Adaptive Interference Alignment with CSI Uncertainty

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Abstract—Interference alignment (IA) is known to significantly increase sum-throughput at high SNR in the presence of multiple interfering nodes, however, the reliability of IA is little known, which is the subject of this paper. We study the error performance of IA and compare it with conventional orthogonal transmission schemes. Since most IA algorithms require extensive channel state information (CSI), we also investigate the impact of CSI imperfection (uncertainty) on the error performance. Our results show that under identical rates, IA attains a better error performance than the orthogonal scheme for practical signal to noise ratio (SNR) values but is more sensitive to CSI uncertainty. We design bit loading algorithms that significantly improve error performance of the existing IA schemes. Furthermore, we propose an adaptive transmission scheme that not only considerably reduces error probability, but also produces robustness to CSI uncertainty.

Index Terms—Interference alignment, bit error rate (BER), bit loading, SVD, spatial multiplexing, MIMO

I. INTRODUCTION

INTERFERENCE alignment (IA) achieves, at high signal-to-noise-ratio (SNR), a sum-rate increasing linearly with the number of user pairs in an interference network [1]. The main idea of IA is to coordinate transmitted signals so that the interferences are concentrated in certain subspaces at the unintended receivers. This opens up an interference-free subspace for each pair. IA achieves the maximum multiplexing gain or degree of freedom (DoF) which is significantly larger than conventional orthogonal schemes (e.g., time division multiple access (TDMA)) or certain non-orthogonal schemes (e.g., treating interference as noise) [1]. This result has inspired a great deal of research activity [2]–[12].

Many of the existing works study IA from a capacity (multiplexing gain) perspective [1]–[10], but very few works analyze the reliability and/or error performance of IA. Motivated by its practical importance, this work investigates the error performance of IA and compare it with conventional methods (e.g. TDMA with singular value decomposition (SVD)-based spatial multiplexing (SM)). In addition, most IA schemes require extensive channel state information (CSI), which is imperfect in practice. This paper also studies the impact of

CSI imperfection (uncertainty/error) on IA. We propose an adaptive method as well as bit loading algorithms for IA schemes, which produce considerable gains over existing IA methods. Specifically,

- We analyze two representative IA algorithms: minimizing interference leakage algorithm (MinIL) and maximizing signal to noise plus interference ratio (SINR) algorithm (Max-SINR) [3]. We find that under perfect CSI, although both algorithms achieve the same multiplexing gain, Max-SINR attains much lower bit error rate (BER) than MinIL (more than 7 dB SNR gain), and it always outperforms the conventional SVD-based SM. In contrast, MinIL has higher BER than SVD-based SM for some network configurations[1], even though it achieves larger multiplexing gain. We also obtain a closed-form BER expression for the MinIL algorithm.
- Our results show that among the compared schemes, Max-SINR is the most sensitive to CSI uncertainty, followed by MinIL, and SVD-based SM is the least sensitive to imperfect CSI. Nevertheless, Max-SINR still achieves the smallest BER if CSI uncertainty is less than 10% (at SNR 20 dB); if the CSI uncertainty exceeds 10%, all the three schemes have almost identical (poor) BER performances. Moreover, we find that IA algorithms exhibit error floors when CSI is imperfect.
- We propose IA bit-loading algorithms which significantly reduce BER for both MinIL and Max-SINR, producing 6 dB and 4 dB SNR gain (at BER of 10^{-2}) respectively.
- We devise an adaptive transmission scheme that switches among the three schemes. Adaptive transmission achieves 5 dB SNR gain compared with the best of the three modes (with bit loading), and is also more robust to CSI uncertainty than Max-SINR.

Some of the related literature on IA is as follows. In the presence of perfect CSI, the feasibility condition of IA in multiple-input-multiple-output (MIMO) interference networks with constant channel coefficients was investigated in [2]. In [3]–[5], different algorithms were proposed to design the precoding and receive combining vectors in IA. Recently, IA with partial CSI has attracted significant attention. For single-input-single-output (SISO) network, Bölcskei and Thukral [6] studied the achievable multiplexing gain of IA when CSI was obtained via limited feedback, which was extended to MIMO network by Krishnamachari and Varanasi [7]. Xie et al. [8]

[1]The network configuration refers to the number of user pairs, the numbers of transmit antennas and receive antennas in an interference network.

found the optimal numbers of feedback bits and cooperative user pairs so that the overall throughput was maximized at high SNR. Most recently, Ning et al. [11] investigated the reliability issue of IA from diversity perspective, showing conditions for achievability of diversity gain in IA. For the IA zero-forcing algorithm, Nosrat-Makouei et al. [12] found approximate SINR expressions in the presence of imperfect CSI and channel correlation.

The rest of the paper is organized as follows: Section II introduces the signal model and the background of IA. Section III discusses transmission schemes for the interference channel and their corresponding BER analyses. In Section IV, the bit loading algorithm and the adaptive transmission scheme are presented. Then, we assess the effects of CSI uncertainty on BER performances in Section V. Section VI presents the simulation results and corresponding discussions, and Section VII concludes this paper.

Notation: Throughout the paper, boldface lower-case letters stand for vectors while upper-case letters represent matrices. $\mathbf{A}^{\dagger}$ indicates the Hermitian transpose of $\mathbf{A}$. $\|\mathbf{a}\|$ means ℓ_2-norm. $\mathcal{CN}(\mathbf{a}, \mathbf{A})$ is complex Gaussian distribution with mean $\mathbf{a}$ and covariance matrix $\mathbf{A}$. $\mathbf{E}[\cdot]$ stands for expectation. $\mathbb{C}^{M \times N}$ is the space of complex $M \times N$ matrices. $\lfloor \cdot \rfloor$ and $\lceil \cdot \rceil$ represent floor and ceiling operation, respectively. $(\mathbf{a})_i$ indicates the ith element of the vector $\mathbf{a}$ and $(\mathbf{A})_{ij}$ indicates the ijth element of matrix $\mathbf{A}$.

II. SIGNAL MODEL

Consider a K-user $N_t \times N_r$ narrowband interference network where there are K user pairs and each transmitter and receiver are equipped with N_t and N_r antennas respectively. Each transmitter uses one DoF or sends one data stream at a time but there are K simultaneous links from the K transmitters within the same band. Assume the channel is block-fading, i.e., the channel remains the same within one frame and changes from one frame to another. $\mathbf{H}_{k\ell} \in \mathbb{C}^{N_r \times N_t}$ is the channel coefficient matrix between the transmitter ℓ and the receiver k of the considered frame (for clarity, we do not introduce the frame index here). Each entity of $\mathbf{H}_{k\ell}$ is independent and identically distributed (i.i.d.) $\mathcal{CN}(0, 1)$. The signal arriving at receiver k is

$$\mathbf{y}_k = \sum_{\ell=1}^{K} \mathbf{H}_{k\ell} \mathbf{v}_\ell s_\ell + \mathbf{w}_k, \quad \text{for } k = 1, 2 \cdots, K \quad (1)$$

where $\mathbf{v}_\ell \in \mathbb{C}^{N_t \times 1}$ is the unit-norm precoding vector associated with s_ℓ, the transmitted signal of transmitter ℓ. Each transmitter has a power constraint P, i.e., $\mathbf{E}[\|s_\ell\|^2] = P$. $\mathbf{w}_k \in \mathbb{C}^{N_r \times 1}$ is additive white Gaussian noise (AWGN) with distribution $\mathcal{CN}(\mathbf{0}, \mathbf{I})$.

