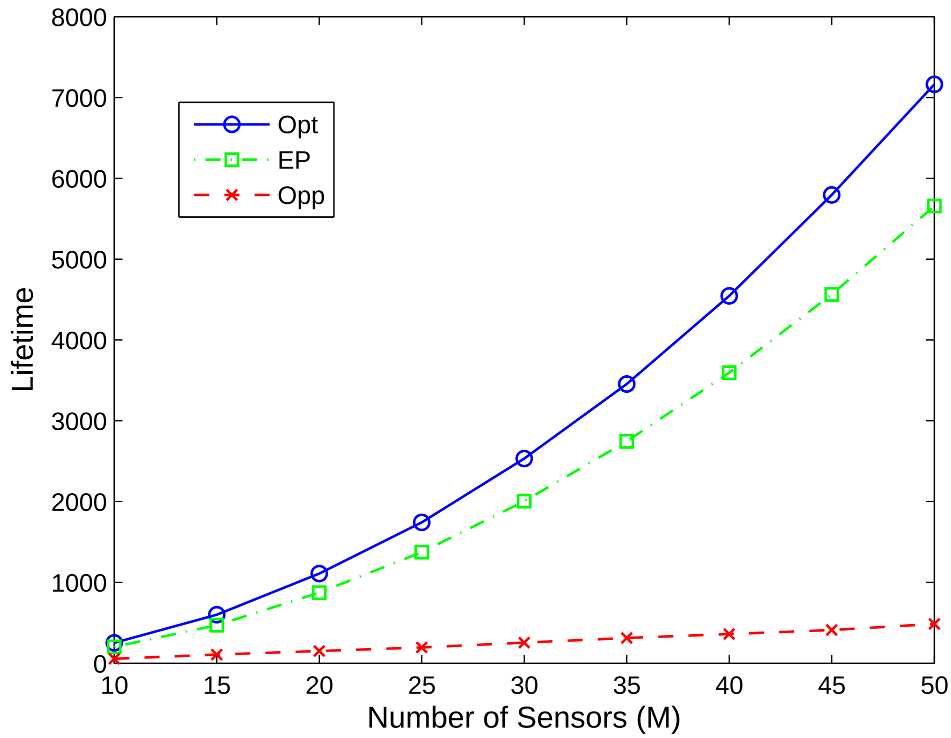

Figure 3.3. Lifetime comparisons for identical sensors

opportunistic (Opp) method and equal-power (EP) strategies. For the opportunistic strategy, at each transmission period only the sensor with the best channel conditions transmits it message. The results are illustrated in Fig. 3.3.

For the second simulation, we consider channels that are not statistically identical. We cannot claim that our minimum power (MP) strategy is optimal in this case. However it is instructive to compare MP with a purely opportunistic (Opp) method and equal-power (EP) strategies. We set all the observation noise variances to 0.02 and the destination noise variance to 0.06. We generate σ_{hi} randomly such that $\sigma_{hi} \in \mathcal{U}[0.1, 0.3]$. The results are illustrated in Fig. 3.4.

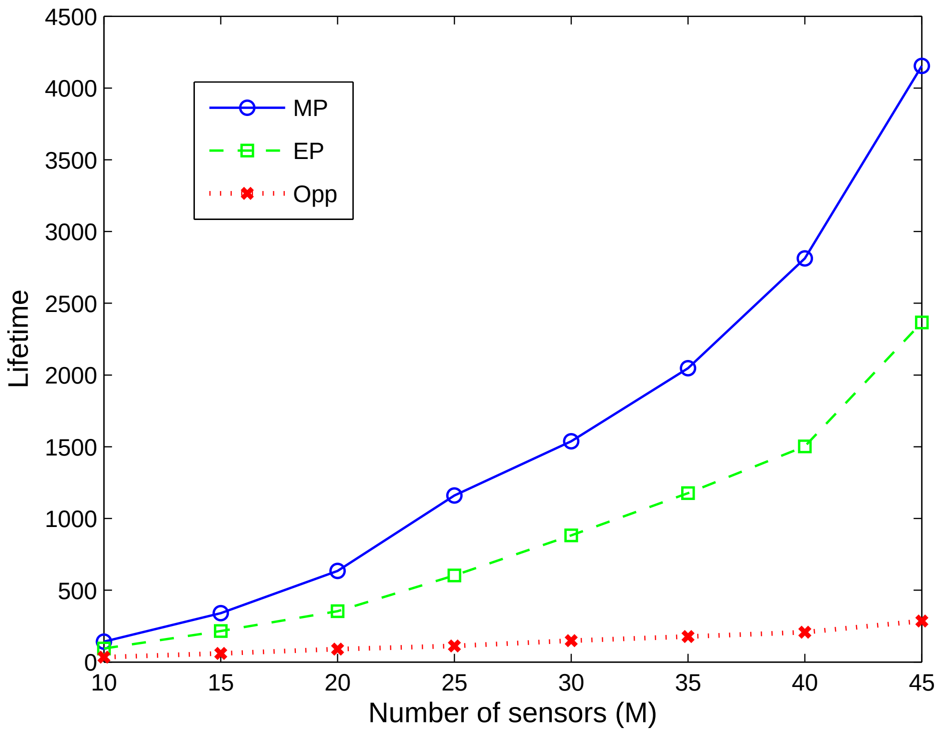

Figure 3.4. Lifetime Comparisons for randomly chosen sensors

CHAPTER 4

SHARED CHANNEL WITHOUT PHASE INFORMATION

4.1 System Model

For this scenario, the received signal model is the same as the previous chapter.

$$y = \sum_{i=1}^{M} h_i w_i(\theta + n_i) + n_d$$

where $n_d \sim \mathcal{CN}(0, \sigma_d^2)$, $n_i \sim \mathcal{CN}(0, \sigma_i^2)$, and $p_i = w_i^2(1 + \sigma_i^2)$. However we assume that the phase of channel coefficients are unknown at the transmitters and thus the signals add incoherently at the fusion center. In this case the MMSE estimator of θ is given by [20]:

$$\hat{\theta} = \frac{\sum_{i=1}^{M} h_i^* w_i}{\sum_{i=1}^{M} w_i^2 |h_i|^2 (1 + \sigma_i^2) + \sigma_d^2} y$$

