

Completion of partially known turbulent flow statistics

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MOTIVATION

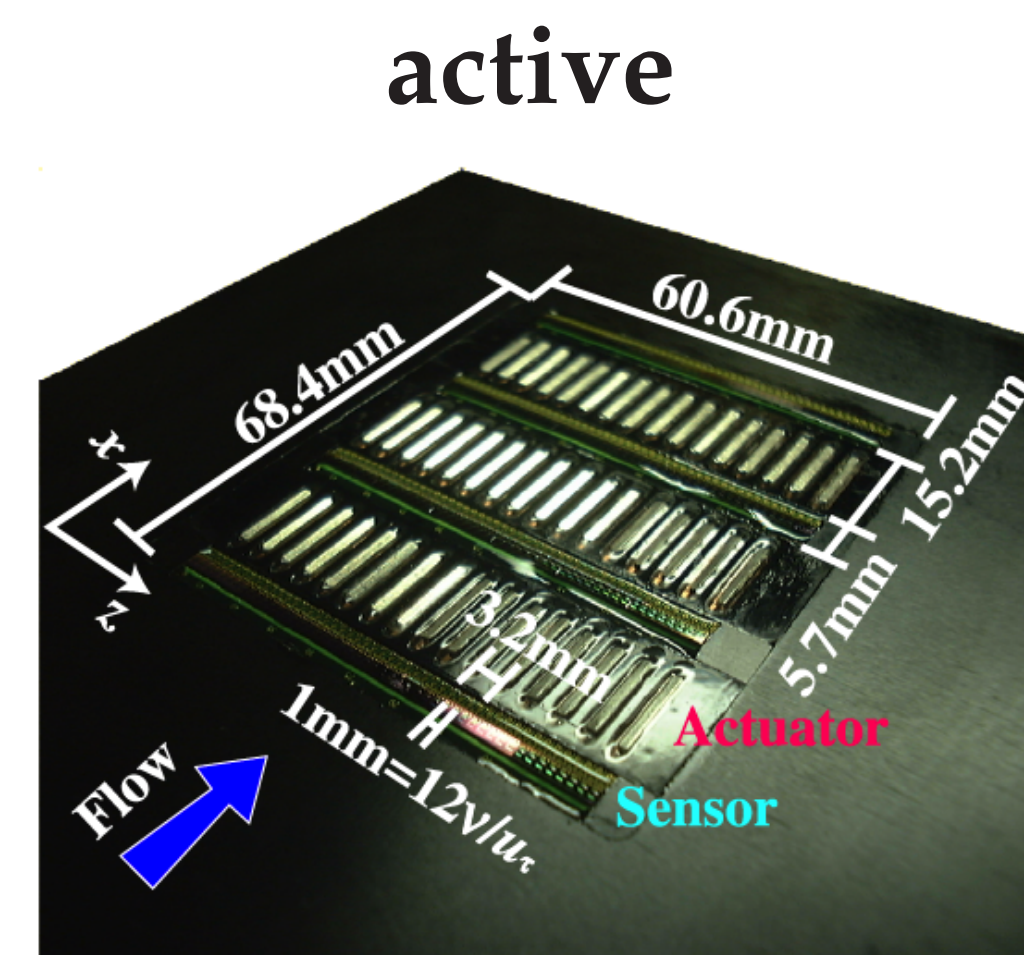
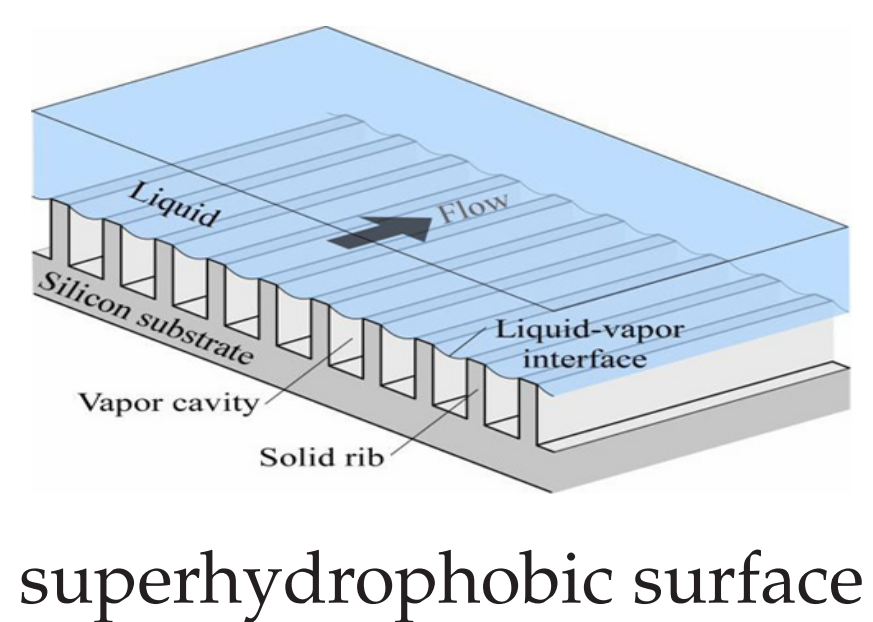
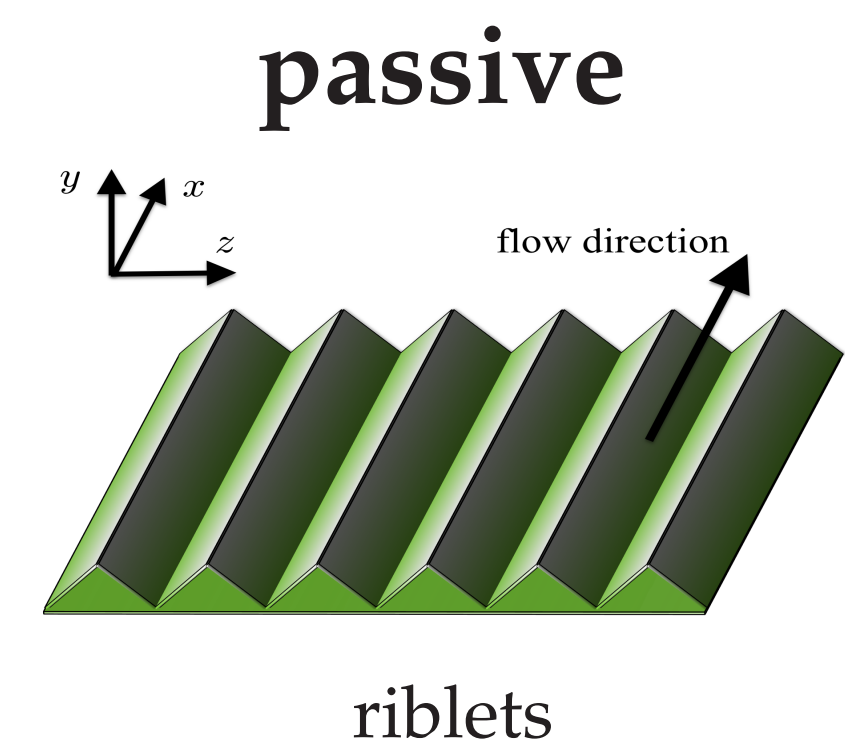
Modeling and control of turbulent flows

