THE UNIVERSITY OF TEXAS AT DALLAS

Electromagnetic Engineering I

EE 4301

Spring 2008
Assignment 2

Due Date:

January 21, 2008, at the beginning of class

Reading:

N. N. Rao, *Elements of Engineering Electromagnetics*, **Sixth Edition**, Chapter 2

Study Professor Syed A. Nasar, *2008+ Solved Problems in Electromagnetics*, Chapter 2

Problems:

Please write your answers to the following problems on engineering paper. No credit will be given for work handed in on other types of paper.

1. You are given the vectors

$$\mathbf{a} = -2\hat{\mathbf{x}} + \hat{\mathbf{y}} + 2\hat{\mathbf{z}}, \quad \mathbf{n} = \frac{1}{3}\mathbf{a}, \quad \mathbf{b} = \hat{\mathbf{x}} + 2\hat{\mathbf{y}} + 2\hat{\mathbf{z}}, \quad \mathbf{c} = 2\hat{\mathbf{x}} + \hat{\mathbf{y}} - 2\hat{\mathbf{z}}$$

   (a) Find $\mathbf{a} \cdot \mathbf{a}$.
   (b) Find $\mathbf{n} \cdot \mathbf{n}$.
   (c) Find $\mathbf{a} \cdot \mathbf{b}$.
   (d) Find $\mathbf{a} \cdot \mathbf{c}$.
   (e) Find $\mathbf{n} \cdot \mathbf{b}$ and draw a figure to scale in the plane defined by $\mathbf{a}$ and $\mathbf{b}$. Your figure should illustrate the fact that $\mathbf{n} \cdot \mathbf{b}$ is equal to the component of $\mathbf{b}$ in the direction of $\mathbf{a}$.
   (f) Find $\mathbf{a} \times \mathbf{b}$ in Cartesian component form.
   (g) Show using only Cartesian components that $\mathbf{a} \cdot (\mathbf{a} \times \mathbf{b}) = 0$.
   (h) Find $\mathbf{b} \times \mathbf{c}$ in Cartesian component form.
   (i) Show that the vectors $\mathbf{a}$ and $\mathbf{b}$ define a parallelogram, and find its area.
   (j) Find $\mathbf{a} \times (\mathbf{b} \times \mathbf{c})$ in Cartesian component form.
   (k) Find $\mathbf{a} \times (\mathbf{b} \times \mathbf{c})$ using the BAC-CAB rule, and verify that your answer is the same as the answer you obtained using Cartesian components.

(l) Show that the vectors **a**, **b** and **c** define a parallelepiped, and find its volume.

(m) Draw the parallelepiped to scale, with correct angles between the sides.

2. State Euler's theorem, and use it to evaluate the real and imaginary parts of $e^{j\pi/6}$ without using a calculator.

3. The voltage across an impedance $Z = 3e^{j\theta}$ Ω is $6e^{j\omega t}$ V. Find the current and the time-averaged power without using a calculator. For what value(s) of θ, if any, is the time-averaged power equal to zero?

4. Find the derivative with respect to x of each of the following functions:

   (a) $\sin(2\pi \times 10^{10}x)$

   (b) $\cos(5 \times 10^{8}x)$

   (c) $e^{-x/3}$

   (d) e^{jx}

   (e) $1/x$

5. Find the integral with respect to x of each of the functions in the preceding problem.

6. A charge of + 10 C is suspended inside a grounded, rectangular, perfectly conducting box that measures $20\text{cm} \times 5\text{cm} \times 10\text{cm}$. What is the value of the total charge on the inner surface of the box?

7. **Design Problem.**[1] For each of the following pairs of electric field intensities, find if possible the location and value of a point charge that produces both fields:

   (a)
   $$\mathbf{E}_1 = 2\hat{\mathbf{x}} + 2\hat{\mathbf{y}} + 1\hat{\mathbf{z}} \quad \text{V/m} \quad \text{at} \quad (2,2,3) \quad \text{and} \quad \mathbf{E}_2 = 1\hat{\mathbf{x}} + 2\hat{\mathbf{y}} + 2\hat{\mathbf{z}} \quad \text{V/m} \quad \text{at} \quad (-1,0,3)$$

   (b)
   $$\mathbf{E}_1 = 2\hat{\mathbf{x}} + 2\hat{\mathbf{y}} + 1\hat{\mathbf{z}} \quad \text{V/m} \quad \text{at} \quad (1,1,1) \quad \text{and} \quad \mathbf{E}_2 = 2\hat{\mathbf{x}} + 1\hat{\mathbf{y}} + 2\hat{\mathbf{z}} \quad \text{V/m} \quad \text{at} \quad (1,2,0)$$

8. You are given a parallel-plate capacitor containing a dielectric with relative permittivity $\epsilon_r = 4$, spacing $d = 0.1$ μm, and area $A = (1.0\ \mu\text{m})^2$.

   (a) Sketch several lines of **E** and several equipotentials.

   (b) Assuming that fringing fields can be neglected, find the capacitance C. (Please remember to use the correct units!)

9. You are given two concentric, perfectly conducting spherical shells. Assume that the inner radius of the outer shell is b, that the outer radius of the inner shell is a, where $a < b$, and that the space between the shells is filled with a dielectric with permittivity $\epsilon = \epsilon_r \epsilon_0$.

   (a) Derive a formula for **D** in the dielectric between the concentric spheres in terms of the charge Q on the inner sphere, the distance r from the center of the inner sphere, and the radial unit vector $\hat{\mathbf{r}}$.

   (b) Sketch several lines of **D**.

   (c) Using the result of (a), derive down a formula for **E** in the dielectric between the concentric spheres.

   (d) Evaluate the formulas derived in (a) and (c) numerically, assuming that $Q = 1$ C, $a = 5 \times 10^{-4}$ m, $b = 10^{-3}$ m, and $\epsilon_r = 2.1$. Be sure to show all units in your calculation!

   (e) Sketch several equipotentials.

10. Work this problem[2] in both Cartesian (rectangular) coordinates and plane polar coordinates: Consider an analog watch that keeps perfect time and assume the origin to be at the center of the dial, the x axis passing through the 12 mark, and the y axis passing through the 3 mark.

    (a) Write the expression for the time-varying unit vector directed along the hour hand of the watch.

    (b) Write the expression for the time-varying unit vector directed along the minute hand of the watch.

    (c) Obtain the specific expression for these unit vectors when the hour hand and the minute hand are aligned exactly and [are] between the 5 and 6 marks.

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[1] Rao, *Elements of Engineering Electromagnetics*, 5th Edition, Problem P2.3.

[2] Rao, *Elements of Engineering Electromagnetics*, 5th Edition, Problem P1.33.

11. Work this problem[3] in both Cartesian (rectangular) coordinates and plane polar coordinates: Find all values of

$$\frac{1}{\sqrt[4]{-7 + j7\sqrt{3}}},$$

and plot them in the complex plane.

12. Plot the field lines of **E** for two charges of $+1$ and -1 Coulomb each, separated by a distance of 1 meter, in a plane that contains both charges.

[3] After Rao, *Elements of Engineering Electromagnetics*, 5th Edition, Problem P1.36.