At the receiver k, a unit-norm receive combining vector $\mathbf{u}_k \in \mathbb{C}^{N_r \times 1}$ is applied to suppress the interference from other streams:

$$\mathbf{u}_k^{\dagger} \mathbf{y}_k = \mathbf{u}_k^{\dagger} \mathbf{H}_{kk} \mathbf{v}_k s_k + \mathbf{u}_k^{\dagger} \sum_{\ell=1, \ell \neq k}^{K} \mathbf{H}_{k\ell} \mathbf{v}_\ell s_\ell + \mathbf{u}_k^{\dagger} \mathbf{w}_k. \quad (2)$$

Assuming interference alignment is feasible [2], the alignment is achieved when the precoding and receive combining vectors

satisfy:

$$\mathbf{u}_k^{\dagger} \mathbf{H}_{k\ell} \mathbf{v}_\ell = 0, \forall \ell \neq k \quad (3)$$

$$\mathbf{u}_k^{\dagger} \mathbf{H}_{kk} \mathbf{v}_k \neq 0, k = 1, \ldots, K. \quad (4)$$

Several algorithms [3]–[5] have been proposed to solve for the precoding and receive combining vectors. Among them, two of the most representative ones are the iterative algorithms proposed by Gomadam, Cadambe and Jafar in [3]: one aims to achieve perfect interference alignment by minimizing the interference leakage (MinIL), the other one intends to maximize the SINR of each user link (Max-SINR). In the following, we use the two algorithms[2] as examples to analyze the BER performance of IA.

III. TRANSMISSION SCHEMES FOR INTERFERENCE CHANNEL AND THEIR BER ANALYSES

A. Minimum Interference Leakage Algorithm

At the receiver k, the total interference power caused by other transmitters is

$$L_k = P \left(\mathbf{u}_k^{\dagger} \sum_{\ell=1, \ell \neq k}^{K} \mathbf{H}_{k\ell} \mathbf{v}_\ell \mathbf{v}_\ell^{\dagger} \mathbf{H}_{k\ell}^{\dagger} \mathbf{u}_k \right). \quad (5)$$

In MinIL algorithm, the precoding vectors $\{\mathbf{v}_k\}$ and the receive combining vectors $\{\mathbf{u}_k\}$ are designed to force $\{L_k\}$ to be zero [3], i.e., satisfying condition (3), so that the interference at each receiver is completely eliminated. Therefore, the post-processing received signal (2) can be rewritten as

$$\mathbf{u}_k^{\dagger} \mathbf{y}_k = \mathbf{u}_k^{\dagger} \mathbf{H}_{kk} \mathbf{v}_k s_k + (\mathbf{u}_k)^{\dagger} \mathbf{w}_k. \quad (6)$$

Denote $z_k \triangleq \mathbf{u}_k^{\dagger} \mathbf{H}_{kk} \mathbf{v}_k$, the effective channel between user pair k, is non-zero with probability 1 [3]. Therefore, in MinIL algorithm, the design of $\{\mathbf{u}_k\}$ and $\{\mathbf{v}_\ell\}$ actually focuses on condition (3) and does not involve the direct channel $\{\mathbf{H}_{kk}\}$. Thus, $\mathbf{u}_k$ and $\mathbf{v}_k$ are independent of $\mathbf{H}_{kk}$. Conditioned on $\{\mathbf{H}_{k\ell}\}_{k \neq \ell}$, and hence $\mathbf{u}_k$ and $\mathbf{v}_k$, z_k is a complex Gaussian random variable with zero mean and variance:

$$\mathbf{E}\left[|z_k|^2\right] = \mathbf{E}\left[\sum_{i=1}^{M} \sum_{j=1}^{M} |(\mathbf{u}_k)_i|^2 |(\mathbf{v}_k)_j|^2 |(\mathbf{H}_{kk})_{ij}|^2\right] \quad (7)$$

$$= \sum_{i=1}^{M} |(\mathbf{u}_k)_i|^2 \sum_{j=1}^{M} |(\mathbf{v}_k)_j|^2 \mathbf{E}\left[|(\mathbf{H}_{kk})_{ij}|^2\right] = 1,$$

where the last step holds because $\mathbf{H}_{kk}$ are i.i.d. $\mathcal{CN}(0, 1)$, and $\mathbf{u}_k$ and $\mathbf{v}_k$ have unit norm. So $|z_k|^2$ is an exponentially distributed random variable with unit mean and the effective channel under MinIL is Rayleigh. In other words, MinIL effectively decomposes the K-user interference network into K equivalent SISO Rayleigh fading channels.

At the receiver k, the post-processing SINR is

$$\gamma_k = |z_k|^2 P. \quad (8)$$

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[2] We assume global CSI is available at transmitters. The transmitters can perform the two IA algorithms and inform the receivers their corresponding receiving combining vectors via control information; or if global CSI is also available at the receivers, both transmitters and receivers can perform the algorithms themselves and hence no extra control information is needed. The CSI can be obtained by pilot and feedback signals, however, the specific mechanism of CSI exchange is out of the scope of this paper.

From (8) and the exact BER expression of $M = I \times J$ rectangular QAM in [13], the BER of IA with MinIL algorithm is

$$
\text{BER}(|z_k|) = \frac{1}{\log_2(I \cdot J)} \left(\sum_{m=1}^{\log_2 I} P_I(m) + \sum_{n=1}^{\log_2 J} P_J(n) \right) \tag{9}
$$

where

$$
P_I(m) = \frac{2}{I} \sum_{i=0}^{(1-2^{-m})I-1} \left\{ \eta(i,m,I)(-1)^{\left\lfloor \frac{i \cdot 2^{m-1}}{I} \right\rfloor} \right.
$$
$$
\left. \times Q\left(\sqrt{\frac{6(2i+1)^2 |z_k|^2 P}{(I^2 + J^2 - 2)}} \right) \right\} \tag{10}
$$

$$
P_J(n) = \frac{2}{J} \sum_{i=0}^{(1-2^{-n})J-1} \left\{ \eta(i,n,J)(-1)^{\left\lfloor \frac{i \cdot 2^{n-1}}{J} \right\rfloor} \right.
$$
$$
\left. \times Q\left(\sqrt{\frac{6(2i+1)^2 |z_k|^2 P}{(I^2 + J^2 - 2)}} \right) \right\} \tag{11}
$$

with $\eta(i,m,I) = \left(2^{m-1} - \left\lfloor \frac{i \cdot 2^{m-1}}{I} + \frac{1}{2} \right\rfloor \right)$. Since $|z_k|$ has Rayleigh distribution, we have

$$
\overline{\text{BER}}_{\text{IA}} = \int_0^{+\infty} \text{BER}(|z_k|) \times 2 |z_k| e^{-|z_k|^2} d|z_k| \tag{12}
$$
$$
= \frac{1}{\log_2(I \cdot J)} \left(\sum_{m=1}^{\log_2 I} \overline{P}_I(m) + \sum_{n=1}^{\log_2 J} \overline{P}_J(n) \right)
$$

where

$$
\overline{P}_I(m) = \frac{1}{I} \sum_{i=0}^{(1-2^{-m})I-1} \left\{ \eta(i,m,I)(-1)^{\left\lfloor \frac{i \cdot 2^{m-1}}{I} \right\rfloor} \right.
$$
$$
\left. \times \left(1 - \left(\frac{(I^2 + J^2 - 2)}{3(2i+1)^2 P} + 1 \right)^{-1/2} \right) \right\} \tag{13}
$$

$$
\overline{P}_J(n) = \frac{1}{J} \sum_{i=0}^{(1-2^{-n})J-1} \left\{ \eta(i,n,J)(-1)^{\left\lfloor \frac{i \cdot 2^{n-1}}{J} \right\rfloor} \right.
$$
$$
\left. \times \left(1 - \left(\frac{(I^2 + J^2 - 2)}{3(2i+1)^2 P} + 1 \right)^{-1/2} \right) \right\}. \tag{14}
$$

As a specific example, consider 4-QAM (QPSK), which leads to

$$
\overline{\text{BER}}_{\text{IA}} = \frac{1}{2} \left(1 - \sqrt{\frac{P}{P+2}} \right) \approx \frac{1}{2P}. \tag{15}
$$

This shows that the diversity order of IA with MinIL is one, which is consistent with the result in [11]. Intuitively, the MinIL algorithm does not have either diversity gain or array gain, since the precoding and receive combining vectors are independent of the direct channel, which leads to the equivalent channel uniformly distributed in an interference-free subspace.