The average SNR for this estimator is:

$$\frac{\sum_{i=1}^{M} p_i |h_i|^2/(1 + \sigma_i^2)}{\sum_{i=1}^{M} p_i \sigma_i^2 |h_i|^2/(1 + \sigma_i^2) + \sigma_d^2}$$

4.2 Power Scheduling

Let N denote the lifetime of our network while maintaining a specified SNR (γ) at the fusion center. So the problem is:

$$\begin{aligned} &\max \ E[N] && && (4.1) \\ \text{s.t.} \ &\frac{\sum_{i=1}^{M} p_{ij} |h_{ij}|^2/(1 + \sigma_i^2)}{\sum_{i=1}^{M} p_{ij} \sigma_i^2 |h_{ij}|^2/(1 + \sigma_i^2) + \sigma_d^2} \geq \gamma && j = 1, \ldots, N \\ &p_{ij} \geq 0 && \forall i, j \\ &\sum_{j=1}^{N} p_{ij} \leq \mathcal{E} && i = 1, \ldots, M \end{aligned}$$

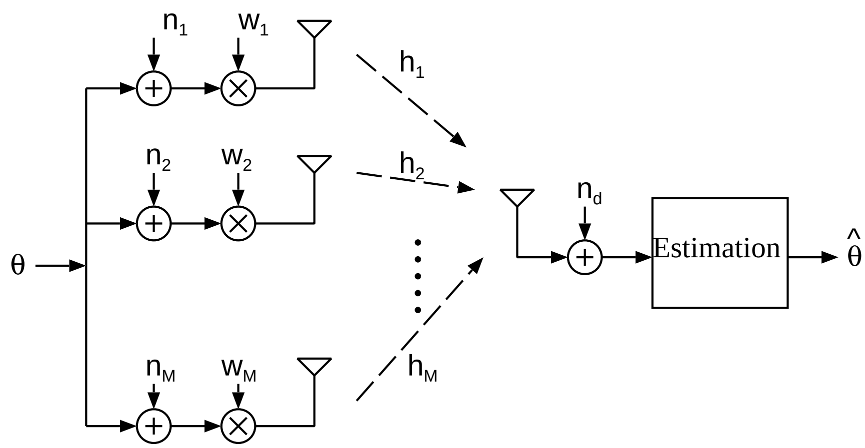

Figure 4.1. Network model for shared channel

where $|p_{ij}|$ is the amplification gain for the i-th sensor during the j-th transmission period. As in the previous chapter we consider a special case of the problem in which all channel coefficients are i.i.d distributed. We also assume that the lifetime of the network will be long enough for the law of large numbers to apply. Under these assumptions, we argue that a power scheduling method that minimizes the power consumption over each transmission period, will maximizes the expected lifetime of the network and all the nodes will run out of energy at approximately the same time. So our original problem (4.1) reduces to the following problem at each transmission period. For simplicity we disregard the time index and show only the sensor index.

$$\begin{aligned} &\min \sum_{i=1}^{M} w_i^2(1+\sigma_i^2) \\ \text{s.t.} \quad &\frac{\sum_{i=1}^{M} |h_i|^2 w_i^2}{\sum_{i=1}^{M} w_i^2 |h_i|^2 \sigma_i^2 + \sigma_d^2} \geq \gamma \\ &-w_i \leq 0 \quad i = 1, 2, \ldots, M \end{aligned} \tag{4.2}$$

This can be expressed in the form of of a standard convex optimization problem.

$$
\begin{aligned}
&\min \sum_{i=1}^{M} w_i^2(1+\sigma_i^2) \\
\text{s.t.} \quad &\gamma \sum_{i=1}^{M} w_i^2 |h_i|^2 \sigma_i^2 + \gamma\sigma_d^2 - \sum_{i=1}^{M} |h_i|^2 w_i^2 \leq 0 \\
&- w_i \leq 0 \quad i = 1, 2, \ldots, M
\end{aligned}
\tag{4.3}
$$

The problem (4.3) is convex in w_i. The Lagrangian of (4.3) is :

$$
\mathcal{L}(\mathbf{w}, \lambda, \mathbf{u}) = \sum_{i=1}^{M} w_i^2(1+\sigma_i^2) + \lambda \left(\sum_{i=1}^{M} w_i^2 |h_i|^2 \sigma_i^2 + \gamma\sigma_d^2 - \sum_{i=1}^{M} |h_i|^2 w_i^2 \right) - \sum_{i=1}^{M} u_i w_i \tag{4.4}
$$

The KKT optimality conditions are:

1. Primary

$$
\gamma \sum_{i=1}^{M} w_i^2 |h_i|^2 \sigma_i^2 + \gamma\sigma_d^2 - \sum_{i=1}^{M} |h_i|^2 w_i^2 \leq 0
$$
$$
-w_i \leq 0 \quad i = 1, 2, \ldots, M
$$

2. Dual

$$
\lambda \geq 0 \qquad \mathbf{u} \geq 0
$$

3. Complimentary slackness

$$
\lambda \left(\sum_{i=1}^{M} w_i^2 |h_i|^2 \sigma_i^2 + \gamma\sigma_d^2 - \sum_{i=1}^{M} |h_i|^2 w_i^2 \right) = 0
$$
$$
u_i w_i = 0 \qquad i = 1, 2, \ldots, M
$$

4. Gradient

$$
\frac{\partial \mathcal{L}}{\partial w_j} = 0 \qquad j = 1, 2, \ldots, M \tag{4.5}
$$

$$
2w_j(1+\sigma_j^2) + 2\lambda\gamma w_j |h_j|^2 \sigma_j^2 - 2\lambda w_j |h_j|^2 - u_j = 0 \qquad j = 1, 2, \ldots, M
$$

