Coded Cooperation in Wireless Communications: Space-Time Transmission and Iterative Decoding

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***Abstract*—When mobiles cannot support multiple antennas due to size or other constraints, conventional space-time coding cannot be used to provide uplink transmit diversity. To address this limitation, the concept of *cooperation diversity* has been introduced, where mobiles achieve uplink transmit diversity by relaying each other's messages. A particularly powerful variation of this principle is *coded cooperation*. Instead of a simple repetition relay, coded cooperation partitions the codewords of each mobile and transmits portions of each codeword through independent fading channels. This paper presents two extensions to the coded cooperation framework. First, we increase the diversity of coded cooperation in the fast-fading scenario via ideas borrowed from space-time codes. We calculate bounds for the bit- and block-error rates to demonstrate the resulting gains. Second, since cooperative coding contains two code components, it is natural to apply turbo codes to this framework. We investigate the application of turbo codes in coded cooperation and demonstrate the resulting gains via error bounds and simulations.**

***Index Terms*—Channel coding, diversity, space-time coding, user cooperation, wireless communications.**

I. Introduction

It has long been known [1] that path diversity improves the effective SNR of a fading wireless channel. In particular, transmitters that possess multiple antennas can use block and trellis space-time codes [2]–[4] to reduce the bit- and block-error probability and improve the end-to-end system performance. However, in many cases, mobiles may not be able to support multiple antennas due to size or other constraints. In these cases, conventional space-time codes cannot be used.

In a single-user scenario, this would be the end of discussion. However, most wireless systems operate in a multiuser mode. Thus, the idea of user cooperation was born, where mobiles share their antennas to achieve uplink transmit diversity. Fig. 1 shows the basic idea behind this concept. Since each of the users sees an independent fading path to the base station, diversity is obtained by transmitting each user's data through both paths.

The fundamental ideas behind cooperation can be found in the literature on the relay channel [5] and the work of Willems on the multiple access channel [6]. However, the earliest work specifically on user cooperation is due to Sendonaris *et al.* [7], [8]. This work presents an information-theoretic model, for which achievable rate regions and outage probabilities are examined, as well as a CDMA implementation, in which two users cooperate by transmitting each bit over two successive bit intervals. In the first interval, the bit is transmitted by a user, and in the second interval, the partner detects and retransmits his estimate of that bit. Laneman and Wornell [9], [10] extended this work by examining outage probabilities and proposing an amplify-and-forward relay as the best option under most SNR scenarios of interest.

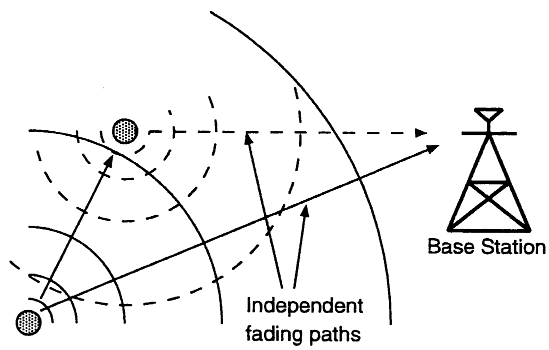

Fig. 1. Cooperation between mobiles.

In the above methods, the partner *repeats* the received bits (via either forwarding or hard detection). Recently, a different framework called *coded cooperation* was proposed [11]–[13], where symbols are not repeated by the partner. Instead, the codeword of each user is partitioned into two sets; one partition is transmitted by the user, and the other by the partner. We review this method further in Section II. Coded cooperation provides significant performance gains for a variety of channel conditions. In addition, by allowing different code rates and partitions, coded cooperation provides a great degree of flexibility to adapt to channel conditions.

In this paper, we introduce two extensions to the coded cooperation framework of [11]–[13] that significantly improve performance under a variety of scenarios. The first extension transmits part of the users' codewords in a manner reminiscent of space-time codes;[1] thus, we refer to this technique as *space-time cooperation*. This extension allows the users to capture better space-time diversity in fast fading, compared with coded cooperation.

Furthermore, since coded cooperation involves two code components, turbo codes are a natural fit. We investigate

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[1]Depending on the multiple access mechanism, an actual space-time code may be used.

turbo-coded cooperation in the context of both original coded cooperation and space-time cooperation. We demonstrate that turbo-coded cooperation improves performance over noncooperative turbo-coded systems that have comparable computational complexity. Our turbo codes are decoded at the base station; the turbo code component does not add to the mobile complexity.

We analyze the performance of space-time cooperation over slow and fast frequency nonselective fading channels. Tight union bounds for bit error rate (BER) and block error rate (BLER), which are verified through simulations, are developed by applying the tools and techniques from Simon and Alouini [14] and Malkamäki and Leib [15]. Full diversity order is achieved in slow fading when both users cooperate. In fast fading, higher diversity order is achieved compared with coded cooperation and no cooperation.

II. Overview of Coded Cooperation

As mentioned above, coded cooperation works by sending each user's codewords via two independent fading paths. The basic idea behind coded cooperation [11]–[13] is that each user tries to transmit incremental redundancy for his partner. Whenever that is not possible, the users automatically revert back to a noncooperative mode. The key to the efficiency of coded cooperation is that all this is managed automatically through code design, and there is no need for feedback between users.

The users segment their source data into blocks that are augmented with a cyclic redundancy check (CRC) code [16], such that there are a total of K bits per source block (including the CRC bits). Each block is then encoded with an error correcting code so that, for an overall rate R code, we have $N = K/R$ total code bits per source block. The two users cooperate by dividing the transmission of their N-bit codewords into two successive time segments, or *frames*. In the first frame, each user transmits a rate $R_1 > R$ codeword with $N_1 = K/R_1$ bits. This higher rate code can be obtained, for example, by puncturing the original codeword. Each user receives and decodes his partner's first frame. If the user successfully decodes the partner's rate R_1 codeword, the user computes and transmits N_2 additional parity bits for the partner's data in the second frame ($N_1 + N_2 = N$). For example, if the first frame was obtained via puncturing, these N_2 bits could be the puncture bits left out of the first frame.

Whenever a user is unable to successfully decode his partner's message, the user will revert to a noncooperative mode by calculating and transmitting N_2 parity bits for his own message. If a user successfully decodes the partner but not vice versa, both users will transmit the partner's bits in the second frame. These bits are optimally combined at the base station prior to decoding (the simulation results in this paper reflect this operation). The base station needs to know whose bits each user is transmitting in the second frame. A simple solution is that the base station can simply decode according to each of the possibilities in succession until successful decoding.[2]

[2]It is possible that the base station runs out of options without correct CRC check. In this case, the codeword with the lowest path metric from the Viterbi algorithm will be chosen.