B. Max-SINR Algorithm

The MinIL algorithm is suboptimal for low and intermediate SNR, because it only focuses on eliminating interference and does not consider the desired signal power. To improve the performance for low and intermediate SNR, the Max-SINR algorithm is proposed [3], where each transceiver pair maximizes its corresponding SINR instead of merely suppressing interference. In the Max-SINR algorithm, the precoding vectors $\{\mathbf{v}_\ell\}$ and the receive combining vectors $\{\mathbf{u}_k\}$ are designed in an iterative manner, so that the instantaneous SINR of the kth pair

$$
\text{SINR}_k = \frac{P \left| \mathbf{u}_k^\dagger \mathbf{H}_{kk} \mathbf{v}_k \right|^2}{1 + L_k}, \quad 1 \le k \le K \tag{16}
$$

is maximized [3] where L_k is defined in (5).

An exact BER analysis of the Max-SINR algorithm is intractable [11] because $\{\mathbf{u}_k\}$ and $\{\mathbf{v}_\ell\}$ depend on $\{\mathbf{H}_{kk}\}$, and the algorithm is iterative. Here, we provide an approximate analysis of the SINR achieved by this algorithm. First, unlike the MinIL algorithm, in the Max-SINR algorithm the interference L_k is not necessarily zero, but it is bounded as P goes to infinity. Intuitively, this can be verified as follows: If $L_k = f(P)$ where $\lim_{P \to \infty} f(P) = \infty$, then the SINR is *not* maximized, since simply forcing L_k to zero leads to higher SINR for sufficiently large P. Thus, we approximate the residual interference as a complex Gaussian random variable with bounded variance δ. Then, the post-processing SINR can be rewritten as

$$
\gamma_k^{\text{M}} = \frac{P \left| z_k^{\text{M}} \right|^2}{1 + \delta} \tag{17}
$$

where $z_k^{\text{M}} = \mathbf{u}_k^\dagger \mathbf{H}_{kk} \mathbf{v}_k$ is the equivalent channel of the kth user pair under the Max-SINR algorithm. On the one hand, the Max-SINR algorithm searches for precoding/receive combining vectors that lead to larger desired signal power relative to the MinIL algorithm, and hence a coherent combining gain is expected. On the other hand, the coherent combining gain of $|z_k^{\text{M}}|$ is upper bounded by that achieved by beamforming [14], since the Max-SINR also needs to suppress the interference.

To support the above analysis, we provide numerical results for the average desired signal power ($|z_k^{\text{M}}|^2$), and interference power (L_k) in Fig. 1, where a 3-user (3×2) interference network is considered. One can see that the residual interference power is bounded (not growing with P), and in fact the interference is at a similar power level as noise. In addition, the average power of the desired signal lies in between the powers of those in MinIL and beamforming.

C. Spatial Multiplexing

We consider spatial multiplexing (SM) as a benchmark. Here, K transceiver pairs are scheduled in a time-division manner (TDMA). When the kth pair is activated, its corresponding channel is decomposed as

$$
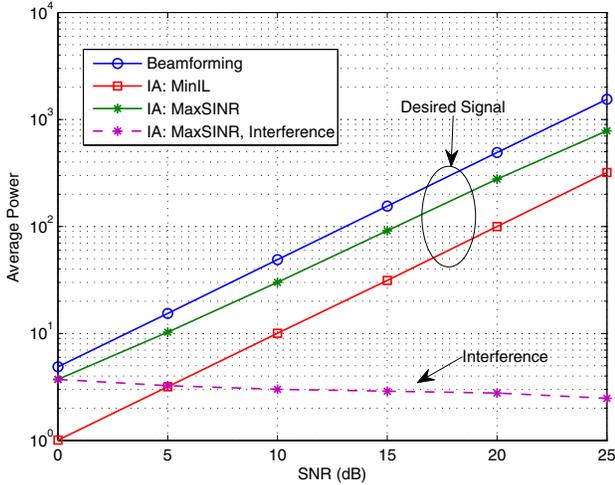\mathbf{H}_{kk} = \mathbf{U} \mathbf{S} \mathbf{V}^\dagger \tag{18}
$$

Fig. 1. Average power of desired signal and interference: $K = 3, N_t = 3, N_r = 2$

where $\mathbf{V}$ and $\mathbf{U}$ are unitary precoding matrix and receive combining matrix, and

$$\mathbf{S} = \mathrm{diag}[\lambda_1, \cdots, \lambda_{N_{\min}}, \underbrace{0, \cdots, 0}_{N_{\max} - N_{\min}}]. \qquad (19)$$

In the above, $N_{\min} = \min(N_t, N_r)$ and $N_{\max} = \max(N_t, N_r)$, and $\lambda_1 \geq \cdots \geq \lambda_{N_{\min}}$ are singular values. In this case, the MIMO channel of this user pair is decomposed into $N_{\min}$ parallel SISO links (multiplexing gain $N_{\min}$).

To communicate at the same rate and power as IA, spatial multiplexing must have constellation size $2^{KR_{\mathrm{IA}}/N_{\min}}$ and power KP, where R_{IA} is the rate used by IA per transceiver pair. The exact BER expression of SVD-based MIMO system is given by [15]; we rewrite it in a general form that is applicable to $I \times J$ rectangular QAM:

$$\overline{\mathrm{BER}}_{\mathrm{SVD}} = \frac{1}{N_{\min} \log_2(I \cdot J)} \left(\sum_{m=1}^{\log_2 I} \widehat{P}_I(m) + \sum_{n=1}^{\log_2 J} \widehat{P}_J(n) \right) \tag{20}$$

where

$$\widehat{P}_I(m) = \frac{1}{I} \sum_{p=1}^{\log_2 I} \sum_{c=0}^{(1-2^{-p})I-1} \sum_{i=1}^{N_{\min}} \sum_{m=0}^{i-1} \sum_{j=m+\Delta}^{m+\Delta+i-1} \sum_{b=0}^{j}$$

$$\frac{(-1)^{2j-\Delta-b+\left\lfloor \frac{c \cdot 2^{p-1}}{I} + \frac{1}{2} \right\rfloor} (i-1)! b!}{(i-1+\Delta)! (j-\Delta)!} \binom{j-\Delta}{m} \binom{i-1+\Delta}{i-1-m}$$

$$\binom{i-1+\Delta}{j-m} \binom{j}{b} \eta(c,p,I) \left(1 - \sum_{k=0}^{b} \binom{2k}{k} \frac{\beta_c^{\frac{1}{2}}}{2^k(\beta_c+2)^{k+\frac{1}{2}}} \right) \tag{21}$$

with $\Delta = N_{\max} - N_{\min}$ and $\beta_c = \frac{6(2c+1)^2 KP}{(I^2+J^2-2)N_{\min}}$. By substituting I with J in (21), $\widehat{P}_J(m)$ can be obtained.

To gain some insights, we consider $N_t = N_r = N$. Note that the BER performance of spatial multiplexing is essentially dominated by the smallest eigenmode. Since the minimum eigenvalue of a Wishart matrix here is exponentially

distributed with parameter N [14] [16], the smallest singular value $\lambda_{\min}$ is Rayleigh distributed with pdf $f_{\lambda_{\min}}(\lambda) = N\lambda e^{-N\lambda^2}$. For 4-QAM, at high SNR the average BER can be approximated as

$$\overline{\mathrm{BER}}_{\mathrm{SVD}} \approx \int_0^{+\infty} Q\left(\sqrt{\frac{\lambda^2 PK}{N}} \right) f_{\lambda_{\min}}(\lambda) d\lambda \tag{22}$$

$$= \frac{1}{2}\left(1 - \sqrt{\frac{PK}{PK+2N^2}} \right) \approx \frac{N^2}{2PK}.$$

One can see that the diversity order is also one in this case.