Due to complimentary slackness, if $u_j > 0$, we must have $w_j = 0$. Now if we substitute $w_j = 0$ in (4.5) we get $u_j = 0$. Thus:

$$u_j > 0 \Longrightarrow u_j = 0$$

which is a contradiction. As a result we have

$$u_j = 0 \quad j = 1, 2, \ldots, M$$

Also if $\lambda = 0$, from (4.5) we have $w_j = 0 \quad \forall j$, which is meaningless. So we conclude $\lambda > 0$ which due to the complimentary slackness would imply that the SNR requirement is active at optimality. Substituting $u_j = 0$ in the gradient condition (4.5), we get:

$$\lambda w_j |h_j|^2 - \lambda \gamma w_j |h_j|^2 \sigma_j^2 = w_j (1 + \sigma_j^2)$$

We know that $w_j(1 + \sigma_j^2) \geq 0$. Thus we have:

$$\lambda w_j |h_j|^2 - \lambda \gamma w_j |h_j|^2 \sigma_j^2 \geq 0$$

$$\lambda w_j |h_j|^2| \left(1 - \gamma \sigma_j^2\right) \geq 0 \tag{4.6}$$

In order to satisfy the condition (4.6) we must have:

$$1 - \gamma \sigma_j^2 < 0 \Longrightarrow w_j = 0$$

$$\sigma_j^2 > \frac{1}{\gamma} \Longrightarrow p_j = 0$$

This result implies that sensors with noise variances larger than a certain value $(1/\gamma)$ will never transmit their message regardless of their channel condition. Let $S = \{1, 2, \ldots, M\}$ denote our original set of sensors. Define a subset of sensors $S' \subseteq S$, in the following manner: $S' = \{i : 1 - \gamma \sigma_i^2 > 0\}$. Let $|S'| = K$ where $| \cdot |$ denotes the cardinality of a set. We now have the following problem (4.7) and its vector form (4.8):

$$\begin{aligned} &\min \sum_{i=1}^{K} p_i \\ \text{s.t.} \quad &\sum_{i=1}^{K} \frac{p_i \left(|h_i|^2 - \gamma \sigma_i^2 |h_i|^2\right)}{1 + \sigma_i^2} = \gamma \sigma_d^2 \end{aligned} \tag{4.7}$$

$$\min \mathbf{1}^T \mathbf{p}$$
$$\text{s.t.} \quad \mathbf{a}^T \mathbf{p} = \gamma \sigma_d^2 \tag{4.8}$$

where $\mathbf{1} = [1 \ldots 1]^T$, $\mathbf{p} = [p_1 \ldots p_K]^T$, and

$$\mathbf{a} = \left[\frac{|h_1|^2(1 - \gamma\sigma_1^2)}{1 + \sigma_1^2} \cdots \frac{|h_K|^2(1 - \gamma\sigma_K^2)}{1 + \sigma_K^2} \right]^T$$

This is a standard linear programming (LP) [22] problem. In general no closed form solution exists for LP problems, but they are readily solved using very efficient algorithms such as the simplex method [23]. However in our case a closed form solution can be obtained for (4.8). Let $\mathbf{a}^T\mathbf{p} = g(\mathbf{p})$, then

$$\frac{\partial g(\mathbf{p})}{\partial p_i} = \frac{|h_i|^2(1 - \gamma\sigma_i^2)}{1 + \sigma_i^2} = \mathbf{a}(i)$$

$g(\mathbf{p})$ is an increasing linear function of $\mathbf{p}$. Now let m denote the index corresponding to the largest element in $\mathbf{a}$ or $m = \arg\max \mathbf{a}$. In that case the optimal solution is:

$$p_i^* = \begin{cases} \frac{(1+\sigma_i^2)\gamma\sigma_d^2}{|h_i|^2(1-\gamma\sigma_i^2)} & i = m \\ 0 & i \neq m \end{cases} \tag{4.9}$$

This problem can also be viewed as an l_1 least norm problem ($\sum p_i = |\mathbf{p}|_1$ for $p_i \geq 0$). It is worth mentioning that if we consider a modified version of this problem which instead of minimizing ($\sum p_i$), we minimize ($\sum p_i^2$), the optimal closed form solution is $\left(\mathbf{a}^T\mathbf{a}\right)^{-1} \mathbf{a}^T \gamma\sigma_d^2$ [24]. It is also worth noting that under the l_2 least norm scenario the nodes with $\sigma_i^2 > 1/\gamma$ will still not transmit.

4.3 Numerical Results

We consider three power scheduling schemes, comparing the solution which minimizes the l_1 norm of the power vector (l_1`min`) versus the solution that minimizes the l_2 norm of the power vector (l_2`min`) and an equal-power method (EP).

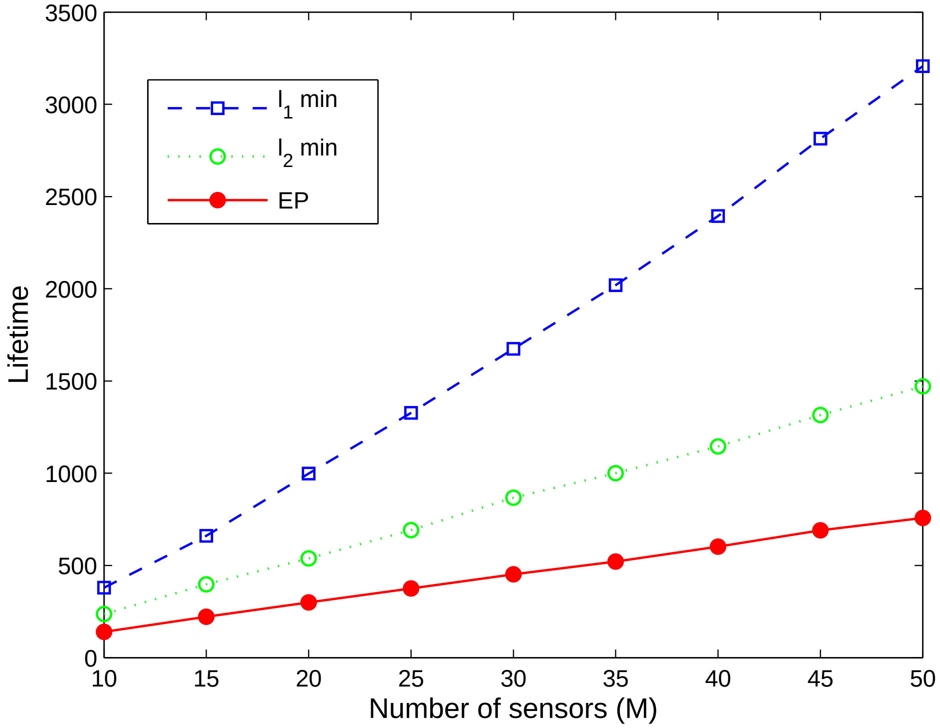

Figure 4.2. Lifetime comparisons for statistically identical shared channels (no phase information)

