Completion of partially known turbulent flow statistics

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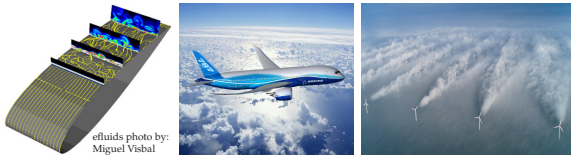
MOTIVATION

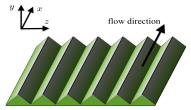
Modeling and control of turbulent flows

prevent/suppress turbulence
reduce turbulent drag
} **Economic impact**

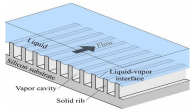


- **Challenges**
  - large number of degrees of freedom
  - complex flow dynamics
- **Objective**
  - control-oriented modeling of turbulent flows
- **Ongoing research**
  - model-based flow control design

passive

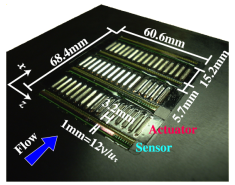


riblets



superhydrophobic surface

active



hot-film sensors and
wall-deformation actuators
(Yoshino *et al.* 2008)

APPROACH

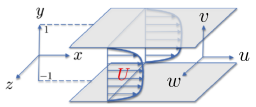
Stochastically forced Navier-Stokes equations

stochastic forcing → linearized dynamics → velocity fluctuations

- embed observed statistics of turbulence in **physics-based** models
- identify forcing statistics to account for available velocity statistics

COMPLETION OF TURBULENT FLOW STATISTICS

Turbulent channel flow

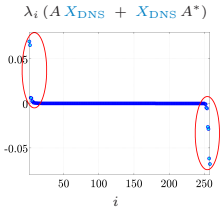


- **Linearized evolution model**

$$\begin{aligned}\dot{\psi} &= A\psi + B\mathbf{d}\\ \mathbf{v} &= C\psi\end{aligned} \qquad A=\begin{bmatrix} A_{os} & 0\\ A_{cp} & A_{sq}\end{bmatrix} \quad \psi=\begin{bmatrix} v\\ \eta\end{bmatrix} \quad \mathbf{v}=\begin{bmatrix} u\\ v\\ w\end{bmatrix}$$

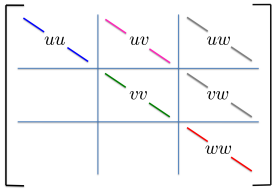
Lyapunov equation: $A\,X + X\,A^* = -B\,\Omega B^*$

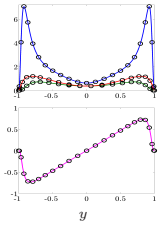
white-in-time excitation too restrictive!



Structured covariance completion problem

available correlations:





- **Convex optimization problem**

$$\begin{aligned}\underset{X,\,Z}{\text{minimize}} \quad & -\log\det(X) + \gamma\,\|Z\|_\star\\ \text{subject to} \quad & AX + XA^* + Z = 0\\ & (CXC^*)_{ij} = \Phi_{ij} \qquad (i,j)\in\mathcal{I}\end{aligned}$$

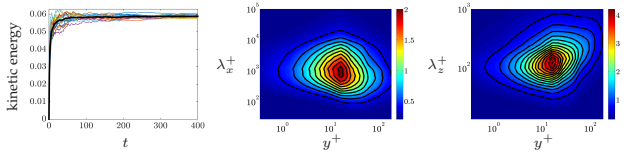
- **Dynamics of colored-in-time forcing**

$\dot{\psi} = A\psi + B\mathbf{d}$: white noise → filter → colored noise → linearized dynamics → velocity fluctuations

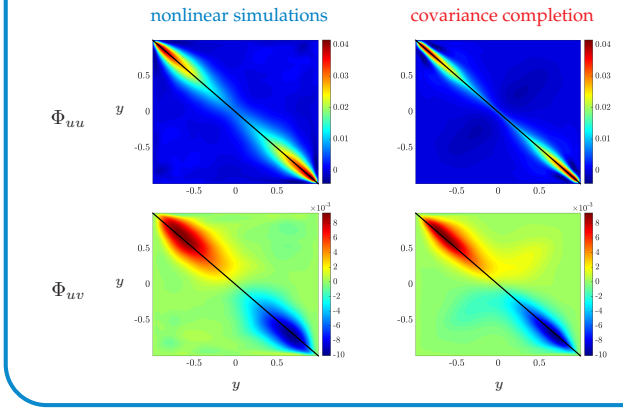
low-rank modification $\dot{\psi} = (A + BC_f)\psi + B\mathbf{w}$: white noise → modified dynamics → velocity fluctuations

COVARIANCE COMPLETION

- linear stochastic simulations



- Recovered off-diagonals - two-point correlations



REMARKS

- **Control-oriented modeling**
  stochastically forced linearized NS equations
  colored-in-time forcing accounts for partially observed statistics
  convex optimization framework
- **Acknowledgments**
  NSF Award CMMI 1363266; Doctoral Dissertation Fellowship
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PUBLICATIONS

[1] A. Zare, M. R. Jovanović, and T. T. Georgiou, "Alternating direction optimization algorithms for covariance completion problems", in *Proceedings of the 2015 American Control Conference*, 2015, pp. 515-520.

[2] A. Zare, Y. Chen, M. R. Jovanović, and T. T. Georgiou, "Low-complexity modeling of partially available second-order statistics via matrix completion", in *IEEE Trans. Automat. Control*, 2014. Note: Submitted; also arXiv:1412.3399v1.

[3] A. Zare, M. R. Jovanović, and T. T. Georgiou, "Completion of partially known turbulent flow statistics via convex optimization", in *Proceedings of the 2014 Summer Program, Center for Turbulence Research*, Stanford University/NASA.