D. Performance Comparison

In this subsection, we summarize and compare the multiplexing gains, diversity orders and the coherent combining gains of the above-mentioned three transmission strategies.

From a capacity perspective, the multiplexing gain of IA is $\frac{KN_{\min}}{2}$ for both MinIL and Max-SINR algorithms whereas SM achieves $N_{\min}$ multiplexing gain. Clearly, when there are more than two user pairs in an interference network, IA has a larger asymptotic capacity.

MinIL has performance similar to a SISO channel under Rayleigh fading and offers neither diversity nor array gain. Max-SINR maximizes the SINR for every user pair and obtains a coherent combining gain. It also achieves a diversity gain when $N_t + N_r \geq K + 2$ [11] since the extra dimensions enable precoding and receive combining vectors to be selected from a group of candidates.

For SVD-based SM, the diversity order is one when $N_t = N_r$, and is $N_{\max} - N_{\min} + 1$ when $N_t \neq N_r$ [17]. The expression for the array gain is complicated and determined due to its dependence on the distributions of the singular values [15].

The above results are summarized in Table I. MinIL and Max-SINR share the same multiplexing gain but the latter also enjoys coherent combining gain and possible diversity gain. Therefore, Max-SINR outperforms MinIL. However, since the diversity order and the coherent combining gain of SVD-based SM depend on the MIMO channel configuration [15], [17], the comparison between IA and SM does not produce a definitive conclusion. Further comparisons will be presented in Section VI via simulations.

IV. BIT LOADING AND ADAPTIVE TRANSMISSION SCHEME

A. Bit Loading

Both interference alignment and SVD-based spatial multiplexing provide a number of equivalent channels or "pipes" for communication. When CSIT is available, one can exploit it by applying adaptive bit loading to those equivalent channels to reduce the error rate or to increase the data rate. In our approach, for data rate R, equal power KP/R is allocated to each bit.

For MinIL, the bit loading is:

TABLE I
PERFORMANCE COMPARISON OF A K-USER $N_t \times N_r$ MIMO INTERFERENCE NETWORK

	Multiplexing Gain	Diversity Order	Coherent Combining Gain
IA with MinIL	$KN_{\min}/2$	1	1
IA with Max-SINR	$KN_{\min}/2$	1, for $N_t + N_r = K + 1$ ≥ 1, for $N_t + N_r \geq K + 2$	$> 1, < \mathbf{E}[\vert\lambda_1\vert^2]$
SM with SVD	$N_{\min}$	$N_{\max} - N_{\min} + 1$	Depends on singular value distribution

TABLE II
ITERATIVE BIT LOADING ALGORITHM

---

1) Initialize the bit allocation scheme as $[M_1, \cdots, M_K] := [1, \cdots, 1]$.
2) Add one bit and its associated power to ith equivalent channel while the modulation sizes of other channels remain the same:

$$[\log_2(M_1), \cdots, \log_2(M_i) \cdots, \log_2(M_K)] \quad (24)$$
$$= [\log_2(M_1), \cdots, \log_2(M_i) + 1 \cdots, \log_2(M_K)],$$

for $i = 1, \cdots, K$. Totally, there are K ways to add the additional bit which are enumerated row-wisely in the following matrix:

$$\begin{pmatrix} \log_2(M_1) + 1 & \log_2(M_2) & \cdots & \log_2(M_K) \\ \log_2(M_1) & \log_2(M_2) + 1 & \cdots & \log_2(M_K) \\ \vdots & \vdots & \ddots & \vdots \\ \log_2(M_1) & \log_2(M_2) & \cdots & \log_2(M_K) + 1 \end{pmatrix}$$

3) Compute the effective BERs of the above K strategies by a weighted sum $\frac{1}{R}\sum_{i=1}^{K} P_b(i, M_i, z_i) \log_2(M_i)$. Choose and update the allocation strategy with the one which returns the minimum weighted BER.
4) Check if $\sum_{i=1}^{K} \log_2(M_i) = R$, if not, go back to 2), otherwise, return the current $[M_1, \cdots, M_K]$.

$$(M_1^\star, \cdots, M_K^\star) = \arg \min_{(M_1, \cdots, M_K)} \frac{1}{R} \sum_{i=1}^{K} P_b(i, M_i, z_i) \log_2(M_i)$$
$$(23)$$

$$s.t. \quad R = \sum_{n=1}^{K} \log_2(M_n), \quad 0 \leq M_n \leq 2^R.$$

$P_b(i, M_i, z_i)$, the instantaneous bit error probability of the ith equivalent channel, is obtained by substituting the modulation size M_i and equivalent SINR $\vert z_i \vert^2 KP/R \log_2(M_i)$ for $\vert z_k \vert^2 P$ in the general M_i-QAM BER expression for AWGN channels (9-11). The above optimization problem can be easily solved by the iterative algorithm described in Table II, which terminates in R steps.

After bit loading, the power transmitted over each equivalent channel may change. However, the directions of $\{\mathbf{u}_k\}$ and $\{\mathbf{v}_\ell\}$ still satisfy conditions (3) and (4). Therefore, $\{\mathbf{u}_k\}$ and $\{\mathbf{v}_\ell\}$ are unaffected.

The bit loading procedure for Max-SINR follows in a similar manner as MinIL:

$$(M_1^\star, \cdots, M_K^\star)$$
$$= \arg \min_{(M_1, \cdots, M_K)} \frac{1}{R} \sum_{i=1}^{K} P_b'(i, M_i, \text{SINR}_i) \log_2(M_i) \quad (25)$$

$$s.t. \quad R = \sum_{n=1}^{K} \log_2(M_n), \quad 0 \leq M_n \leq 2^R,$$

where $P_b'(i, M_i, \text{SINR}_i)$ is the BER of the equivalent channel i, obtained similarly as the corresponding value in (23) except

using SINR_i from (16). To do so a Gaussian assumption is used on interference, which is an approximation.

Unlike MinIL, in Max-SINR the precoding and receive combining vectors must be re-designed after bit loading. This is because the Max-SINR transmit and receive vectors depend on the power allocated to equivalent channels. Our approach is to apply Max-SINR (with updated bit/power allocation) once again[3] to obtain a new set of $\{\mathbf{u}_k\}$ and $\{\mathbf{v}_\ell\}$.

The bit loading for SVD-based SM is performed similarly by replacing K with $N_{\min}$:

$$\left(M_1^\star, \cdots, M_{N_{\min}}^\star\right)$$
$$= \arg \min_{\left(M_1, \cdots, M_{N_{\min}}\right)} \frac{1}{R} \sum_{i=1}^{N_{\min}} P_b(i, M_i, \lambda_i) \log_2(M_i) \quad (26)$$

$$s.t. \quad R = \sum_{n=1}^{N_{\min}} \log_2(M_n), \quad 0 \leq M_n \leq 2^R.$$

The diversity order of SVD-based SM after the above bit loading is $N_t N_r$, since the bit loading algorithm for SM includes beamforming as a special case [14]. For example, in the low-SNR regime it allows the transmitter to spend all the power in the dominant eigenmode.

B. Adaptive Transmission Scheme

As will be shown in Section VI-B, the bit loading algorithm significantly reduces the error rates of MinIL, Max-SINR and SVD-based SM. Since the performance of none of these schemes dominates at all SNR, one may consider an adaptive scheme so that for each channel condition, the best of the three is selected and used. This requires knowledge of CSI, but the three schemes already assume existence of CSI, so no further assumptions are introduced. To summarize, an adaptive transmission scheme will be designed which can switch among MinIL, Max-SINR and SVD-based SM, all with bit loading.