The comparisons are made in the context of two network geometries. First we consider a network in which all the channels are Rayleigh distributed with the same variance. This reflects a physical situation where the nodes are approximately the same distance from the fusion center. Results are shown in Fig. 4.2. We also consider Rayleigh distributed channels which are not identically distributed. We set all the observation noise variances to 0.03 and the destination noise variance to 0.02. The required SNR at the fusion center is set to 20. σ_{hi}'s are generated randomly such that $\sigma_{hi} \in \mathcal{U}[0.2, 0.4]$. The results are illustrated in Fig. 4.3.

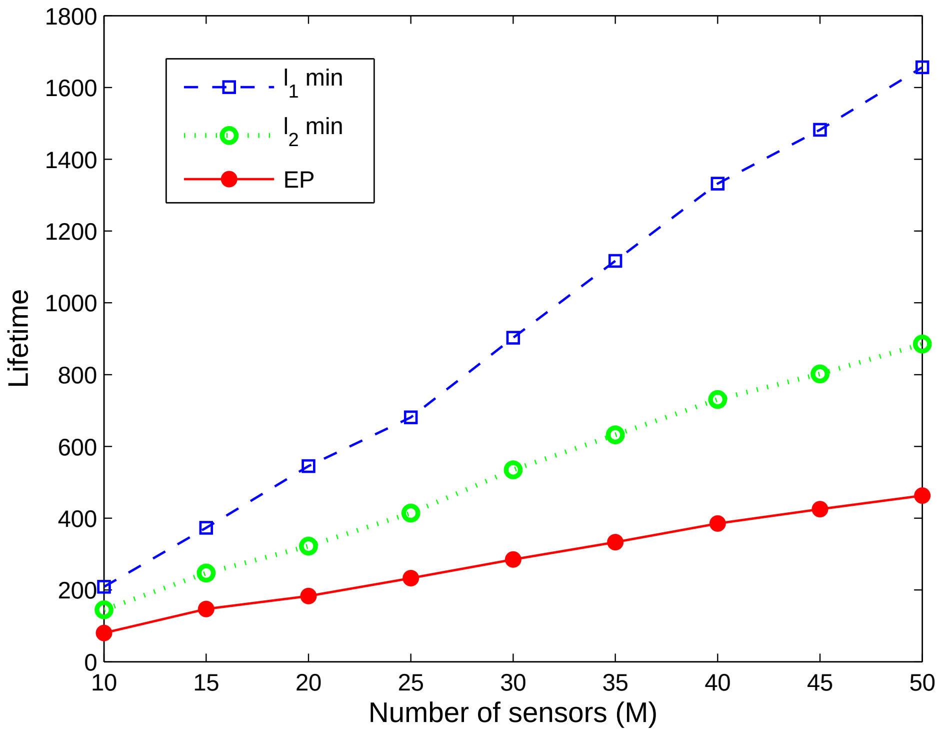

Figure 4.3. Lifetime Comparisons for randomly distributed shared channels(no phase information)

CHAPTER 5

RESOURCE ALLOCATION FOR DISTRIBUTED DETECTION IN SENSOR NETWORKS

5.1 System Model

We consider a binary hypothesis testing situation. Our network consists of M sensors. Each sensor makes a measurement $x_i = \theta + n_i$, where $n_i \sim \mathcal{N}(0, \sigma_i^2)$ is additive Gaussian noise. We assume that under hypothesis $\mathcal{H}_0$, $\theta \sim \mathcal{N}(a, \sigma^2)$ and under hypothesis $\mathcal{H}_1$, $\theta \sim \mathcal{N}(b, \sigma^2)$. For simplicity and without loss of generality we assume $a = 0$, $b = 1$, and $\sigma^2 = 1$. Each sensor takes the log of its local likelihood ratio (LLR) and sends it to the fusion center via analog communication, which is received with additive noise. Let n_{id} denote the destination noise for the ith sensor. We assume noises are uncorrelated and normally distributed according to $n_{id} \sim \mathcal{N}(0, \sigma_{id}^2)$. Let h_i denote the channel gain between the ith sensor and the fusion center. Also let w_i denote the amplification factor for the ith sensor. We assume that sensors are communicating to the fusion center over orthogonal channels. The received signal from the ith sensor at the fusion center is :

$$y_i = \ln\left(\frac{p(x_i; \mathcal{H}_1)}{p(x_i; \mathcal{H}_0)}\right) w_i h_i + n_{id} \quad i = 1, \ldots, M$$

We consider the Neyman-Pearson approach [25]. Given a required false alarm probability $P_{FA} = \alpha$ and an average power constraint P, our goal is to maximize the detection probability (β). for simplicity we also assume that $\mathcal{H}_0$ is the dominant hypothesis ($Pr(\mathcal{H}_0) \gg Pr(\mathcal{H}_1)$).