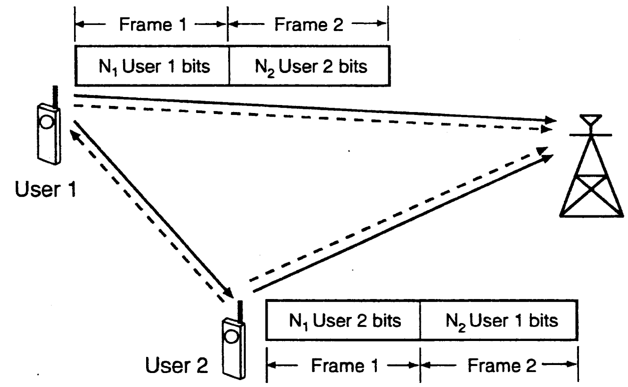

Fig. 2. Coded cooperation transmission scheme.

This strategy maintains the overall system performance and the rate at the cost of some added complexity at the base station. In [13], it is shown that this added complexity is minimal if implemented carefully (50% or less under most conditions).

In coded cooperation, each user always transmits a total of N bits per source block over the two frames, and the users only transmit in their own multiple access channels. We define the level of cooperation as N_2/N, which is the percentage of the total bits per each source block that the user transmits for his partner. Fig. 2 illustrates the operation of the scheme. In addition, half-duplex operation for mobiles is usually necessary, which is possible by assigning orthogonal channels to users [10], [13].

Various channel coding methods can be used within the coded cooperation framework. For example, the overall code may be a block or convolutional code or a combination of both. The partitioning of the code bits for the two frames may be achieved through puncturing, product codes, or other forms of concatenation. Performance results reported in [12] and [13] were obtained via a simple but effective implementation of coded cooperation using rate-compatible punctured convolutional (RCPC) codes [17]. In this implementation, the overall rate R code is selected from a given RCPC code family (for example, the mother code). The codeword for the first frame is then obtained by applying the puncturing matrix corresponding to rate R_1, and the additional parity bits transmitted in the second frame are those bits that are punctured in the first frame.

Coded cooperation achieves full diversity when both users cooperate (i.e., when both users successfully decode each other) and gives impressive gains in BER for the case of slow fading. However, due to the nature of the framework, under fast fading, spatial and temporal diversity are not simultaneously exploited by the coded cooperation of [12] and [13]. This is the key motivation for applying space-time coding principles to coded cooperation, which we describe in the following section.

III. Coded Cooperation With Space-Time Transmission

A. System Model

For the purposes of exposition, we consider a cellular system in which two mobiles are communicating with a base station. The channels between each user and the base station (uplink channels) are independent of each other and independent of the

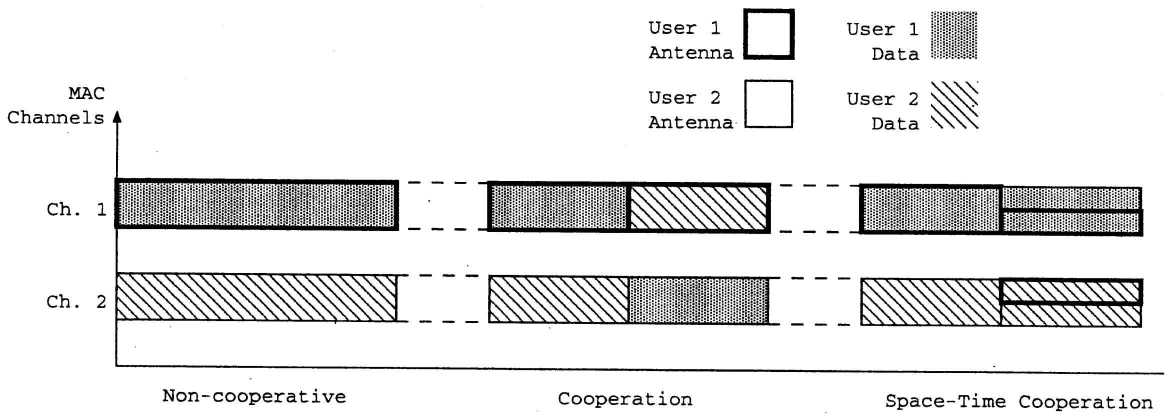

Fig. 3. Space-time cooperation compared to the original coded cooperation and no cooperation.

channel between the users (interuser channel). All channels are subject to flat (frequency nonselective) fading. We consider flat fading in order to isolate the benefits of spatial diversity; however, the cooperation techniques presented here easily extend to wideband systems that experience frequency selective fading. Furthermore, these techniques apply equally well to wireless local area networks and *ad hoc* networks (as in [9] and [10]).

The users are allocated orthogonal channels as part of an overall multiple access scheme. We consider the case in which the receivers maintain channel state information and employ coherent detection. For modulation using binary phase-shift keying (BPSK), we model the baseband-equivalent discrete-time signal transmitted by user $i \in \{1,2\}$ as $s_i(n) = \sqrt{E_{b,i}} \cdot b_i(n)$, where $E_{b,i}$ is the transmitted energy per bit for user i, and $b_i(n) \in \{-1,+1\}$ is the BPSK-modulated code bit at time n. The corresponding signal received by user $j \in \{0,1,2\}$ ($j \neq i$, and $j = 0$ denotes the base station) is

$$r_{i,j}(n) = \alpha_{i,j}(n)s_i(n) + z_j(n) \tag{1}$$

where $\alpha_{i,j}(n)$ is the fading coefficient magnitude between users i and j, and $z_j(n)$ accounts for noise and other additive interference at the receiver. We model $\alpha_{i,j}(n)$ as independent samples of a Rayleigh-distributed random variable characterized by mean-square value $\Omega_{i,j} = E\left[\alpha_{i,j}^2(n)\right]$, where the value of $\Omega_{i,j}$ accounts for large-scale path loss and shadowing. For slow (quasistatic) fading, the fading coefficients remain constant ($\alpha_{i,j}(n) = \alpha_{i,j}$) over the transmission of each source block, whereas for fast fading, they are i.i.d. for each transmitted symbol. The noise term $z_j(n)$ is modeled as independent, zero-mean additive white Gaussian noise with variance $N_j/2$.