We first optimize each of the three transmission modes according to the CSI. Then all users select a single transmission mode which achieves the minimum BER. The adaptation rate is the same as the rate of CSI update. To elaborate, for each transmission mode, bit loading is applied to obtain the optimal constellation vectors $\{M_i^\star\}$ as described in the previous subsection. This also provides the corresponding average BER of all users at the given CSI for each mode. We then select the mode with the lowest BER:

$$m^\star = \arg \min_{m \in \{\text{IA:MinIL, IA:Max-SINR, SM:SVD}\}}$$
$$\{P_{\text{BL}}^{\text{MinIL}}, P_{\text{BL}}^{\text{SVD}}, P_{\text{BL}}^{\text{Max-SINR}}\} \quad (27)$$

---

[3]Further iterations do not produce appreciable gain.

where

$$P_{\text{BL}}^{\text{SVD}} \triangleq \frac{1}{R} \sum_{i=1}^{N_{\min}} P_b(i, M_i^\star, \lambda_i) \log_2(M_i^\star), \qquad (28)$$

$$P_{\text{BL}}^{\text{MinIL}} \triangleq \frac{1}{R} \sum_{i=1}^{K} P_b(i, M_i^\star, z_i) \log_2(M_i^\star), \qquad (29)$$

$$P_{\text{BL}}^{\text{Max-SINR}} \triangleq \frac{1}{R} \sum_{i=1}^{K} P_b'(i, M_i^\star, \text{SINR}_i^\star) \log_2(M_i^\star). \qquad (30)$$

Note that the selected mode $m^\star$ is the same for all users and remains the same during one frame. The corresponding bit allocations $\{M_i^\star\}$ in (28)-(30) could be different across K users.

The details of calculation of the bit-loading information as well as the receive and transmit filters can vary according to the system requirements. If, for example, the transmitters are base stations and the receivers are mobiles (downlink), it is reasonable that the global CSI is aggregated at the transmitter where calculations are also made, and communicated with the receivers. In an uplink scenario, the situation would be reversed.

The performance of the adaptive transmission scheme will be presented in Section VI.

V. CSI UNCERTAINTY

In practice, perfect CSI may not be available, therefore one is often interested in the performance of the communication system under partial CSI. We model the channel as [18]:

$$\mathbf{H}_{k\ell} = \sqrt{1-\epsilon}\hat{\mathbf{H}}_{k\ell} + \sqrt{\epsilon}\mathbf{W}_{k\ell} \qquad (31)$$

where $\mathbf{H}_{k\ell}$ is the channel between the transmitter ℓ and the receiver k, which is known by the receiver k, $\hat{\mathbf{H}}_{k\ell} \in \mathbb{C}^{N_r \times N_t}$ is the channel *estimate* known by all the transmitters, and $\mathbf{W}_{k\ell}$ is related to the channel error matrix and is independent of $\hat{\mathbf{H}}_{k\ell}$. All elements of $\hat{\mathbf{H}}_{k\ell}$ and $\mathbf{W}_{k\ell}$ are i.i.d. $\mathcal{CN}(0,1)$. The rationale behind assuming perfect channel state information at receiver (CSIR) but imperfect CSIT is because it is always easier to obtain CSIR relative to CSIT.[4] The parameter $0 \leq \epsilon \leq 1$ reflects uncertainty of CSIT: $\epsilon = 0$ corresponds to perfect CSIT whereas $\epsilon = 1$ means that CSIT is completely unreliable.

In the following, we will first discuss the effects of CSI uncertainty based on equal power allocation, and then describe the bit loading and adaptive transmission scheme under CSI uncertainty in the last subsection.

A. Minimum Interference Leakage Algorithm

Based on available CSIT, the transmitters design the precoding and receive combining vectors to satisfy

$$\hat{\mathbf{u}}_k^\dagger \hat{\mathbf{H}}_{k\ell} \hat{\mathbf{v}}_\ell = 0, \forall \ell \neq k \qquad (32)$$

$$\hat{\mathbf{u}}_k^\dagger \hat{\mathbf{H}}_{kk} \hat{\mathbf{v}}_k \neq 0, k = 1, \ldots, K. \qquad (33)$$

---

[4]For example, the receivers first estimate the CSI and feed it back to transmitters. Due to limited feedback or feedback delay, the transmitters only have the access to partial CSI $\hat{\mathbf{H}}_{k\ell}$ whereas the receivers always know the actual channel $\mathbf{H}_{k\ell}$ (since they can keep on estimating the CSI).

In this case, there is residual interference due to imperfect CSI and the resulting received signal is:

$$\hat{\mathbf{u}}_k^\dagger \mathbf{y}_k = \hat{\mathbf{u}}_k^\dagger \mathbf{H}_{kk} \hat{\mathbf{v}}_k s_k + \sqrt{\epsilon}\hat{\mathbf{u}}_k^\dagger \sum_{\ell=1,\ell\neq k}^{K} \mathbf{W}_{k\ell}\hat{\mathbf{v}}_\ell s_\ell + (\hat{\mathbf{u}}_k)^\dagger \mathbf{w}_k. \qquad (34)$$

Denote the equivalent channel as $\hat{z}_k = \hat{\mathbf{u}}_k^\dagger \mathbf{H}_{kk}\hat{\mathbf{v}}_k$. The instantaneous SINR of the kth user pair is

$$\begin{aligned}
\text{SINR}_k &= \frac{P|\hat{z}_k|^2}{\left(1 + P\sum_{\ell=1,\ell\neq k}^{K}\left|\hat{\mathbf{u}}_k^\dagger \mathbf{H}_{k\ell}\hat{\mathbf{v}}_\ell\right|^2\right)} \\
&= \frac{P|\hat{z}_k|^2}{\left(1 + \epsilon P\sum_{\ell=1,\ell\neq k}^{K}\left|\hat{\mathbf{u}}_k^\dagger \mathbf{W}_{k\ell}\hat{\mathbf{v}}_\ell\right|^2\right)}
\end{aligned} \qquad (35)$$

Since both $\hat{\mathbf{u}}_k$ and $\hat{\mathbf{v}}_k$ are independent of $\hat{\mathbf{H}}_{kk}$ (as mentioned in Section III-A) and $\mathbf{H}_{kk}$ (based on the channel model (31)), $\hat{z}_k$ has the same distribution as z_k, and the corresponding analysis can be adopted for $\hat{z}_k$. In addition, $\hat{\mathbf{u}}_k$ and $\hat{\mathbf{v}}_k$ are independent of $\mathbf{W}_{kk}$, and hence the interference power of the data stream k is

$$\begin{aligned}
\hat{L}_k &= \epsilon P \sum_{\ell=1,\ell\neq k}^{K} \mathbf{E}\left[\left|\hat{\mathbf{u}}_k^\dagger \mathbf{W}_{k\ell}\hat{\mathbf{v}}_\ell\right|^2\right] \\
&= \epsilon P \sum_{\ell=1,\ell\neq k}^{K} \mathbf{E}\left[\sum_i^M |(\hat{\mathbf{u}}_k)_i|^2 \sum_j^M |(\hat{\mathbf{v}}_\ell)_j|^2 |(\mathbf{W}_{k\ell})_{ij}|^2\right] \\
&= \epsilon(K-1)P,
\end{aligned} \qquad (36)$$

which suggests that the interference power grows with the transmit power P, the number of users in the network K, and the level of CSI uncertainty ϵ. As a result, the post-processing SINR can be written as

$$\hat{\gamma}_k = \frac{|\hat{z}_k|^2 P}{\epsilon(K-1)P + 1}. \qquad (37)$$

Note that (37) reduces to (8) if $\epsilon = 0$.

