Low-complexity stochastic modeling of turbulent flows

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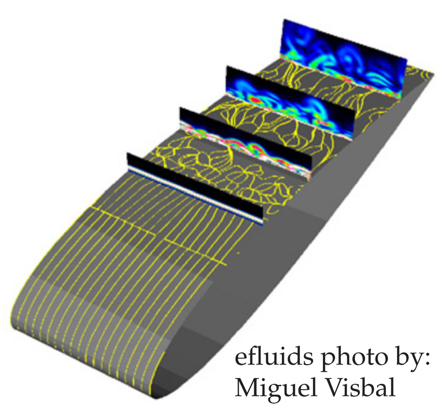
Motivation

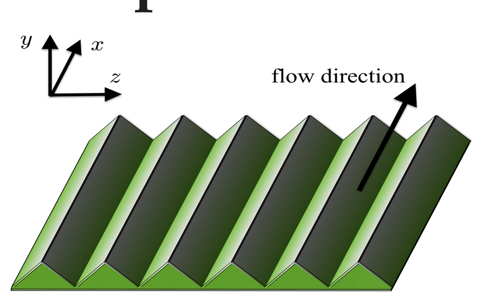
Control of turbulent flows

prevent/suppress turbulence
reduce turbulent drag
} Economic impact

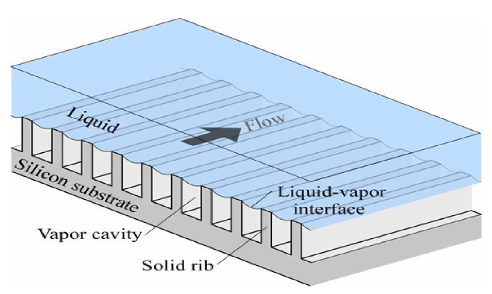


- **Challenges**
  - large number of degrees of freedom
  - complex flow dynamics
- **Objectives**
  - control-oriented modeling of turbulent flows
- **Ongoing research**
  - model-based flow control design

passive

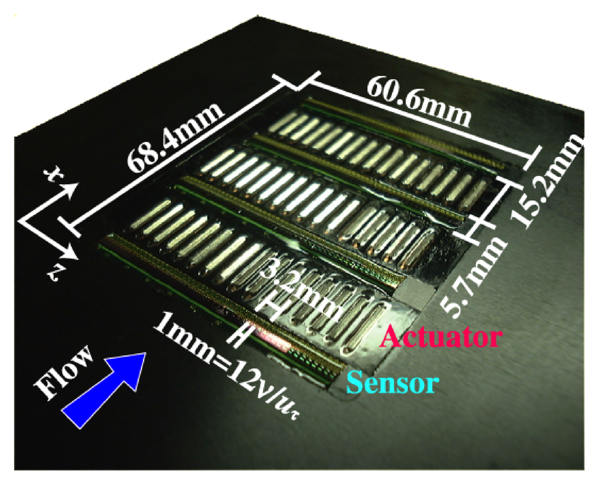


riblets



superhydrophobic surface

active

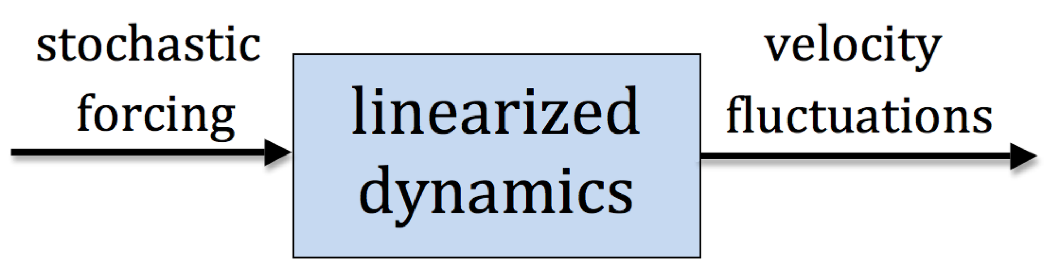


hot-film sensors and wall-deformation actuators

(Yoshino *et al.* 2008)