The instantaneous received SNR for the channel between users i and j given by $\gamma_{i,j}(n) = \alpha_{i,j}^2(n)E_{b,i}/N_j$. For $\alpha_{i,j}(n)$ Rayleigh distributed, $\gamma_{i,j}(n)$ has an exponential distribution with mean

$$\overline{\gamma_{i,j}} = E_{\alpha_{i,j}}\left[\gamma_{i,j}(n)\right] = E_{\alpha_{i,j}}\left[\frac{\alpha_{i,j}^2(n)E_{b,i}}{N_j}\right] = \Omega_{i,j}\frac{E_{b,i}}{N_j}. \tag{2}$$

We assume that the channel statistics, e.g., $\Omega_{i,j}$ and $\overline{\gamma_{i,j}}$, do not change with time. We quantify the overall quality of each channel by its corresponding average received SNR as given by (2). We consider cases in which the average received SNR for the two uplink channels $\overline{\gamma_{1,0}}$ and $\overline{\gamma_{2,0}}$ are equal (statistically similar channels) and unequal (statistically dissimilar channels). This symmetry or asymmetry may result from the relative proximity of the cooperating users to each other and to the base station.

In [7]–[10], the channel between users i and j is assumed to be reciprocal, i.e., $\alpha_{i,j}(n) = \alpha_{j,i}(n)$, which is justifiable for time division multiple access (TDMA) and code division multiple access (CDMA) systems with slow fading. The results for slow fading presented in this work reflect this assumption, noting that in [13], it is shown that reciprocity or lack of it does not significantly affect the results. For fast fading, $\alpha_{i,j}(n)$ and $\alpha_{j,i}(n)$ are considered independent (unless the user's first frame transmissions are perfectly synchronized, which is definitely not true for TDMA and generally not true for CDMA).

B. Space-Time Cooperation

Our extension of the coded cooperation framework is illustrated in Fig. 3. Unlike the original coded cooperation framework, where users transmit their partner's data in the second frame (whenever possible), in the new method, which we call space-time cooperation, the users send both their own as well as their partner's parity bits in the second frame. This strategy is effective in the fast fading channel for the two reasons given below.

Under fast fading, a user's uplink channel sees independent fading between the first and second frames; thus, using the partner's channel in the second frame does not provide any added benefit. In space-time cooperation, the second frame by itself enjoys path diversity because each user transmits *both* users' parities during the second frame.

Furthermore, the users, by sending both their own as well as their partner's data, are hedging their bets against adverse conditions in the interuser channel. Recall that each user makes independent decisions on cooperation, based on the reception of partner's data. If only one of the cooperating users receives the other correctly but not vice versa, then one of the users will benefit from the transmissions of *both* second frames, whereas the other will not. It has been shown [13] that in the context of slow fading, especially with a reciprocal inter-user channel, the impact of these imbalances are minimal because the probability of imbalances are very small. In fast fading, that is not so. By using part of their power in the second frame for their own data, the users reduce the impact of such adverse conditions.

The details of the space-time cooperation are as follows. The encoded block for User i $\boldsymbol{y_i}$ is divided into the two frames ($\boldsymbol{v_i}$ and $\boldsymbol{s_i}$) such that $\boldsymbol{y_i} = [\boldsymbol{v_i}, \boldsymbol{s_i}]$, $i = 1, 2$. For second frame transmission, if User 1 successfully decodes $\boldsymbol{v_2}$, User 1 transmits $\boldsymbol{s_1}$ using User 1's channel and $\boldsymbol{s_2}$ using User 2's channel. Otherwise, User 1 transmits $\boldsymbol{s_1}$ only. In order to maintain the same average power, User i divides his power in the second frame according to the ratio β_i so that User i's own bits $\boldsymbol{s_i}$ are transmitted with energy $\beta_i E_{b,i}$, and the partner User j's bits are transmitted with energy $(1 - \beta_i)E_{b,i}$.

We denote the additional parity bits of User i, which are transmitted by User j as $\boldsymbol{s}_{i,j}$. For the purposes of this work, we assume that the multiple access and coding schemes are such that $\boldsymbol{s}_{i,i}$ and $\boldsymbol{s}_{i,j}$ can be coherently combined at the base station. We discuss the implementation of this framework for different multiple access and coding schemes in more detail in Section III-C.

The users act independently in the second frame, with no knowledge of whether their first frame was correctly decoded by their partner. As a result, there are four possible cooperative cases for the transmission of the second frame. In Case 1, both users successfully decode their partners so that they each send both their own and their partner's second set of coded bits in the second frame, resulting in the fully cooperative scenario depicted in Fig. 3. In Case 2, neither user successfully decodes their partner's first frame, and the system reverts to the noncooperative case for that pair of source blocks, i.e., $\beta_i = 1$, $i = 1, 2$. In Case 3, User 2 successfully decodes User 1, but User 1 does not successfully decode User 2. Consequently, User 1 transmits only his own bits in the second frame, i.e., $\beta_1 = 1$, whereas User 2 splits his power and transmits the additional parity bits for both himself and User 1. Case 4 is identical to Case 3 with the roles of User 1 and User 2 reversed. Note that although a user's bits are transmitted only in that user's multiple access channel, unlike original coded cooperation, the base station still must know which case has occurred in order to apply optimum combining weights and decoding metrics (β_i are assumed known *a priori*). We can use the same method described in Section II for coded cooperation.

In Section V, we present an analytical methodology for evaluating the performance of space-time cooperation, showing that we achieve full diversity in slow fading (for Cases 1 and 3) and improved diversity over the original coded cooperation in fast fading. We demonstrate the validity of these bounds via simulations as part of the performance results presented in Section VI.

C. Implementation Issues

In the case of CDMA, transmission in each channel requires only the use of a different chip sequence. Since the data of each user is transmitted by two different users, the two transmissions are not time coherent, but they may be resolved at the base station and coherently combined via RAKE fingers. We note that this method was first suggested in the context of the CDMA uncoded cooperation scenario of Sendonaris *et al.* [7], [8].

In the case of FDMA, the new method entails transmission on two different frequency bands. Each bit in the second frame is transmitted by two antennas; thus, the simple but effective code of Alamouti [2] may be used. Alamouti's space-time code requires that the symbols be transmitted from the two antennas in synchronization; otherwise, intersymbol interference will erode the performance of the system. Tight uplink synchronization between users exists in orthogonal frequency division multiple access (OFDMA) systems [18], for example, where training symbols with cyclic prefix and other techniques are used, whose details fall outside the scope of this paper.