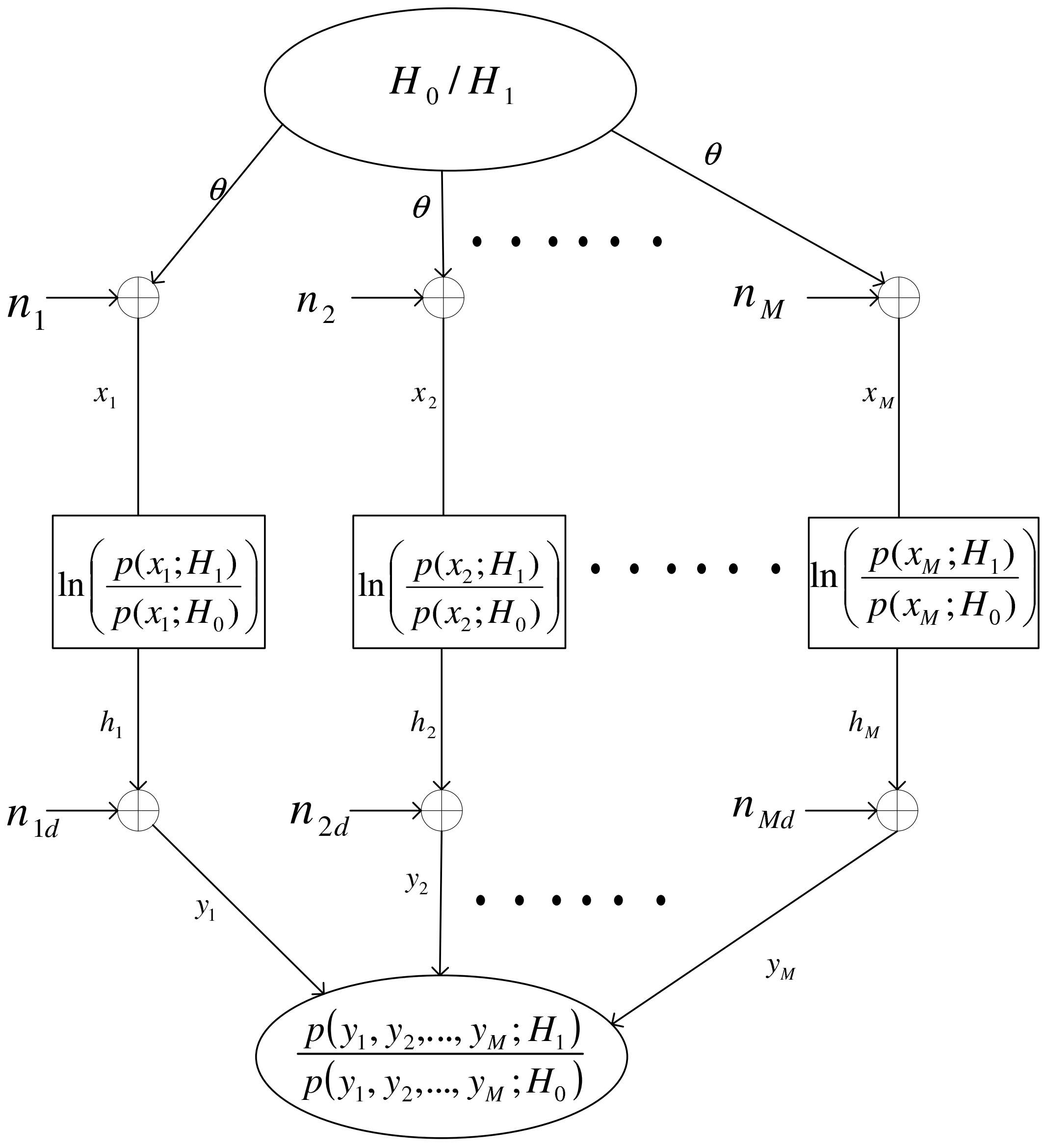

Figure 5.1. Network model for binary detection

5.2 Detector

As mentioned in the previous section, we assume that under $\mathcal{H}_0$, $\theta \sim \mathcal{N}(0,1)$ and under $\mathcal{H}_1$, $\theta \sim \mathcal{N}(1,1)$. Let $L(x_i)$ denote the likelihood function of x_i.

$$L(x_i) = \frac{p(x_i;\mathcal{H}_1)}{p(x_i;\mathcal{H}_0)} = \exp\left(\frac{2x_i - 1}{2(1+\sigma_i^2)}\right)$$

The *log* of the local likelihood function is sent to the fusion center. So the received value at the fusion center is:

$$y_i = \left(\frac{2x_i - 1}{2(1+\sigma_i^2)}\right) w_i h_i + n_{id} \quad i = 1, \ldots, M$$

The transmission power for the ith sensor is $p_i = \left(\frac{2x_i-1}{2(1+\sigma_i^2)}\right)^2 w_i^2$. It can be shown that the likelihood function for y_i is:

$$L(y_i) = \frac{p(y_i;\mathcal{H}_1)}{p(y_i;\mathcal{H}_0)} = \exp\left(\frac{y_i w_i h_i}{w_i^2 h_i^2 + \sigma_{id}^2(1+\sigma_i^2)}\right)$$

At the fusion center we form the log-likelihood ratio of all the received signals which we denote by LL_{FC}.

$$LL_{FC}(y_1, \ldots, y_M) = \sum_{i=1}^{M} \frac{y_i w_i h_i}{w_i^2 h_i^2 + \sigma_{id}^2(1+\sigma_i^2)}$$

Thus we will have the following detector at the fusion center:

$$\sum_{i=1}^{M} \frac{y_i w_i h_i}{w_i^2 h_i^2 + \sigma_{id}^2(1+\sigma_i^2)} \underset{\mathcal{H}_0}{\overset{\mathcal{H}_1}{\gtrless}} \tau \tag{5.1}$$

5.3 Power scheduling

The false alarm probability is:

$$\alpha = \Pr\ (LL_{FC} > \tau|\mathcal{H}_0) = Q\left(\frac{\tau + \sum_{i=1}^{M} \frac{w_i^2 h_i^2}{2(1+\sigma_i^2)\left(w_i^2 h_i^2 + \sigma_{id}^2(1+\sigma_i^2)\right)}}{\sqrt{\sum_{i=1}^{M} \frac{w_i^2 h_i^2}{(1+\sigma_i^2)\left(w_i^2 h_i^2 + \sigma_{id}^2(1+\sigma_i^2)\right)}}}\right) \tag{5.2}$$

Similarly, the detection probability, β, can be calculated as:

$$\beta = \Pr\ (LL_{FC} > \tau|\mathcal{H}_1) = Q\left(\frac{\tau - \sum_{i=1}^{M} \frac{w_i^2 h_i^2}{2(1+\sigma_i^2)\left(w_i^2 h_i^2 + \sigma_{id}^2(1+\sigma_i^2)\right)}}{\sqrt{\sum_{i=1}^{M} \frac{w_i^2 h_i^2}{(1+\sigma_i^2)\left(w_i^2 h_i^2 + \sigma_{id}^2(1+\sigma_i^2)\right)}}}\right) \tag{5.3}$$