When CSIT is imperfect, with the assumption of Gaussian interference, the BER expression of MinIL can be represented by (12) but with modified $\overline{P}_I(m)$ and $\overline{P}_J(n)$:

$$\begin{aligned}
\overline{P}_I(m) = \frac{1}{I} \sum_{i=0}^{(1-2^{-m})I-1} &\left\{\eta(i,m,I)(-1)^{\left\lfloor\frac{i\cdot2^{m-1}}{I}\right\rfloor}\right. \\
&\left.\left(1 - \left(\frac{(I^2+J^2-2)(\epsilon(K-1)P+1)}{3(2i+1)^2P}+1\right)^{-1/2}\right)\right\}
\end{aligned} \qquad (38)$$

$$\begin{aligned}
\overline{P}_J(n) = \frac{1}{J} \sum_{i=0}^{(1-2^{-n})J-1} &\left\{\eta(i,n,J)(-1)^{\left\lfloor\frac{i\cdot2^{n-1}}{J}\right\rfloor}\right. \\
&\left.\left(1 - \left(\frac{(I^2+J^2-2)(\epsilon(K-1)P+1)}{3(2i+1)^2P}+1\right)^{-1/2}\right)\right\}.
\end{aligned} \qquad (39)$$

B. Max-SINR Algorithm

For Max-SINR with imperfect CSIT, the precoding $\{\hat{\mathbf{u}}_k\}$ and receive combining vectors $\{\hat{\mathbf{v}}_k\}$ are designed to maximize

$$\text{SI}\hat{\text{N}}\text{R}_k = \frac{P \left| \hat{\mathbf{u}}_k^\dagger \hat{\mathbf{H}}_{kk} \hat{\mathbf{v}}_k \right|^2}{\left(1 + P \sum_{\ell=1, \ell \neq k}^K \left| \hat{\mathbf{u}}_k^\dagger \hat{\mathbf{H}}_{k\ell} \hat{\mathbf{v}}_\ell \right|^2 \right)}. \quad (40)$$

Using the above design, the actual equivalent channel is

$$\hat{\mathbf{u}}_k^\dagger \mathbf{H}_{kk} \hat{\mathbf{v}}_k = \sqrt{1-\epsilon} \hat{\mathbf{u}}_k^\dagger \hat{\mathbf{H}}_{kk} \hat{\mathbf{v}}_k + \sqrt{\epsilon} \hat{\mathbf{u}}_k^\dagger \mathbf{W}_{kk} \hat{\mathbf{v}}_k$$
$$= \sqrt{1-\epsilon} \hat{z}_k^{\text{M}} + \sqrt{\epsilon} \hat{\mathbf{u}}_k^\dagger \mathbf{W}_{kk} \hat{\mathbf{v}}_k. \quad (41)$$

The actual interference power is

$$\hat{L}_k^{\text{M}} = P \sum_{\ell=1, \ell \neq k}^K \mathbf{E} \left[\left| \hat{\mathbf{u}}_k^\dagger \mathbf{H}_{k\ell} \hat{\mathbf{v}}_\ell \right|^2 \right] \quad (42)$$

$$= P \sum_{\ell=1, \ell \neq k}^K \mathbf{E} \left[\left| \sqrt{1-\epsilon} \hat{\mathbf{u}}_k^\dagger \hat{\mathbf{H}}_{k\ell} \hat{\mathbf{v}}_\ell + \sqrt{\epsilon} \hat{\mathbf{u}}_k^\dagger \mathbf{W}_{k\ell} \hat{\mathbf{v}}_\ell \right|^2 \right]$$

$$= (1-\epsilon)\hat{\delta} + \epsilon(K-1)P. \quad (43)$$

The last step follows the analysis that led to (36), since $\hat{\mathbf{u}}_k$ and $\hat{\mathbf{v}}_\ell$ are independent of $\mathbf{W}_{k\ell}$. Note that $\hat{z}_k^{\text{M}} = \hat{\mathbf{u}}_k^\dagger \hat{\mathbf{H}}_{kk} \hat{\mathbf{v}}_k$ and $\hat{\delta} = P \sum_{\ell=1, \ell \neq k}^K \left| \hat{\mathbf{u}}_k^\dagger \hat{\mathbf{H}}_{k\ell} \hat{\mathbf{v}}_\ell \right|^2$ have the same distributions as z_k^{M} and δ in (17), respectively. Therefore, the resulting actual post-processing SINR is:

$$\hat{\gamma}_k^{\text{M}} = \frac{\left| \hat{\mathbf{u}}_k^\dagger \mathbf{H}_{kk} \hat{\mathbf{v}}_k \right|^2 P}{\hat{L}_k^{\text{M}} + 1} = \frac{(1-\epsilon) \left| \hat{z}_k^{\text{M}} \right|^2 P + \epsilon P}{(1-\epsilon)\hat{\delta} + \epsilon(K-1)P + 1}. \quad (44)$$

Compared with MinIL (see (37)), in Max-SINR, CSI uncertainty has a bigger impact on the post-processing SINR. For MinIL in (37), the CSI uncertainty causes a residual interference of power $\epsilon(K-1)P$ but does not reduce the equivalent desired signal power. On the contrary, for Max-SINR in (44), CSI uncertainty affects two interference terms: $\epsilon(K-1)P$ and $(1-\epsilon)\hat{\delta}$. Moreover, the average desired signal power is influenced by imperfect CSIT. The desired signal power now splits into two parts—-one comes from the equivalent desired power $\left| \hat{z}_k^{\text{M}} \right|^2$ based on CSIT but is scaled by $(1-\epsilon)$ and the other one relates to the CSI imperfection. As such, Max-SINR is more sensitive to CSI uncertainty than MinIL.

C. Spatial Multiplexing

With CSI uncertainty, the exact BER expression of $I \times J$ rectangular QAM in SVD-based MIMO system is given as below [15]:

$$\overline{\text{BER}}_{\text{SVD}} = \frac{2e^{\frac{\epsilon}{1-\epsilon}}}{N_{\min} \log_2(I \cdot J)} \left(\sum_{m=1}^{\log_2 I} \widehat{P}_I(m) + \sum_{n=1}^{\log_2 J} \widehat{P}_J(n) \right) \quad (45)$$

where

$$\widehat{P}_I(m) = \frac{1}{I} \sum_{p=1}^{\log_2 I} \sum_{c=0}^{(1-2^{-p})I-1} \sum_{i=1}^{N_{\min}} \sum_{m=0}^{i-1} \sum_{j=m+\Delta}^{m+\Delta+i-1} \sum_{b=0}^{j} \binom{j-\Delta}{m}$$

$$\frac{(-1)^{2j-\Delta-b+\left\lfloor \frac{c \cdot 2^{p-1}}{I} + \frac{1}{2} \right\rfloor} (i-1)! \left(\frac{\epsilon}{1-\epsilon} \right)^{2(j-b)}}{(i-1+\Delta)!(j-\Delta)!} \binom{i-1+\Delta}{i-1-m}$$

$$\binom{i-1+\Delta}{j-m} \binom{j}{b} \eta(c, p, I) \int_{\frac{\epsilon}{1-\epsilon}}^{\infty} Q\left(\sqrt{\beta_c \lambda} \right) \lambda^b e^{-\lambda} d\lambda \quad (46)$$

with a modified $\beta_c = \frac{6(2c+1)^2(1-\epsilon)}{(I^2+J^2-2)(N_{\min}/(KP)+(N_{\min}-1)\epsilon)}$. Similarly, $\widehat{P}_J(m)$ is obtained by replacing I with J in (46).