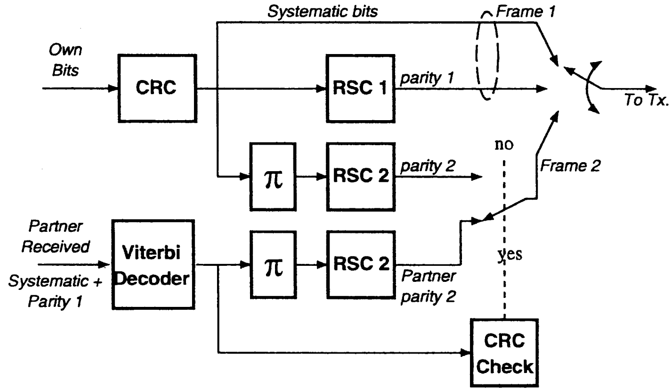

Fig. 4. Turbo encoding in a coded cooperation scheme.

In the case of TDMA, once again, the Alamouti space-time code may be used, and once again, the issue of synchronization arises. Unlike OFDMA, in TDMA, symbol-level synchronization between users in the uplink is not guaranteed. However, coarse synchronization is usually present: GSM, for example, provides synchronization up to 0.5 of a symbol interval [19], which is not enough for our purposes, but suggests that tighter synchronization is possible. Progress is already being made on the relay synchronization issue [20], but more work is needed in this area.

IV. Turbo-Coded Cooperation

The implementation of coded cooperation using turbo codes is shown in Fig. 4. Turbo codes employ two constituent recursive systematic convolutional (RSC) codes with interleaving [21], [22]. The users and the base station have the same random interleaver, shown as π in Fig. 4. The codeword for the first frame is obtained using the first RSC code. Upon successful decoding of the partner, the user interleaves the source bits over the K-bit block and transmits the parity bits corresponding to the second RSC code.

Turbo coding can be used with either coded cooperation or space-time cooperation. The difference between the two cases, as before, is in the the second frame. In turbo-coded cooperation, each user transmits his partner's parity bits in Frame 2 using all available power. In space-time turbo-coded cooperation, each user transmits his own as well as his partner's second set of parity bits by splitting the available power. In either case, if the first frame of the partner is not successfully decoded, the user will interleave, encode, and transmit the second set of parity bits for his own source block using all of his power.

The scheme presented above has a fixed cooperation percentage of 33%. It is possible to have flexible cooperation percentage, as well as better performance, by using punctured turbo codes or rate compatible punctured turbo codes (RCPT) [23].

However, the mobiles must then perform turbo decoding on the partner's bits, increasing the complexity. In this paper, we only consider the simpler case with a fixed cooperation percentage, where a user employs the conventional Viterbi decoding for the partner, as shown in Fig. 4.

At the base station, the combination of the first and second frames offers the possibility of turbo decoding. The low-complexity iterative decoder [21], [24] offers near-optimum decoding performance for turbo codes.

For the sake of brevity, we omit explanation of the single-input single-output (SISO) modules and iterative decoding of turbo codes. See the rich existing literature, e.g., [24], [25], and references therein.

V. Performance Analysis

In this section, we present an analytical methodology for evaluating the performance of coded cooperation. In developing pairwise error probabilities in Section V-A, we use tools and techniques from Craig [26], Simon and Alouini [14], and Malkamäki and Leib [15]. We then determine union bounds for the overall bit- and block-error probabilities in Section V-B using weight enumerating functions. The validity of the resulting bounds is demonstrated via simulations in Section VI.

A. Pairwise Error Probability

The pairwise error probability (PEP) for a coded system is the probability of detecting an erroneous codeword $\boldsymbol{e} = [e(1), e(2), \ldots, e(N)]$ when codeword $\boldsymbol{c} = [c(1), c(2), \ldots, c(N)]$ is transmitted. In general, for a binary code with BPSK modulation, coherent detection, and maximum-likelihood decoding, the PEP conditioned on the set of instantaneous received SNR values $\boldsymbol{\gamma} = [\gamma(1), \gamma(2), \ldots, \gamma(N)]$ can be written as [14, (12.13)]

$$P(\boldsymbol{c} \rightarrow \boldsymbol{e}|\boldsymbol{\gamma}) = Q\left(\sqrt{2\sum_{n\in\eta}\gamma(n)}\right) \quad (3)$$

where $Q(x)$ denotes the Gaussian Q-function [27, (2-1-97)], and $\gamma(n)$ is the instantaneous received SNR for code bit n. The set η is the set of all n for which $c(n) \neq e(n)$, and the cardinality of η is equal to the Hamming distance d between codewords $\boldsymbol{c}$ and $\boldsymbol{e}$. The erroneous detection of $\boldsymbol{e}$, instead of $\boldsymbol{c}$, is known as an error event, and thus, d is typically referred to as the corresponding error event Hamming weight.

In the following, we restrict ourselves to the class of linear codes and assume the transmitted codeword $\boldsymbol{c}$ is the all-zero codeword. Consequently, the PEP depends only on d and not the particular codewords $\boldsymbol{c}$ and $\boldsymbol{e}$, and the conditional PEP will be denoted simply by $P(d|\boldsymbol{\gamma})$.

1) Slow Fading: In this case, the fading coefficients for each user uplink channel remain constant over a codeword, i.e., $\alpha_{i,0}(n) = \alpha_{i,0}$ and $\gamma_{i,0}(n) = \gamma_{i,0}$ for $n = 1, \ldots, N$ for User i's uplink channel (subscript 0 denotes the base station). We can rewrite (3) for User 1's codeword as

$$P(d|\gamma_{1,0}\gamma_{2,0}) = Q\left(\sqrt{2d_1\gamma_{1,0} + 2d_2\beta_1\gamma_{1,0} + 2d_2(1-\beta_2)\gamma_{2,0}}\right) \quad (4)$$

where d_1 and d_2 are the numbers of bits in the Hamming weight d that are transmitted through User 1's channel and both users' channels, respectively, such that $d_1 + d_2 = d$.

To obtain the the unconditional PEP, we must take the expected value of (4) over the distributions of $\gamma_{1,0}$ and $\gamma_{2,0}$. Using the tools developed by Simon and Alouini [14], we can obtain the following result:

$$P(d) = \frac{1}{\pi}\int_0^{\pi/2}\left(1+\frac{(d_1+\beta_1 d_2)\overline{\gamma_{1,0}}}{\sin^2\theta}\right)^{-1}\left(1+\frac{d_2(1-\beta_2)\overline{\gamma_{2,0}}}{\sin^2\theta}\right)^{-1} d\theta \quad (5)$$

where $\overline{\gamma_{i,0}}$ is the average uplink SNR between User i and the base station. Note that (5) is an exact expression for the unconditional PEP and is easily evaluated with numerical integration techniques.