prevent/suppress turbulence
reduce turbulent drag } **Economic impact**



efluids photo by: Miguel Visbal

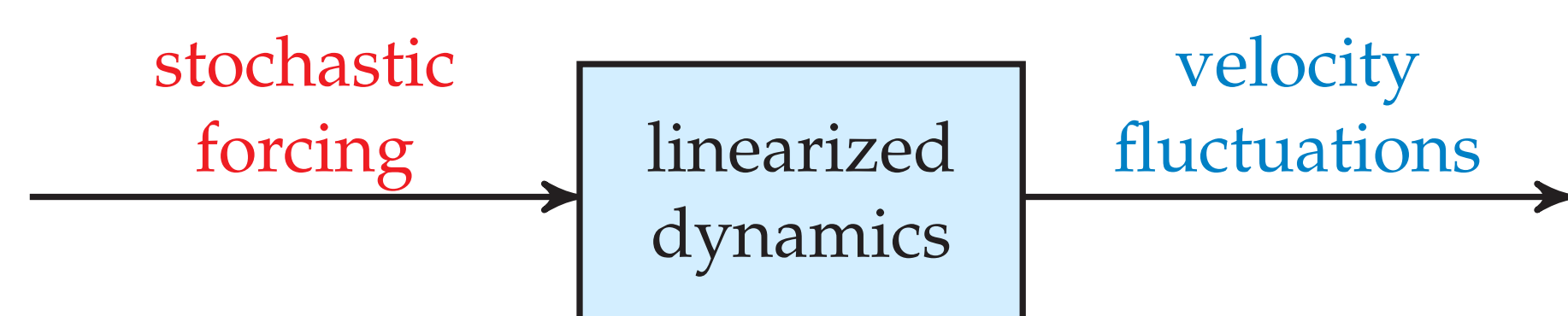
- **Challenges**
 - large number of degrees of freedom
 - complex flow dynamics
- **Objective**
 - control-oriented modeling of turbulent flows
- **Ongoing research**
 - model-based flow control design



hot-film sensors and wall-deformation actuators
(Yoshino et al. 2008)

APPROACH

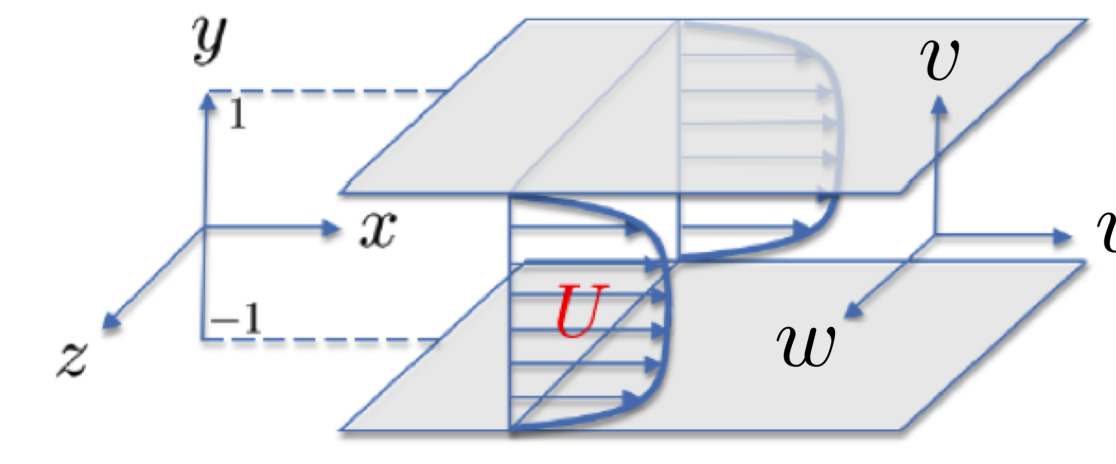
Stochastically forced Navier-Stokes equations



- embed observed **statistics** of turbulence in **physics-based** models
- identify **forcing statistics** to account for **available velocity statistics**