If there were no power constraint on the network, each sensor would transmit at a very high power to neutralize the effect of the observation noise. The error performance of a such network will asymptotically approach the performance of a network with ideal links. However in our case we have an average power constraint of P. Furthermore, since we are considering the NP criteria, we are not given prior probabilities for $\mathcal{H}_0$ and $\mathcal{H}_1$. However if we assume that $\mathcal{H}_0$ is the dominant hypothesis ($\Pr(\mathcal{H}_0) \gg \Pr(\mathcal{H}_1)$), we can write our average power constraint as:

$$\sum_{i=1}^{M} \frac{w_i^2\left(1+4(1+\sigma_i^2)\right)}{4\left(1+\sigma_i^2\right)^2} \leq P$$

This is due to the fact that

$$E[p_i] = E[p_i|\mathcal{H}_0]Pr[\mathcal{H}_0] + E[p_i|\mathcal{H}_1]Pr[\mathcal{H}_1] \approx E[p_i|\mathcal{H}_0] = \frac{w_i^2\left(1+4(1+\sigma_i^2)\right)}{4(1+\sigma_i^2)}$$

The approximation above is a direct result of our assumption that $Pr(\mathcal{H}_0) \gg Pr(\mathcal{H}_1)$. Now we have the following optimization problem:

$$\begin{aligned} &\max \quad \beta \\ \text{s.t.} \quad & Q\left(\frac{\tau + \sum_{i=1}^{M} \frac{w_i^2 h_i^2}{2(1+\sigma_i^2)\left(w_i^2 h_i^2 + \sigma_{id}^2(1+\sigma_i^2)\right)}}{\sqrt{\sum_{i=1}^{M} \frac{w_i^2 h_i^2}{(1+\sigma_i^2)\left(w_i^2 h_i^2 + \sigma_{id}^2(1+\sigma_i^2)\right)}}}\right) \leq \alpha \\ & \sum_{i=1}^{M} \frac{w_i^2\left(1+4(1+\sigma_i^2)\right)}{4\left(1+\sigma_i^2\right)^2} \leq P \end{aligned} \tag{5.4}$$

Where β has the form from equation (5.3). We take advantage of the fact that Q-function is a monotonically decreasing function, so in general in order to maximize $Q\big(f(x)\big)$, all we need to do is minimize $f(x)$. We can also rewrite the first constraint in (5.4). So we can rewrite our original problem (5.4), in the following form:

$$\begin{aligned}
&\min \quad \frac{\tau - \sum_{i=1}^{M} \frac{w_i^2 h_i^2}{2(1+\sigma_i^2)\left(w_i^2 h_i^2+\sigma_{id}^2(1+\sigma_i^2)\right)}}{\sqrt{\sum_{i=1}^{M} \frac{w_i^2 h_i^2}{(1+\sigma_i^2)\left(w_i^2 h_i^2+\sigma_{id}^2(1+\sigma_i^2)\right)}}} \\
&\text{s.t.} \quad Q^{-1}(\alpha) - \frac{\tau + \sum_{i=1}^{M} \frac{w_i^2 h_i^2}{2(1+\sigma_i^2)\left(w_i^2 h_i^2+\sigma_{id}^2(1+\sigma_i^2)\right)}}{\sqrt{\sum_{i=1}^{M} \frac{w_i^2 h_i^2}{(1+\sigma_i^2)\left(w_i^2 h_i^2+\sigma_{id}^2(1+\sigma_i^2)\right)}}} \leq 0 \\
&\sum_{i=1}^{M} \frac{w_i^2\left(1+4(1+\sigma_i^2)\right)}{4\left(1+\sigma_i^2\right)^2} - P \leq 0
\end{aligned} \tag{5.5}$$

In order to solve the problem (5.5), we start by introducing an axillary variable t into the problem. So we have the following problem,

$$\begin{aligned}
&\min \quad \frac{\tau - t/2}{\sqrt{t}} \\
&\text{s.t.} \quad Q^{-1}(\alpha) - \frac{\tau + t/2}{\sqrt{t}} \leq 0 \\
&\sum_{i=1}^{M} \frac{w_i^2\left(1+4(1+\sigma_i^2)\right)}{4\left(1+\sigma_i^2\right)^2} - P \leq 0 \\
&\sum_{i=1}^{M} \frac{w_i^2 h_i^2}{\left(1+\sigma_i^2\right)\left(w_i^2 h_i^2+\sigma_{id}^2(1+\sigma_i^2)\right)} = t
\end{aligned} \tag{5.6}$$

The Lagrangian of (5.6) is:

$$\mathcal{L}(\mathbf{w}, t, \tau, \lambda_1, \lambda_2, \nu) = \frac{\tau - t/2}{\sqrt{t}} + \lambda_1 \left(Q^{-1}(\alpha) - \frac{\tau + t/2}{\sqrt{t}}\right) + \lambda_2 \left(\sum_{i=1}^{M} \frac{w_i^2\left(1+4(1+\sigma_i^2)\right)}{4\left(1+\sigma_i^2\right)^2} - P\right) + \nu \left(\sum_{i=1}^{M} \frac{w_i^2 h_i^2}{\left(1+\sigma_i^2\right)\left(w_i^2 h_i^2+\sigma_{id}^2(1+\sigma_i^2)\right)} - t\right) \tag{5.7}$$

The KKT optimality conditions for (5.6) are:

1. Primary

$$Q^{-1}(\alpha) - \frac{\tau + t/2}{\sqrt{t}} \leq 0$$
$$\sum_{i=1}^{M} \frac{w_i^2 \left(1 + 4(1+\sigma_i^2)\right)}{4\left(1+\sigma_i^2\right)^2} - P \leq 0$$
$$\sum_{i=1}^{M} \frac{w_i^2 h_i^2}{\left(1+\sigma_i^2\right)\left(w_i^2 h_i^2 + \sigma_{id}^2(1+\sigma_i^2)\right)} = t$$