The following upper bound is obtained for (5) by noting that the integrand is maximized for $\sin^2\theta = 1$

$$P(d) \leq \frac{1}{2}\left(\frac{1}{1+(d_1+\beta_1 d_2)\overline{\gamma_{1,0}}}\right)\left(\frac{1}{1+d_2(1-\beta_2)\overline{\gamma_{2,0}}}\right). \quad (6)$$

For the cases in which $\beta_2 \neq 1$ (Cases 1 and 3), we see from (6) that, for large SNR, the PEP is inversely proportional to the product of the average SNR for the two uplink channels. Thus, provided that d_1 and d_2 are both greater than zero, full diversity order of two is achieved. This is also the same as the original coded cooperation framework, for which we have (6) with $\beta_1 = \beta_2 = 0$.

For no cooperation (which also corresponds to Case 2), we have that $\beta_1 = \beta_2 = 1$ in (6), and thus, we see that we only have diversity order one. For Case 4, we have $\beta_1 < 1$ and $\beta_2 = 1$ so that again, we only have diversity order one.

2) Fast Fading: For fast fading, the fading coefficients are no longer constant over the codeword but are i.i.d. across the coded bits. Thus, we can generalize (4) as

$$P(d|\boldsymbol{\gamma}_{1,0}, \boldsymbol{\gamma}_{2,0}) = Q\left(\sqrt{2\sum_{n\in\eta_1}\gamma_{1,0}(n) + 2\beta_1\sum_{n\in\eta_2}\gamma_{1,0}(n) + 2(1-\beta_2)\sum_{n\in\eta_2}\gamma_{2,0}(n)}\right)$$

where the set η_i is the portion of bits of the Hamming weight d transmitted through User i's channel. The cardinalities of η_1 and η_2 are d_1 and d_2, respectively, where again $d_1 + d_2 = d$.

We use the techniques of [14] once more to obtain the following expression for the unconditional PEP:

$$P(d) = \frac{1}{\pi}\int_0^{\pi/2}\left(1+\frac{\overline{\gamma_{1,0}}}{\sin^2\theta}\right)^{-d_1}\left(1+\frac{\beta_1\overline{\gamma_{1,0}}}{\sin^2\theta}\right)^{-d_2}\left(1+\frac{(1-\beta_2)\overline{\gamma_{2,0}}}{\sin^2\theta}\right)^{-d_2} d\theta \quad (8)$$

$$\leq \frac{1}{2}\left(\frac{1}{1+\overline{\gamma_{1,0}}}\right)^{d_1}\left(\frac{1}{1+\beta_1\overline{\gamma_{1,0}}}\right)^{d_2}\left(\frac{1}{1+(1-\beta_2)\overline{\gamma_{2,0}}}\right)^{d_2}. \quad (9)$$

From (9), the diversity order for fast fading in Cases 1 and 3 ($\beta_2 \neq 1$) is $d_1 + 2d_2 = d + d_2$. In contrast, for Cases 2 and 4 ($\beta_2 = 1$), as well as for the original coded cooperation framework ($\beta_1 = \beta_2 = 0$), and for no cooperation ($\beta_1 = \beta_2 = 1$), the diversity order is equal to d. Thus, we see that our modified framework involving space-time transmission does indeed provide increased diversity in fast fading.

B. Bit and Block Error Rate

We can obtain union bounds for the BER and BLER as a function of the PEP using well-known weight enumerating techniques. Note that in order to get tight bounds for the case of slow fading, we can use the limit-before-average technique from [15].

1) Convolutional Codes: Since, in our framework, we always consider terminated convolutional codes with a finite uncoded block length K and coded block length N, we can obtain bounds for the BER and BLER using the weight enumerating function of the equivalent block code as

$$P_b(\boldsymbol{\gamma}) \le \sum_{d=d_f}^{N} \sum_{w=1}^{K} \frac{w}{K} a_{w,d} P(d|\boldsymbol{\gamma})$$
$$P_{block}(\boldsymbol{\gamma}) \le \sum_{d=d_f}^{N} \sum_{w=1}^{K} a_{w,d} P(d|\boldsymbol{\gamma}) \quad (10)$$

where d_f is the free distance of the code, and $a_{w,d}$ is the multiplicity of codewords corresponding to input weight w and output weight d.

2) Turbo Codes: We can similarly consider the weight enumerating function (WEF) for the equivalent block code, as shown in [22] and [28]. Using the concept of *uniform interleaver*[3] [22], the WEF of the overall concatenated code is given based on the WEF of the constituent codes. We follow the same direction of [22] with a minor modification for a turbo code with C_1 and C_2 as the constituent systematic recursive convolutional codes and an interleaver with size K.

The conditional WEF of a block code $A_w^C(Z)$ gives all possible codewords generated by the set of input sequences with weight w (note that Z is only a dummy variable). Assume that $A_w^{C_1}(Z)$ is the conditional WEF of C_1 and $A_w^{C_2}(Y)$ for C_2. Then, using the probabilistic uniform interleaver, the conditional WEF of the turbo code is [22]

$$A_w^C(Z,Y) = \frac{A_w^{C_1}(Z) \times A_w^{C_2}(Y)}{\binom{K}{w}}. \quad (11)$$

Although we employ the original type of turbo code [21], [22] that has similar constituent convolutional codes, keeping the WEF of C_1 and C_2 separate (with two dummy variables Z and Y) makes it possible to deal with the four different scenarios in the cooperation schemes, which as a subcase has the analysis of [22]. The BER and BLER of the turbo code are obtained using the union bound argument [22]

$$P_b(\boldsymbol{\gamma}) \le \sum_{z=0}^{K}\sum_{y=0}^{K}\sum_{w=1}^{K} \frac{w}{K} a_{w,z,y} P(d|\boldsymbol{\gamma})$$
$$P_{block}(\boldsymbol{\gamma}) \le \sum_{z=0}^{K}\sum_{y=0}^{K}\sum_{w=1}^{K} a_{w,z,y} P(d|\boldsymbol{\gamma}) \quad (12)$$

[3]A uniform interleaver with size K maps a codeword of weight w into all its distinct $\binom{K}{w}$ permutations with equal probability $1/\binom{K}{w}$[22].

where $a_{w,z,y}$ denotes the multiplicity of codewords corresponding to input weight w and parity weights z and y, which are obtained from the corresponding code WEF $W^w A_w^C(Z,Y)$, and $P(d|\boldsymbol{\gamma})$ is the corresponding PEP expression from Section V-A. The expressions above assume $R_1 = R_2 = 1/2$. Note that d_1 is equal to the summation of the exponents of W and Z (w and z), and d_2 is equal to the exponent of Y (y).