COMPLETION OF TURBULENT FLOW STATISTICS

Turbulent channel flow



- **Linearized evolution model**

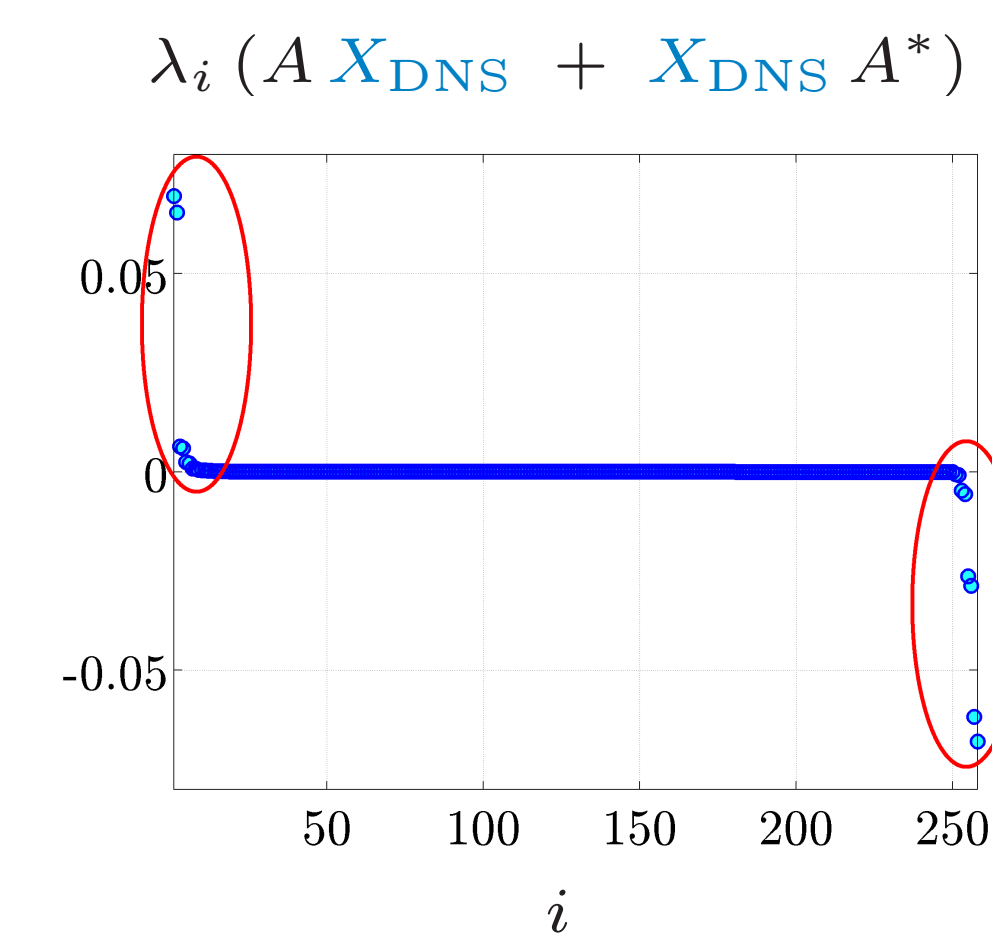
$$\dot{\psi} = A\psi + B\mathbf{d}$$

$$\mathbf{v} = C\psi$$

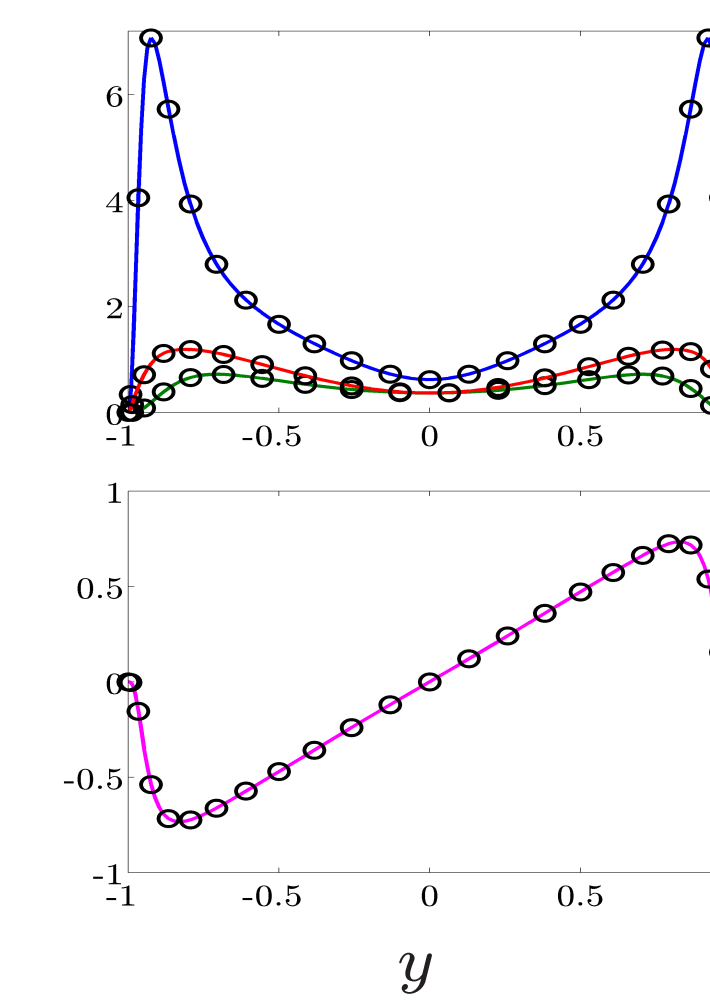