2. Dual

$$\lambda_1 \geq 0 \qquad \lambda_2 \geq 0$$

3. Complementary slackness

$$\lambda_1 \left(Q^{-1}(\alpha) - \frac{\tau + t/2}{\sqrt{t}} \right) = 0$$
$$\lambda_2 \left(\sum_{i=1}^{M} \frac{w_i^2 \left(1 + 4(1+\sigma_i^2)\right)}{4\left(1+\sigma_i^2\right)^2} - P \right) = 0$$

4. Gradient

$$\frac{\partial \mathcal{L}}{\partial w_i} = \frac{\lambda_2 \left(1 + 4(1+\sigma_i^2)\right)}{2\left(1+\sigma_i^2\right)^2} + \frac{\nu h_i^2 \sigma_{id}^2}{\left(w_i^2 h_i^2 + \sigma_{id}^2(1+\sigma_i^2)\right)^2} = 0 \quad i = 1, \ldots, M$$
$$\frac{\partial \mathcal{L}}{\partial \tau} = \frac{1}{\sqrt{t}} - \frac{\lambda_1}{\sqrt{t}} = 0$$
$$\frac{\partial \mathcal{L}}{\partial t} = \frac{(-1/2)\sqrt{t} - (1/2)t^{-1/2}(\tau - t/2)}{t} - \lambda_1 \frac{(1/2)\sqrt{t} - (1/2)t^{-1/2}(\tau + t/2)}{t} - \nu = 0$$

From the second gradient condition we can see that $\lambda_1 = 1$. Also since $\lambda_1 > 0$ due to complementary slackness we can conclude that at optimality the first constraint is active:

$$\tau = Q^{-1}(\alpha)\sqrt{t} - \frac{t}{2} \tag{5.8}$$

Substituting $\lambda_1 = 1$ in the third gradient equation and after some simplification, we will get:

$$\nu = \frac{-1}{\sqrt{t}}$$

We can also show λ_2 is strictly greater than 0 ($\lambda_2 = 0$ will lead to a contradiction). So again using complementary slackness we can conclude that at optimality the second constraint of (5.6) is also active. Substituting $\nu = \frac{-1}{\sqrt{t}}$ into our first gradient condition and rewriting out results we get $M+2$ non-linear equations with $M+2$ unknowns $(t, \lambda_2, w_1, \ldots, w_M)$:

$$\begin{aligned}
&\frac{\lambda_2 \left(1 + 4(1+\sigma_i^2)\right)}{2\left(1+\sigma_i^2\right)^2} - \frac{h_i^2 \sigma_{id}^2}{\sqrt{t}\left(w_i^2 h_i^2 + \sigma_{id}^2(1+\sigma_i^2)\right)^2} = 0 \\
&\sum_{i=1}^{M} \frac{w_i^2 \left(1 + 4(1+\sigma_i^2)\right)}{4\left(1+\sigma_i^2\right)^2} - P = 0 \\
&\sum_{i=1}^{M} \frac{w_i^2 h_i^2}{(1+\sigma_i^2)\left(w_i^2 h_i^2 + \sigma_{id}^2(1+\sigma_i^2)\right)} - t = 0
\end{aligned}$$

The above equations can be solved numerically. Having found the variables, we can use (5.8) to determine the optimal threshold for our detector.

5.4 Numerical Results

For our numerical results, we a consider a network with randomly chosen channel coefficients. Given an average power constraint, we compare the detection probability of our optimal power scheduling method versus a uniform method in which all the sensors use the same amount of power. The channel coefficients and noise variances are chosen randomly according to $h \in \mathcal{U}[0.2, 1]$, $\sigma_i^2 \in \mathcal{U}[0.1, 0.3]$, and $\sigma_d^2 \in \mathcal{U}[0.1, 0.3]$. We set $\alpha = 0.01$ and average power constraint at 5. The results are shown in Fig. 5.2.

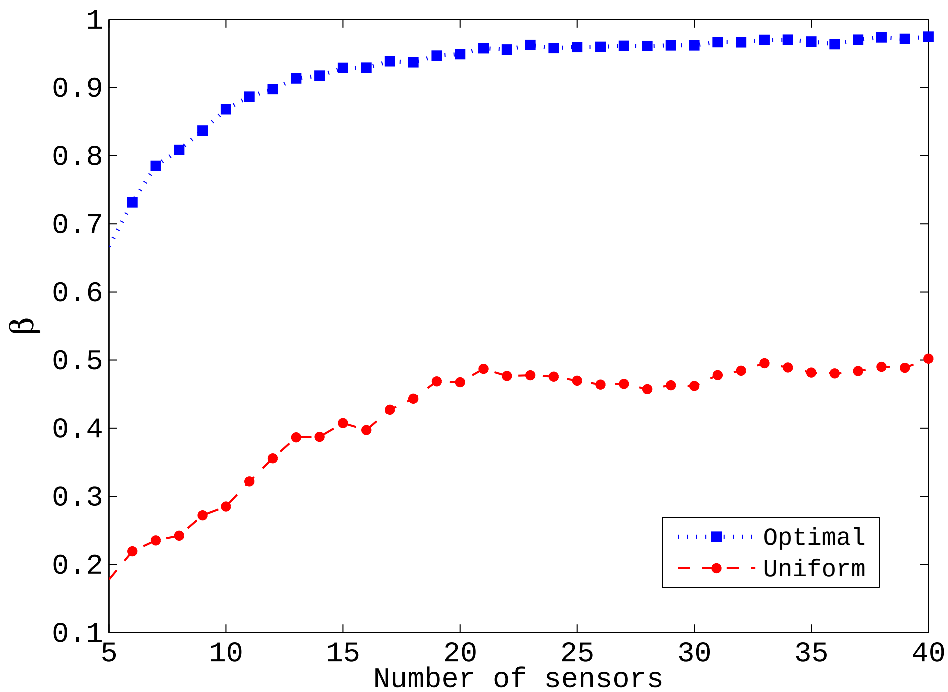

Figure 5.2. Detection Probability comparison for uniform versus optimal power scheduling

BIBLIOGRAPHY

[1] I. Kang and R. Poovendran, "Maximizing static network lifetime of wireless broadcast adhoc networks," in *Proc. IEEE IOC 2003*, Anchorage, AK, May 2003.