3) Overall Bit and Block Error Rate: The overall end-to-end unconditional BER is equal to the average of the unconditional BERs over the four possible transmission cases discussed in Section III-B

$$P_b = \sum_{i=1}^{4} P_b(\text{Case } i) P(\text{Case } i) \quad (13)$$

where $P_b(\text{Case } i)$ denotes the BER corresponding to Case i, and $P(\text{Case } i)$ is the probability of occurrence of Case i. The end-to-end BLER has an identical expression. Bounds on the probabilities $P(\text{Case } i)$ for each of the four cases are obtained from the BLER corresponding to the code used for the first frame transmissions. The calculation of $P(\text{Case } i)$ is omitted here but can be found in [13]. Based on (13), the overall end-to-end diversity achieved via cooperation is similarly a weighted average of the diversity corresponding to each of the four cases, where the relative weights are determined by the interuser channel conditions. This behavior is illustrated in the performance results given in Section VI.

VI. Performance Evaluation

For our simulations, we use a 16-bit CRC code with generator polynomial given by coefficients 15 935 (hexadecimal notation). We computed via computer enumeration the WEF of our codes, including the partitioning of the Hamming weights d into d_1 and d_2, corresponding to the source block length K. All comparisons are between systems with equal information rate K and equal code rate R; therefore, we plot the error probabilities against channel SNR. Plotting BER or BLER versus the information bit SNR, or E_b/N_0, yields identical results up to an additive constant in the log-SNR domain. The plots apply equally to each of the multiple access schemes, for the following reason: Using an orthogonal space-time block code is equivalent to coherently combining multiple copies of each information symbol [2], [4]. Thus, the results for space-time cooperation presented below are equivalent for the case of CDMA, in which coherent combining is achieved via the signature correlation properties, and for TDMA and FDMA, in which an orthogonal space-time block code would be used, as discussed in Section III-C. In addition, all results presented in this section for space-time cooperation are for $\beta_1 = \beta_2 = 0.5$.

A. Rate-Compatible Punctured Convolutional Codes

We use the family of RCPC codes with memory $M = 4$, puncturing period $P = 8$, rate 1/4 mother code, and generator polynomials G$(23, 35, 27, 33)$ (octal) given by Hagenauer [17]. For slow fading, we choose overall code rate $R = 1/4$, whereas for fast fading, we use $R = 2/5$. In all cases, the source block size is $K = 128$ bits.

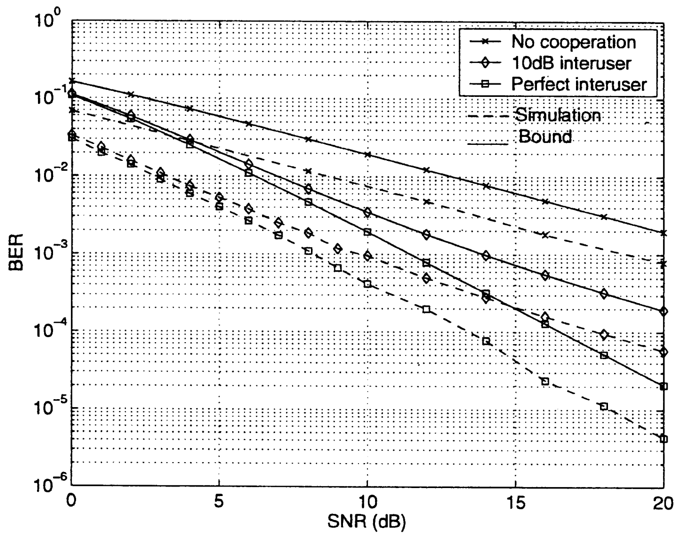

Fig. 5. Slow Rayleigh fading results. Equal uplink SNR; cooperation at 50%.

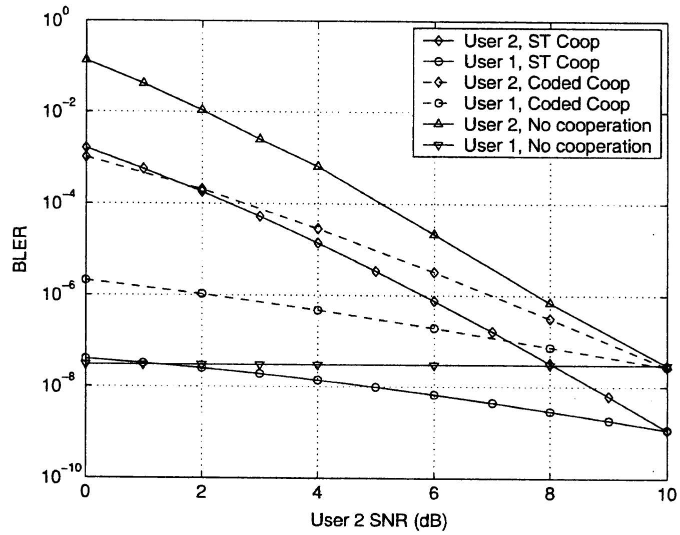

Fig. 7. Comparing block error rates of coded cooperation versus space-time cooperation (analytical bounds). Interuser channel at 10 dB, cooperation at 30%, and the users have unequal uplink SNR (User 1 at 10 dB and User 2 varies 0–10 dB).

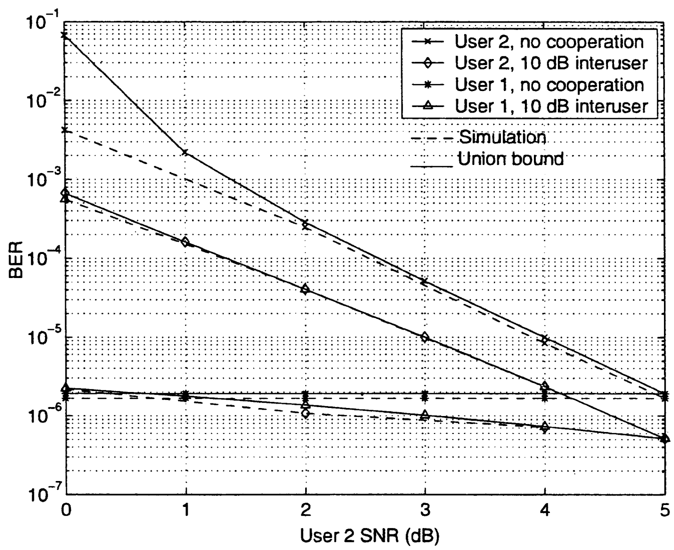

Fig. 6. Fast Rayleigh fading results. Unequal uplink SNR: User 1 is fixed at 5 dB, whereas User 2 varies 0–5 dB.