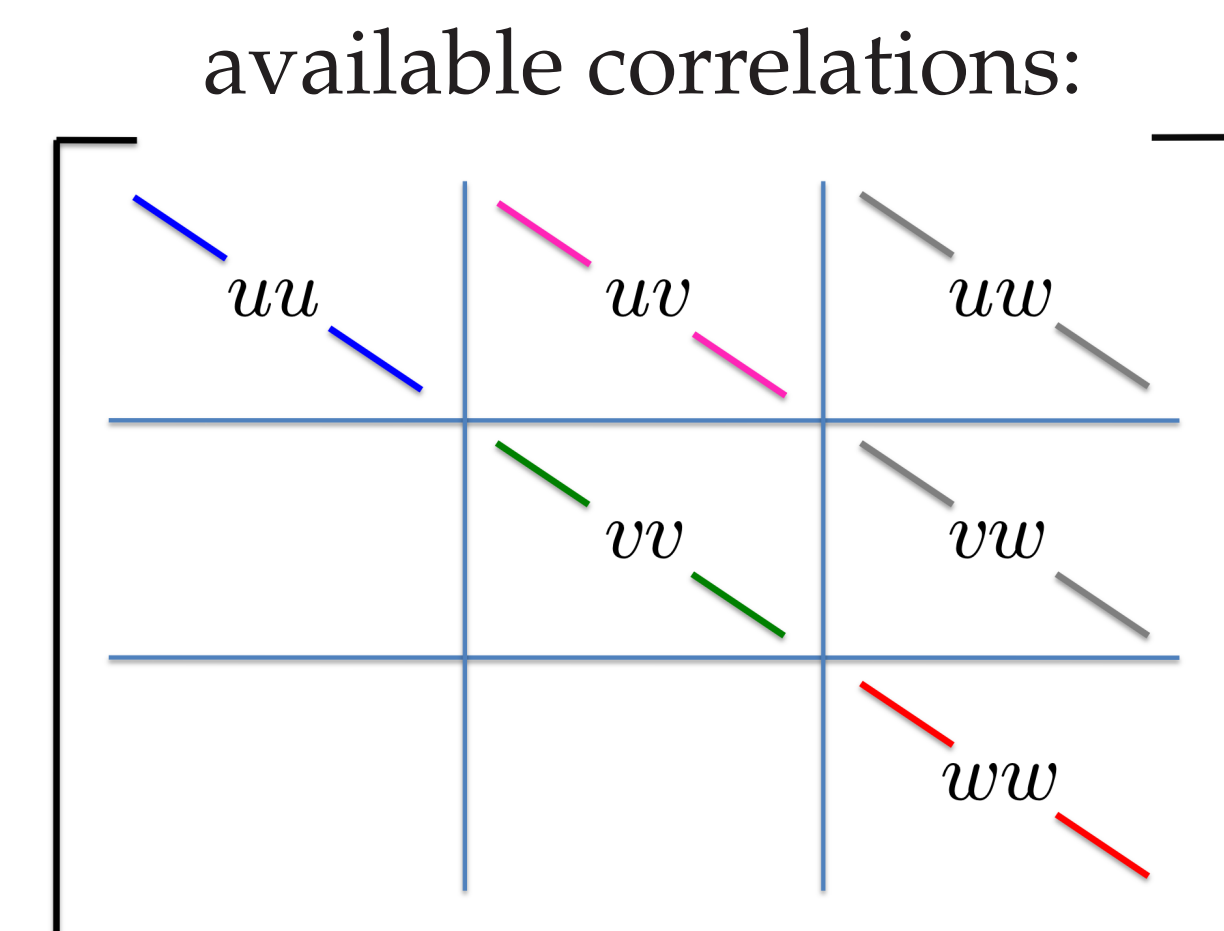
$$A = \begin{bmatrix} A_{os} & 0 \\ A_{cp} & A_{sq} \end{bmatrix} \quad \psi = \begin{bmatrix} v \\ \eta \end{bmatrix} \quad \mathbf{v} = \begin{bmatrix} u \\ v \\ w \end{bmatrix}$$