[2] A. Ephremides, "Energy concerns in wireless networks," *IEEE Wireless Communications*, vol. 9, no. 4, 2002.

[3] Yunxia Chen and Qing Zhao, "On the lifetime of wireless sensor networks," *IEEE Communications Letters*, vol. 9, no. 11, pp. 976–978, Nov. 2005.

[4] S. Cui, J.-J. Xiao, A. Goldsmith, and and H.Poor Z.-Q.Lou, "Energy-efficient joint estimation in sensor networks:analog vs. digital," in *Proc. ICASSP*, Philadelphia, PA, March 2005, pp. 745–748.

[5] J.-J.Xiao, S. Cui, and and A. Goldsmith Z.-Q.Lou, "Joint estimation in sensor networks under energy constraints," in *Proc. IEEE 1st Conf. Sensor and Ad Hoc Communi. and Networks,*, Santa Clara, CA, Oct. 2004, pp. 264–271.

[6] Y. Chen, Q. Zhao, Vikram Krishnamurthy, and Dejan Djonin, "Transmission scheduling for optimizing sensor network lifetime: A stochastic shortest path approach," *IEEE Trans. Signal Processing*, vol. 55, pp. 2294–2309, May 2007.

[7] Haining Shu, Qilian Liang, and Jean Gao, "Wireless sensor network lifetime analysis using interval type-2 fuzzy logic systems," *IEEE Transactions on Fuzzy Systems*, vol. 16, no. 2, pp. 416–427, April 2008.

[8] Jin Zhu and S. Papavassiliou, "On the energy-efficient organization and the lifetime of multi-hop sensor networks," *IEEE Communications Letters*, vol. 7, no. 11, pp. 537–539, Nov. 2003.

[9] J.-H. Chang and L. Tassiulas, "Maximum lifetime routing in wireless sensor network," *IEEE/ACM Trans. on Networking*, vol. 12, no. 4, pp. 609–619, Aug. 2004.

[10] Yong Ma and J. H. Aylor, "System lifetime optimization for heterogeneous sensor networks with a hub-spoke technology," *IEEE Transactions on Mobile Computing*, vol. 3, no. 3, pp. 286–294, July-Aug. 2004.

[11] R. Madan and S. Lall, "Distributed algorithms for maximum lifetime routing in wireless sensor networks," *IEEE Transactions on Wireless Communications*, vol. 5, no. 8, pp. 2185–2193, Aug 2006.

[12] M. Gatzianas and L. Georgiadis, "A distributed algorithm for maximum lifetime routing in sensor networks with mobile sink," *IEEE Transactions on Wireless Communications*, vol. 7, no. 3, pp. 984–994, March 2008.

[13] J.-F. Chamberland and V. V. Veeravalli, "Decentralized detection in sesnor networks," *IEEE Trans. Signal Process.*, vol. 51, no. 2, pp. 407–416, Feb. 2003.

[14] J.-F. Chamberland and V. V. Veeravalli, "Asymptotic results for decentralized detection in power constrained wireless sensor networks," *IEEE J. Select. Areas Commun., Special Issue on Wirelss Sesnor Networks*, vol. 22, no. 6, pp. 1001–1015, Jun 2004.

[15] Sooskan Panichppiboon, Gianluigi Ferrari, and Ozan K. Tonguz, "Optimal transmit power in wireless sensor networks," *IEEE Trans. on Mobile Computing*, vol. 5, no. 10, pp. 1432–1447, Oct. 2006.

[16] Constantino Rago, Peter Willet, and Yaakov Bar Shalom, ," .

[17] Wee Peng Tay, John N. Tsitsiklis, and Moe Z. Win, "Asymptotic performance of a censoring sensor network," *IEEE Trans. on Information Theory*, vol. 53, no. 11, pp. 4191–4209, Nov. 2007.

[18] Swaroop Appadwedula, Venugopal V. Veeravalli, and Douglas L. Jones, "Decentralized detection with censoring sensors," *IEEE Trans. Signal Process.*, vol. 56, no. 4, pp. 1362–1373, Apr. 2008.

[19] Channel aware distributed detection in wireless sensor networks, "Optimal transmit power in wireless sensor networks," *IEEE Signal Processing Magazine*, vol. 23, no. 4, pp. 16–26, July 2006.

[20] Steven M. Kay, *Foundamentals of statistical signal processing : estimation theory*, Prentice Hall, Upper Saddle River , New Jersey, 1993.

[21] Henry Stark and John W. Woods, *Probabilty and Random Processes with Applications to Signal Processing*, Prentice Hall, Upper Saddle River, New Jersey, third edition, 2002.

[22] S. Boyd and L. Vanderberghe, *Convex Optimization*, Cambridge University Press, Cambridge, UK, 2003.

[23] Dimitris Bertsimas and John N. Tsitsiklis, *Introduction to Linear Optimization*, Athena Scientific, Belmont, Massachusetts, 1997.

[24] Todd K. Moon and Wynn C. Stirling, *Mathematical Methods and Algorithms for Signal Processing*, Prentice Hall, Upper Saddle River , New Jersey, 1999.

[25] Steven M. Kay, *Foundamentals of statistical signal processing : detection theory*, Prentice Hall, Upper Saddle River , New Jersey, 1998.

VITA

Frank (Farhad) Azadi Namin was born in Denton, Texas, on October 25, 1977, to Behnam Azadi Namin and Farzaneh Ebrahim. He returned to Iran in 1978, where he spent most of his formative years. After returning to United States, he enrolled at the University of Texas at Dallas. He received a degree of Bachelor of Science with a major in Electrical Engineering in August 2006. In the fall of 2006 he entered the Graduate School of The University of Texas at Dallas, studying towards a Master of Science degree in Electrical Engineering.

Permanent address: 17817 Coit Road,
Dallas, TX 75252,
USA.

This thesis was typeset with LaTeX.