Fig. 5 shows analytical bound and simulation results of BER for slow Rayleigh fading with 10 dB average SNR interuser channel and perfect interuser channel. Both users' uplink channels have the same average SNR (symmetric uplink channels), and the level of cooperation is 50%. Under slow fading, space-time cooperation achieves significant gain over noncooperative systems: gains that are similar to coded cooperation. As an example, at BER of 10^{-3}, a coding gain of 9 dB is achieved over the noncooperative baseline system of similar rate, bandwidth, and power when the interuser channel is at 10 dB. The perfect (error free) interuser channel demonstrates the limits of the gains, which at BER $= 10^{-3}$ is about 11 dB.

Fig. 6 shows BER results for fast Rayleigh fading. The cooperation percentage is at 30%. User 1's uplink channel is fixed at 5 dB, whereas User 2's channel varies from 0 to 5 dB. As shown in Fig. 6, User 2 realizes a gain of 1 dB if the interuser channel average SNR is 10 dB. Interestingly, User 1, which has a better channel, also improves somewhat. The union bounds match well to the simulation results.

Fig. 7 compares the BLER of coded cooperation and space-time coded cooperation, using analytical bounds, for fast Rayleigh fading with 10–dB interuser SNR. Cooperation is at 30%, and User 1's SNR is fixed at 10 dB. This figure shows that not only does User 2 improve significantly, but in addition, User 1 does not lose performance by cooperating with User 2, even though User 1 has better SNR to start with.

B. Turbo Codes

We employ the best reported turbo code with rate-1/2 eight-state constituent codes with generator polynomials $\mathrm{G}(1, 17/13)$ (octal) from [29]. The overall code rate is 1/3. The source block has $K = 128$ bits. The cooperation percentage is 33%. The baseline for all comparisons is a noncooperative *turbo coded* system; therefore, comparisons are fair on the basis of computational complexity as well as rate.

Fig. 8 shows the simulation results for the BER of turbo coded cooperation compared with noncooperative turbo coding with various interuser channel conditions. As shown in Fig. 8, cooperation yields significant gain in slow fading due to the increased diversity. The gain at BER $= 10^{-3}$ is from 5 dB (for the case of 6 dB interuser channel) to 8 dB (for the perfect interuser channel).

Fig. 9 shows the union bounds and simulation results for the BLER of turbo coded cooperation in fast fading. User 1 has a fixed average uplink SNR at 5 dB, and the interuser channel has SNR of 10 dB. The union bounds match the simulated results except at very low SNR in the noncooperative case, which is a behavior already reported in literature, e.g., [28]. The gain for User 2 is about 3 dB at BLER $= 10^{-4}$, decreasing gradually as User 2's SNR approaches the SNR of User 1. User 1 (who has a better channel) sacrifices performance by cooperation. Nevertheless, provided that User 1's performance remains acceptable, this constitutes a better overall system performance since the worst user has improved significantly.

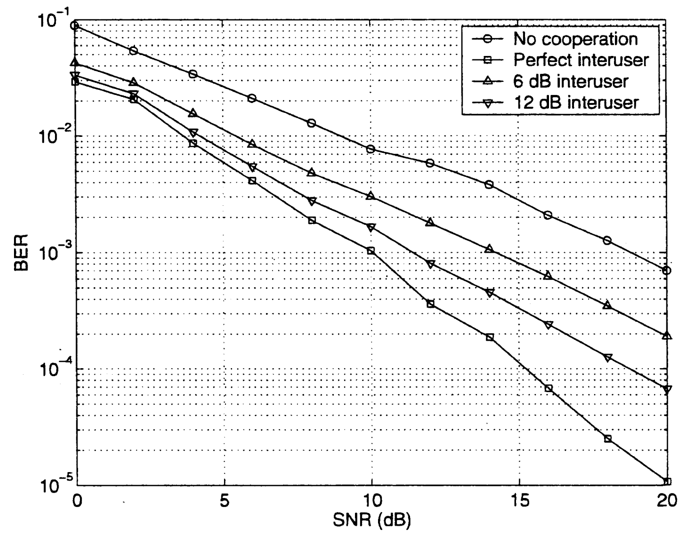

Fig. 8. Turbo coded cooperation in slow fading. Users have equal uplink SNR.

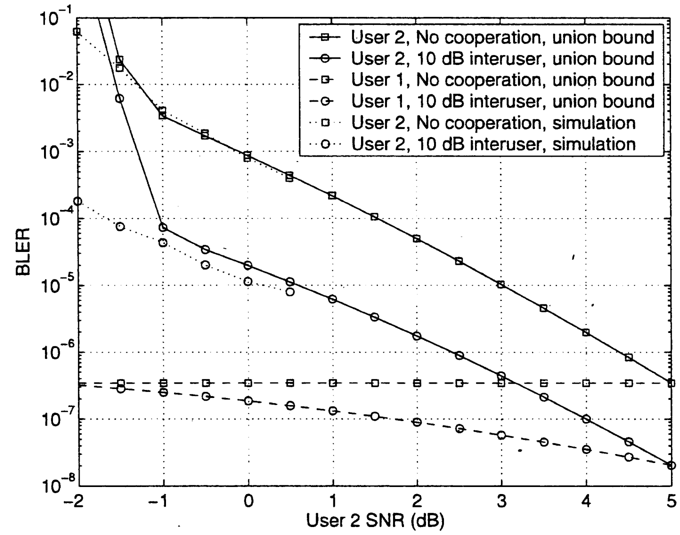

Fig. 10. Turbo coded *space-time* cooperation in fast fading, User 1 SNR = 5 dB.

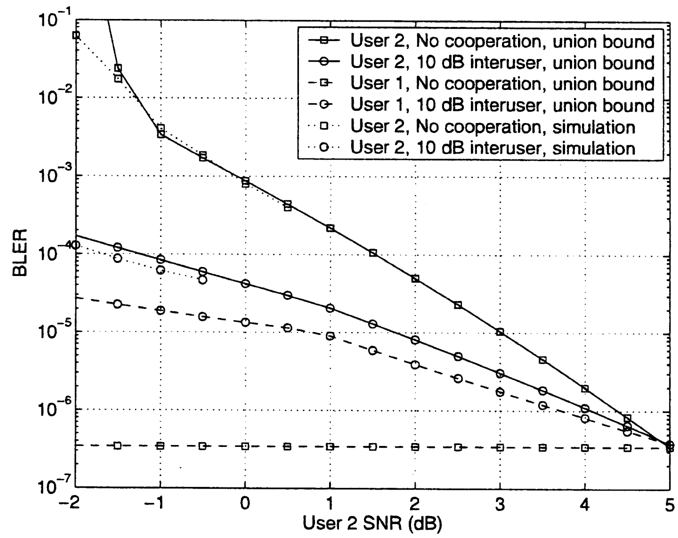

Fig. 9. Turbo coded cooperation in fast fading, User 1 SNR = 5 dB.