Lyapunov equation: $AX + XA^* = -B\Omega B^*$

white-in-time excitation too restrictive!



Structured covariance completion problem



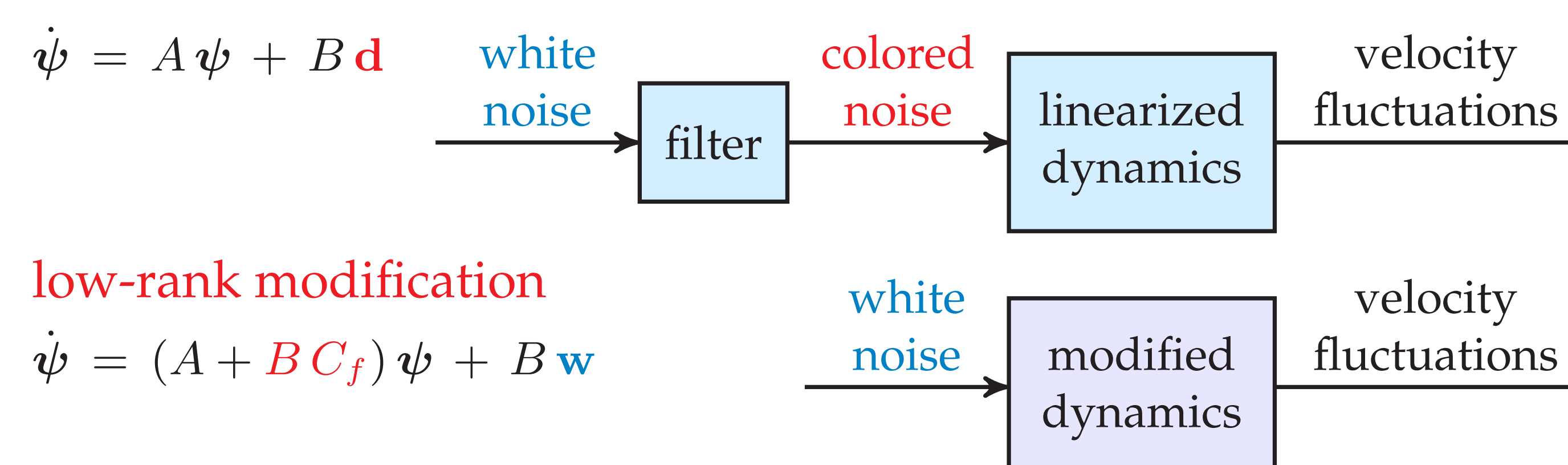
- **Convex optimization problem**

$$\text{minimize}_{X, Z} \quad -\log \det(X) + \gamma \|Z\|_*$$

$$\text{subject to} \quad AX + XA^* + Z = 0$$

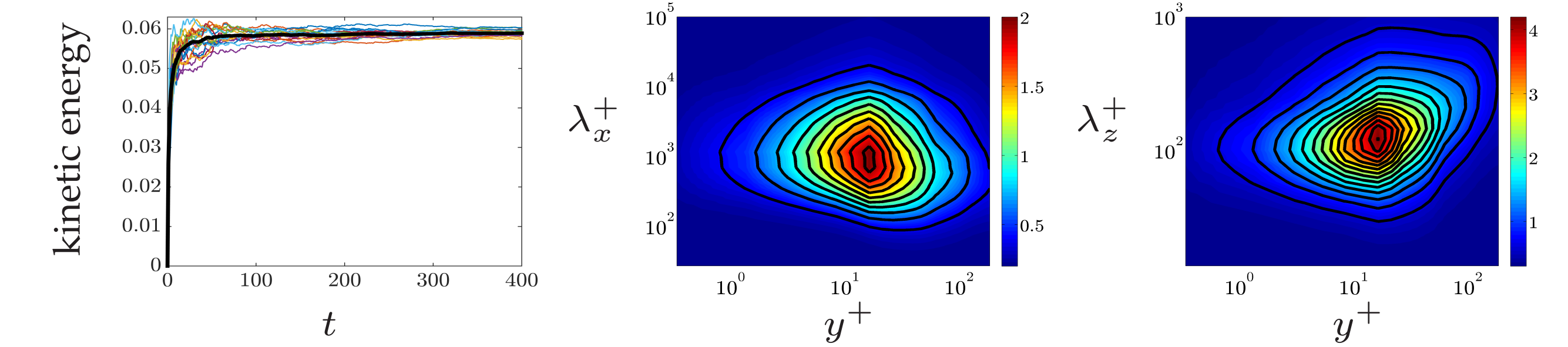
$$(CXC^*)_{ij} = \Phi_{ij} \quad (i, j) \in \mathcal{I}$$