D. Bit Loading and Adaptive Transmission Scheme

In the presence of CSI uncertainty, the proposed bit loading (Section IV-A) and the adaptive algorithm (Section IV-B) can still be applied, where all the calculations and selection are based on the available CSIT. To fit in the channel model in this section, z_i is substituted by $\hat{z}_i$ in (23), SINR$_i$ in (25) is replaced by SI$\hat{\text{N}}$R$_i$ in (40) and λ_i is substituted by $\hat{\lambda}_i$ in (26) (where $\hat{\lambda}_i$ is the singular value of $\hat{\mathbf{H}}_{kk}$). Although the CSIT is not perfect, the bit loading and the adaptive transmission scheme still provide additional gains compared with the non-bit-loaded case, as shown in Section VI.

VI. SIMULATION RESULTS AND DISCUSSION

In this section, the BER performances of IA with MinIL and Max-SINR as well as the SVD-based SM are evaluated via Monte Carlo simulations. This section is divided into two parts: the performances with and without bit loading. For each part, we first focus on the perfect CSIT cases and then discuss the effects of CSI uncertainty. In our simulations, the transmit power of all the schemes are kept the same. Moreover, the data rate is maintained the same among the three transmission modes: IA transmits at rate R_{IA} per transceiver pair, while SM transmits at rate KR_{IA}. The numbers of iterations of MinIL and Max-SINR are both set to be 100.

A. Performances of IA and SM without Bit Loading

1) Perfect CSIT: The solid lines in Fig. 2 are the BERs of a 3-user 2×2 interference network with perfect CSIT. The analytical BER expression of IA with MinIL agrees very well with the simulation. The diversity order of MinIL, Max-SINR and SM are all one, which verifies the analyses in Section III.

As shown in Fig. 2, IA with MinIL outperforms SM with SVD with about 2 dB SNR gain. This is because a smaller modulation size is used by MinIL while achieving the same data rate. In particular, MinIL uses 4-QAM in each user pair, while SM uses 8-QAM on every spatial channel. Moreover, even if both modes use the same modulation, 4-QAM, recalling (15) and (22) in Section III, the effective power gain ratio of MinIL and SM is approximately $\frac{N^2}{K} = \frac{4}{3}$. This power gain also reduces the BER of MinIL relative to SM.

Compared with the above two modes, Max-SINR has much lower BER. For instance, given BER around 10^{-2}, Max-SINR has more than 7 dB SNR gain over MinIL. This is
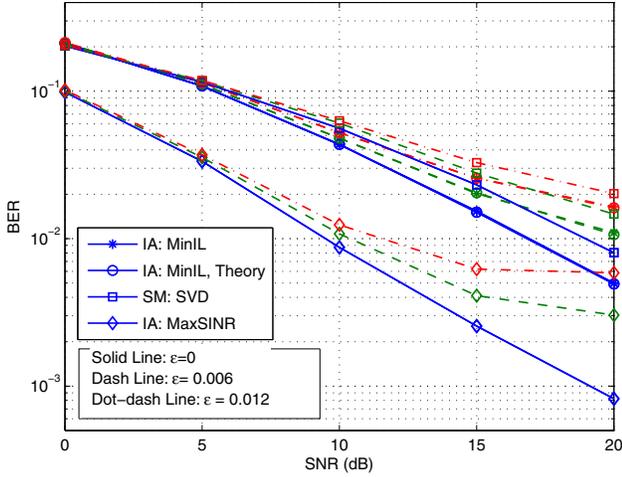
Fig. 2.   BER performances of the three transmission modes without bit loading: $K = 3, N_t = N_r = 2$; ϵ represents CSIT uncertainty: $\epsilon = 0$ corresponds to perfect CSIT and $\epsilon = 1$ means the CSIT is completely unreliable.
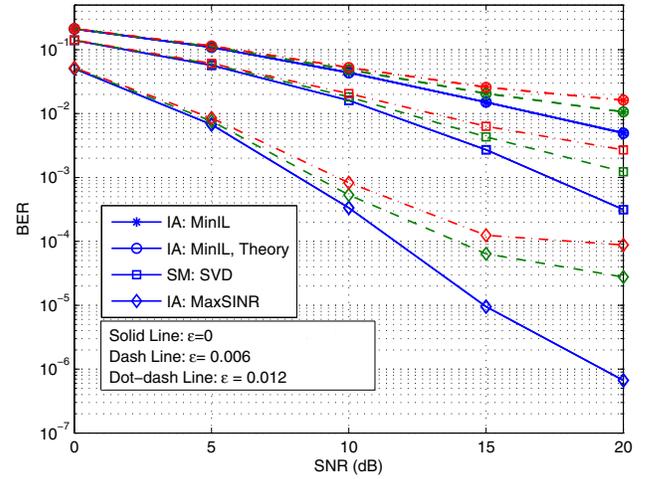

Fig. 3.   BER performances of the three transmission modes without bit loading: $K = 3, N_t = 3, N_r = 2$.
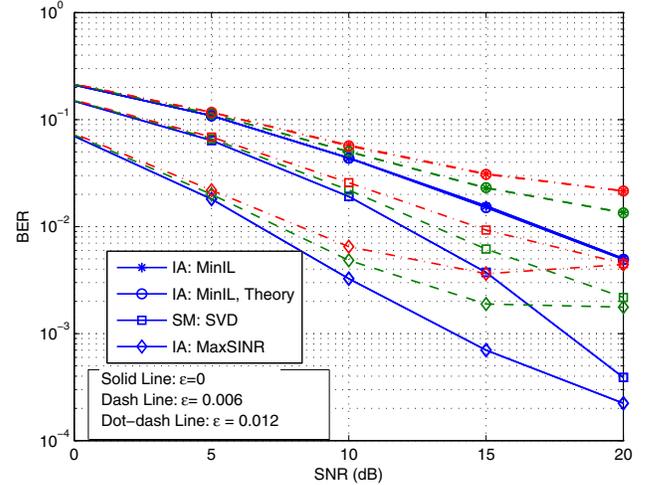

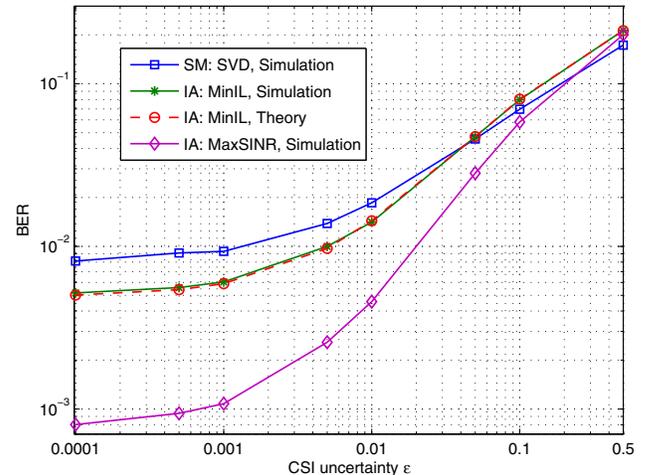Fig. 4.   BER performances of the three transmission modes without bit loading: $K = 4, N_t = 3, N_r = 2$.

Fig. 5.   BER performances of the three transmission modes in the presence of CSI uncertainty: $K = 3, N_t = N_r = 2, \mathrm{SNR} = 20$ dB

because Max-SINR makes more effective use of the desired channel, giving rise to the substantial coherent combining gain. Therefore, although MinIL and Max-SINR achieve the same multiplexing gain, Max-SINR is much superior than MinIL in terms of reliability.

Next, we consider a 3-user asymmetric 3×2 interference network. From Fig. 3, both Max-SINR and SM achieve diversity gain, while the diversity order of MinIL is still limited to one. Since in such a system configuration, $N_t + N_r = K + 2$, Max-SINR extracts diversity gain when operating at multiplexing gain of three [11]. Max-SINR attains the best BER performance among the three, showing around 9 dB SNR gain relative to SM. For SVD-based SM, diversity order $(N_{\max} - N_{\min} + 1) = 2$ is achieved. Due to the diversity gain as well as coherent combining gain produced by an extra transmit antenna [15], SM achieves smaller BER than MinIL for all considered SNRs.