The performance of the space-time turbo coded cooperation in fast fading channel is demonstrated in Fig. 10. The channel conditions are the same as Fig. 9. The gain for User 2 at BLER $= 10^{-4}$ is similar to turbo coded cooperation, but the gain is maintained over a wider range of SNR. Moreover, User 1 gains from cooperation as well, unlike the previous case.

VII. Minimax Power Splitting

In many applications, the wireless system is required to provide and maintain a minimum quality of service for all users. In a noncooperative system, one user may have a very good channel that provides a quality of service significantly better than that required for the application. Another user may have a poor channel such that the user is in outage or must significantly increase power to meet the quality of service requirement. This in turn has a detrimental affect on the other users in the system. This scenario represents a poor allocation of system resources.

When the users cooperate, they can share their resources such that all the cooperating users achieve the minimum quality of service more reliably and with less power. Specifically, for the space-time coded cooperation framework introduced in this paper, we would like to find the user power splitting ratios β_1 and β_2 such that the combined resources for the two users are shared in the most effective way. Guaranteeing a minimum quality of service for both users corresponds to the following minimax criterion for determining β_1 and β_2:

$$\min_{\beta_1,\beta_2}[\max(P_{b_1}, P_{b_2})] \tag{14}$$

where P_{b_1} and P_{b_2} are the end-to-end BER of User 1 and User 2, respectively.

In order to solve this optimization problem, we note that P_{b_1} is a monotonically increasing function of β_1 and a monotonically decreasing function of β_2 (similarly, P_{b_2} increases with β_2 and decreases with β_1). Thus, the optimum point in (14) corresponds to a point for which P_{b_1} and P_{b_2} are equal. If P_{b_1} and P_{b_2} are not equal, clearly, we can alter either β_1 or β_2 or both to make them equal and thus reduce the maximum of P_{b_1} and P_{b_2}. We can therefore simplify the criterion of (14) as

$$\min_{\beta_1,\beta_2}[\max(P_{b_1}, P_{b_2})] = \min_{\beta_1,\beta_2}(P_{b_1}) \quad \text{Subject to } P_{b_1} = P_{b_2}. \tag{15}$$

Using the method of Lagrange multipliers, we can write

$$\min_{\beta_1,\beta_2}[P_{b_1} - \lambda(P_{b_1} - P_{b_2})] = \min_{\beta_1,\beta_2}[(1-\lambda)P_{b_1} + \lambda P_{b_2}]. \tag{16}$$

Taking derivatives of (16) with respect to β_1 and β_2, after some algebraic manipulation, we obtain

$$\begin{cases} (P_{b_1})'_{\beta_1} = \frac{(P_{b_1})'_{\beta_2}(P_{b_2})'_{\beta_1}}{(P_{b_2})'_{\beta_2}} \\ P_{b_2} = P_{b_1} \end{cases} \tag{17}$$

where $(f)'_x$ is the derivative of f with respect to x. The two equations above can be solved to obtain desired values for β_1 and β_2. These optimal values for β_1 and β_2 are functions of the average interuser and uplink SNR, as well as the overall coding scheme and percent cooperation. For the results presented in Fig. 11 and Table I, we solved (17) numerically by searching

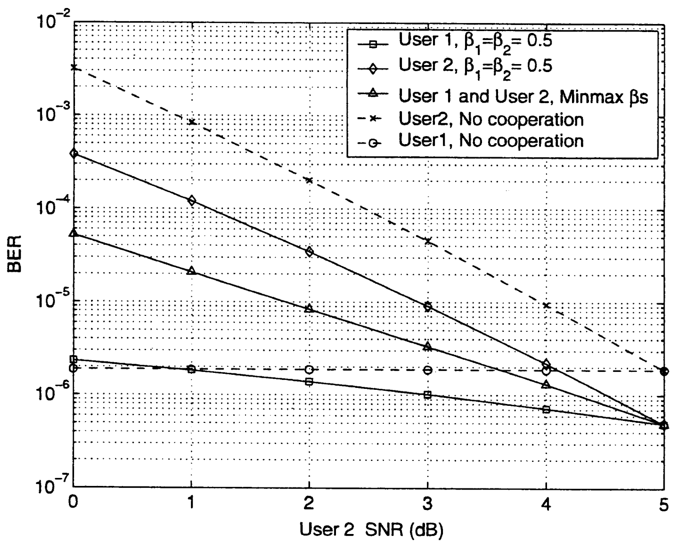

Fig. 11. Union bounds with minimax criteria for unequal uplink SNR in fast fading.

TABLE I
OPTIMUM β_1 AND β_2 VALUES FOR MINIMAX CRITERIA, WHERE $\overline{\gamma_1} = 5$ dB

$\overline{\gamma_2}$	0dB	1dB	2dB	3dB	4dB	5dB
β_1	0.2	0.2	0.3	0.4	0.4	0.5
β_2	1	0.9	0.8	0.7	0.6	0.5

over the region $(\beta_1, \beta_2) \in [0,1] \times [0,1]$. We use the bounds derived in Section V to obtain P_{b_1}, P_{b_2}, and their derivatives.

Fig. 11 shows the union bounds with the minimax optimization in fast fading with the RCPC code implementation. User 1's average uplink SNR ($\overline{\gamma_1}$) is fixed at 5 dB, whereas User 2's ($\overline{\gamma_2}$) varies from 0 to 5 dB. The interuser channel has 10 dB average SNR, and the overall code rate is $R = 2/5$ with 30% cooperation. The optimum values of β_1 and β_2 for each SNR point are shown in Table I. We see that as User 2's uplink channel deteriorates relative to User 1, both users allocate more of their power to User 2's information. The optimum are $\beta_1 = \beta_2 = 0.5$ when both user's uplink SNR are equal, as expected.

VIII. CONCLUSIONS

This paper introduces two extensions to coded cooperation: space-time and turbo-coded cooperation. Coded cooperation is an effective framework for achieving uplink transmit diversity for single-antenna mobiles. The extensions presented in this paper make coded cooperation even more effective. The motivation for space-time cooperation is to capture space-time diversity in fast fading. The motivation for turbo coded cooperation is to leverage the natural two-code structure of coded cooperation to achieve better performance. We develop tight bounds on bit- and block-error rate of the proposed methods. These bounds, as well as simulations, demonstrate the performance of the proposed extensions: space-time cooperation yields gains in fast fading and turbo coded cooperation gains over noncooperative turbo coded systems.

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