- **Dynamics of colored-in-time forcing**

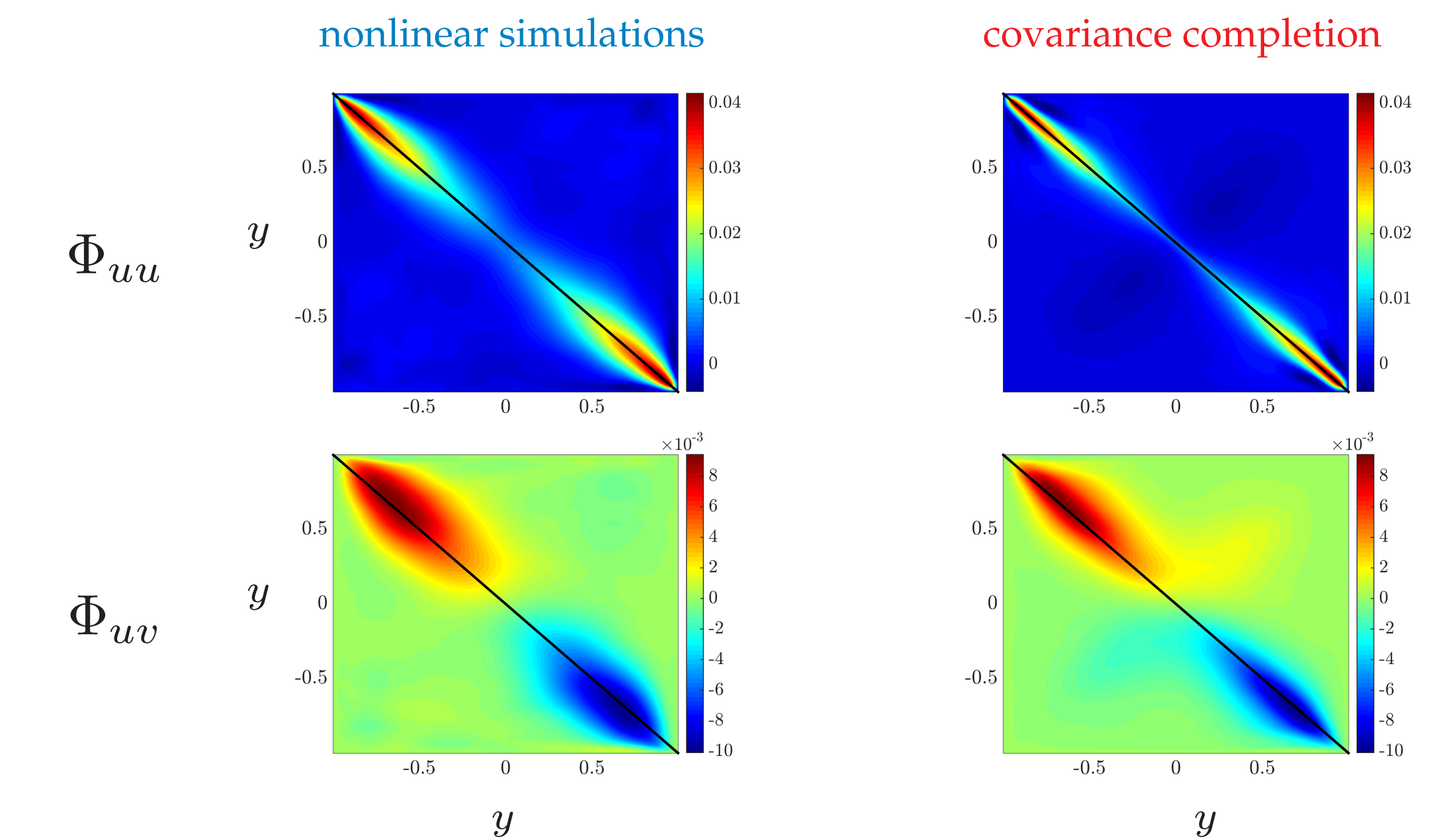


COVARIANCE COMPLETION

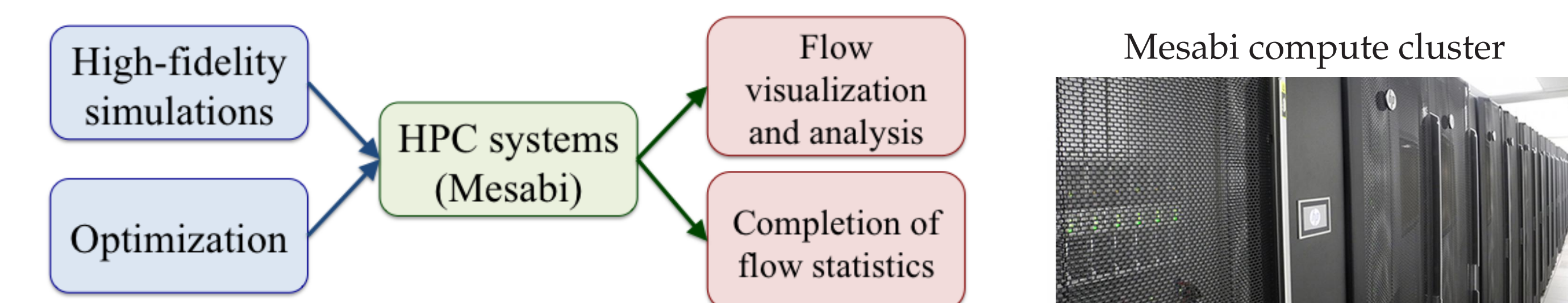
- linear stochastic simulations



- Recovered off-diagonals - two-point correlations



MSI RESOURCES



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PUBLICATIONS

- [1] A. Zare, M. R. Jovanović, and T. T. Georgiou, "Color of turbulence", in *J. Fluid Mech.*, 2016. Note: Submitted; also arXiv:1602.05105.
- [2] A. Zare, M. R. Jovanović, and T. T. Georgiou, "Alternating direction optimization algorithms for covariance completion problems", in *Proceedings of the 2015 American Control Conference*, 2015, pp. 515-520.
- [3] A. Zare, Y. Chen, M. R. Jovanović, and T. T. Georgiou, "Low-complexity modeling of partially available second-order statistics via matrix completion", in *IEEE Trans. Automat. Control*, 2014. Note: Submitted; also arXiv:1412.3399v1.
- [4] A. Zare, M. R. Jovanović, and T. T. Georgiou, "Completion of partially known turbulent flow statistics via convex optimization", in *Proceedings of the 2014 Summer Program, Center for Turbulence Research, Stanford University/NASA*.