Note that with a 3×2 MIMO configuration, IA is capable to accommodate 4 user pairs based on the feasibility condition [2]. With the full multiplexing gain being exploited, the diversity order of IA with either MinIL or Max-SINR is one. However, in this case SM has diversity order two. Fig. 4 shows that although SM has a steeper slope of BER curve (higher diversity order), Max-SINR achieves lower BER than SM for low and moderate SNR. Intuitively, this is because Max-SINR has large multiplexing gain as well as substantial coherent combining gain.

2) CSI Uncertainty: Now, we investigate the effects of imperfect CSIT. Based on the dash and dot-dash lines in Figs. 2 - 4, with imperfect CSIT, all three modes have error floors at high SNR. As the CSI uncertainty increases, the SNR where the error floor starts to occur is reduced. To provide an alternative view, Fig. 5 presents the BER curves versus difference levels of CSI uncertainty for a symmetric 3-user 2×2 interference network at SNR of 20 dB. The BER performances of the three modes all degrade as CSI uncertainty grows. Among the three, Max-SINR is most sensitive to CSI

uncertainty, while SM is the most robust one. However, Max-SINR still outperforms the other two modes for $\epsilon \leq 0.1$. When $\epsilon > 0.1$, all the three modes have almost the same level of poor BER.

To sum up, given the same data rate and sum power, MinIL outperforms SM for symmetric MIMO channels, while SM is better than MinIL for asymmetric MIMO channels. Max-SINR attains the smallest BER for low to intermediate SNR, regardless of channel configuration, but it is most vulnerable to CSI uncertainty.

B. Performances of IA and SM with Bit Loading

1) Perfect CSIT: From Fig. 6 where bit loading is applied, BER is significantly reduced compared with no bit loading (Fig. 2). For example, with perfect CSIT, for SNR at 15 dB, the BERs of MinIL, Max-SINR, and SVD-based SM decrease to 2×10^{-3}, 2×10^{-4}, and 7×10^{-4}, from 1.5×10^{-2}, 2.5×10^{-3}, and 2.5×10^{-2}, respectively. In fact, bit loading improves the error performances of MinIL and Max-SINR by 6 dB and 4 dB SNR gain, respectively, given an BER of 10^{-2}. Moreover, the diversity orders of all three modes go beyond one. Diversity gains are achieved for all three modes: the bit loading algorithm allows the transmitter to transmit along only one equivalent channel if others are under deep fades. After bit loading, SM outperforms MinIL, even for a symmetric MIMO channel. To see this, consider the case when only one equivalent channel is activated for both MinIL and SM. Now, IA reduces to point-to-point MIMO channel with a single equivalent Rayleigh fading channel $\mathbf{u}_k^\dagger \mathbf{H}_{kk} \mathbf{v}_k$. In contrast, SM transmits at the maximum eigenmode, achieving a larger diversity gain and power gain [17].

For Max-SINR with bit loading, its BER remains the smallest among all three schemes for low to intermediate SNR, but is inferior to SM for high SNR. This is because after bit loading the diversity order of SM is about $N_t N_r = 4$, which is larger than that of Max-SINR. The interference alignment constraints reduce the capability of Max-SINR to explore the diversity.

The adaptive transmission scheme attains the lowest BER among all the modes. For example, in Fig. 6, about 5 dB SNR gain is achieved for BER at 2×10^{-5}. This additional gain of the adaptive scheme comes from the better exploitation of the available CSI as well as the selection of the best transmission mode for each channel realization.

2) CSI Uncertainty: In Fig. 7, we illustrate the BER performance for bit loaded cases when there is CSI uncertainty. Here, we consider a 3-user 2×2 interference network with SNR 15 dB. Since the bit loading algorithms are all based on the imperfect CSIT, all the three modes and the adaptive transmission scheme degrade as the CSI uncertainty grows. It is interesting to note that the adaptive transmission scheme not only reduces the error rate but also enhances robustness to CSI uncertainty compared with Max-SINR. In this example, the adaptive transmission scheme is better than SM when $\epsilon < 0.15$, while Max-SINR outperforms SM only when $\epsilon \leq 0.05$.

VII. CONCLUSION

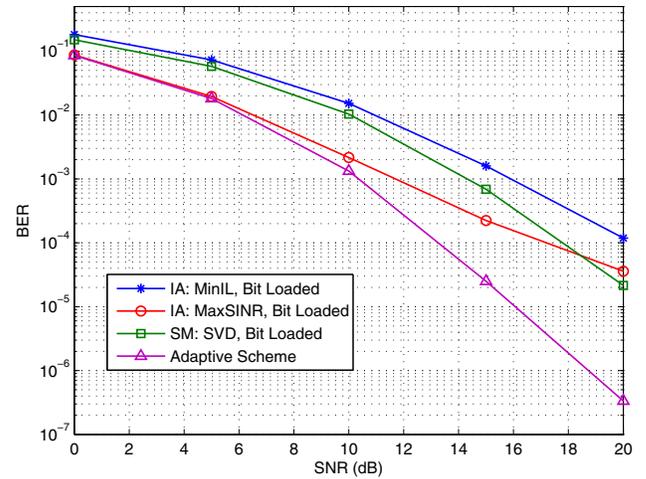This paper investigates the BER performances of IA schemes and the impact of CSI uncertainty, and in addition

Fig. 6. BER performances of the three transmission modes with bit loading and the adaptive transmission scheme for perfect CSIT: $K = 3, N_t = N_r = 2$
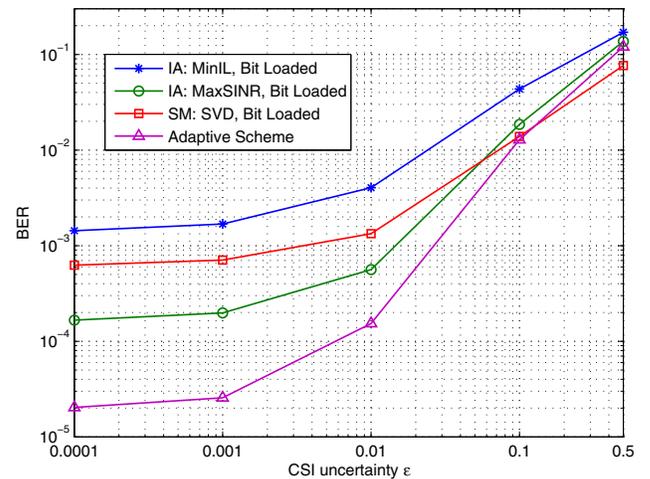

Fig. 7. BER performances of the three transmission modes with bit loading and the adaptive transmission scheme in the presence of CSI uncertainty: $K = 3, N_t = N_r = 2, \mathrm{SNR} = 15$ dB

proposes bit loading algorithms for IA as well as an adaptive transmission scheme. Two representative IA algorithms, MinIL and Max-SINR, are studied. We compare the BER performances of the two with another transmission mode, SM with SVD. Max-SINR always outperforms the other two for low to intermediate SNR but it is sensitive to CSI uncertainty. Specifically, Max-SINR is superior to other schemes as long as the CSI uncertainty is less than 10%. If the CSI uncertainty is above 10%, all three schemes have approximately the same (poor) BER performance. Our proposed IA bit-loading algorithm significantly improves the error performances of MinIL and Max-SINR. Adaptive transmission achieves an even better performance than the best of the three individual transmission modes (with bit loading) and offers robustness to CSI uncertainty as well.

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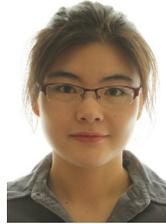
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