Approach

Stochastically forced Navier-Stokes equations

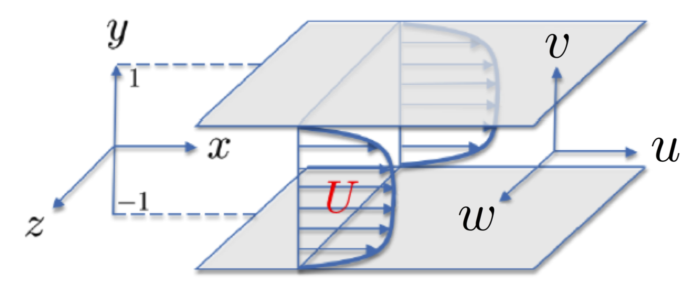


- embed observed statistical features of turbulence in control-oriented models

Completion of turbulent flow statistics

- view second-order statistics as data for an inverse problem
- identify forcing statistics to account for partially known turbulent statistics

Turbulent channel flow

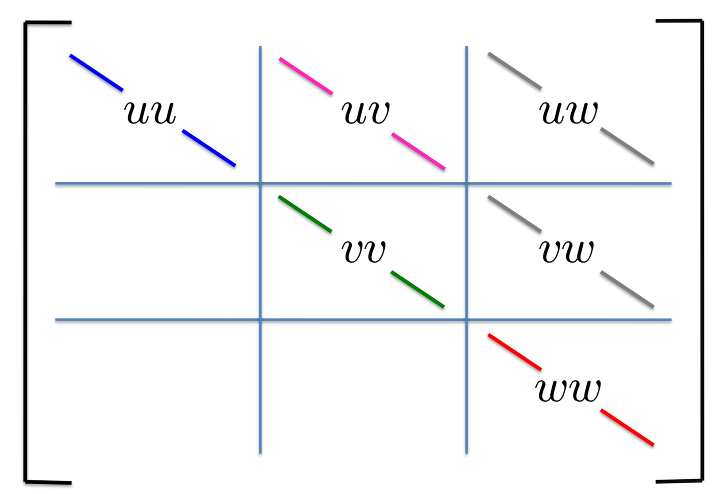


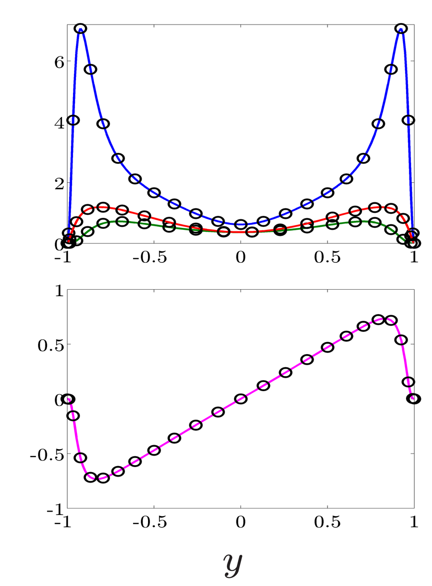
- **Linearized evolution model**

$$\begin{aligned}\psi_t &= A\,\psi + \mathbf{f}\\ \mathbf{v} &= C\,\psi\end{aligned} \qquad A = \begin{bmatrix} A_{\mathrm{os}} & 0\\ A_{\mathrm{cp}} & A_{\mathrm{sq}}\end{bmatrix} \quad \psi = \begin{bmatrix} v\\ \eta\end{bmatrix} \quad \mathbf{v} = \begin{bmatrix} u\\ v\\ w\end{bmatrix}$$

Covariance matrix completion problem

available correlations:

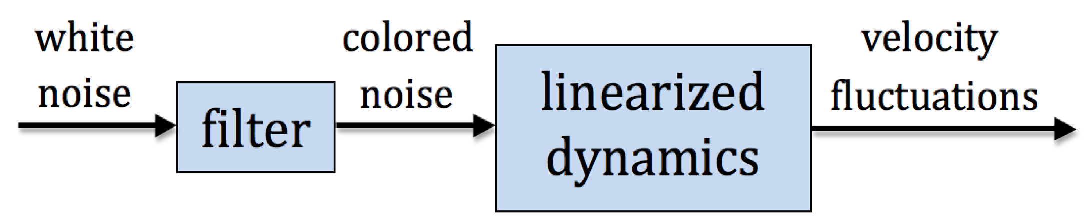




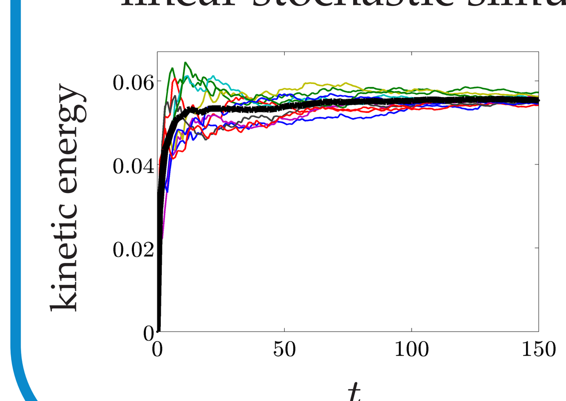
- **Convex optimization problem**

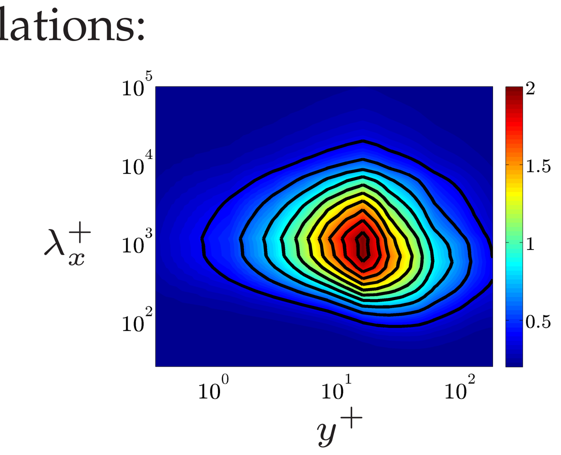
$$\begin{array}{ll} \underset{X,\,Z}{\text{minimize}} & \|Z\|_* \\ \text{subject to} & A X + X A^* + Z = 0 \\ & \mathrm{trace}\,(T_i X) = g_i, \qquad i = 1,\dots,N \\ & X \succeq 0 \end{array}$$

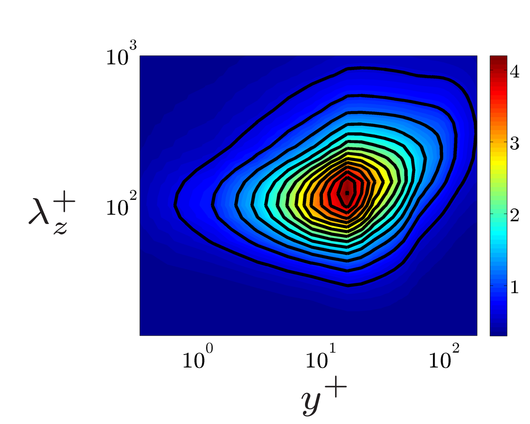
- white-in-time excitation $\longrightarrow$ too restrictive!
- **Filter design**



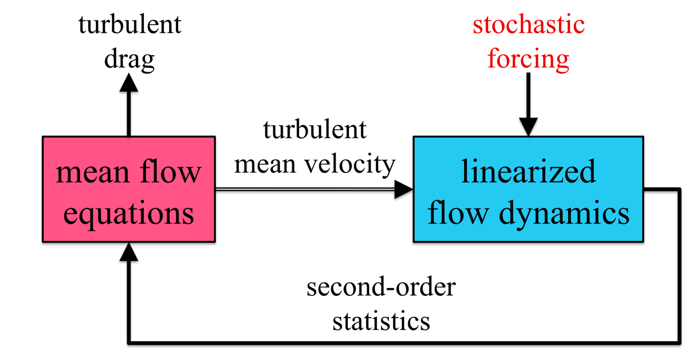
linear stochastic simulations:





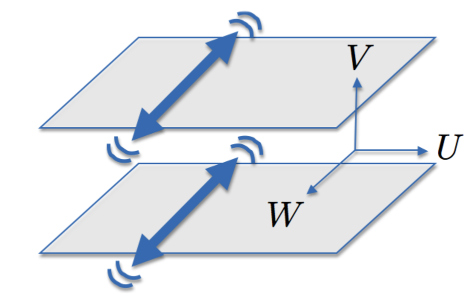


Model-based control



Spanwise wall oscillations

$W(y = \pm 1, t) = 2\alpha \sin(\frac{2\pi}{T} t)$



simulations and experiments:

sustained drag reduction:	up to $\approx 45\%$
"optimal" period of oscillations:	$T^+ \approx 100$

Perturbation analysis:

$$X(\kappa) = X_0(\kappa) + \alpha^2 X_2(\kappa) + \mathcal{O}(\alpha^4)$$

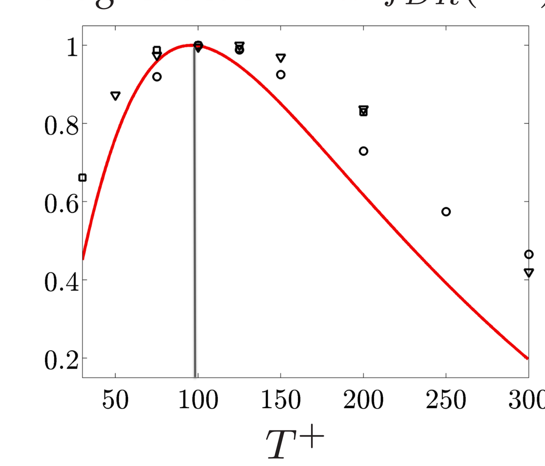
$$uv_2(y,\kappa) = \mathrm{diag}\{C_u\, X_2(\kappa)\, C_v^*(\kappa)\}$$

$$uv(y,\kappa) \approx uv_0(y,\kappa) + \alpha^2\, uv_2(y,\kappa)$$

$$\Downarrow$$

$$\begin{aligned} U &\approx U_0 + \alpha^2 U_2\\ W &\approx W_0 + \alpha^2 W_2 \end{aligned}$$

drag reduction $\approx \alpha^2 f_{DR}(T^+)$



Symbols: numerical simulations (Quadrio & Ricco, JFM '04)

Remarks

- **Control-oriented modeling**
  - stochastically forced linearized NS equations
  - colored-in-time forcing accounts for partially observed statistics
- **Sensor-free control**
  - optimal period of oscillations captured by perturbation analysis
  - simulation-free approach to predicting full-scale results
- **Acknowledgments**
  - NSF Award CMMI 1363266; UMII Transdisciplinary Fellowship; 2014 CTR Summer Program

Publications

[1] A. Zare, M. R. Jovanović, and T. T. Georgiou, "Completion of partially known turbulent flow statistics via convex optimization", in *Proceedings of the 2014 Summer Program, Center for Turbulence Research*, Stanford University/NASA.

[2] A. Zare, M. R. Jovanović, and T. T. Georgiou, "Completion of partially known turbulent flow statistics", in *Proceedings of the 2014 American Control Conference*, 2014, pp. 1680-1685.

[3] Y. Chen, M. R. Jovanović, and T. T. Georgiou, "State covariances and the matrix completion problem", in *Proceedings of the 52nd IEEE Conference on Decision and Control*, 2013, pp. 1702-1707.