THE UNIVERSITY OF TEXAS AT DALLAS

Electromagnetic Engineering I

EE 4301

Spring 2008
Assignment 9

Due Date and Time:

At the beginning of class, March 24, 2008

Reading:

N. N. Rao, *Elements of Engineering Electromagnetics*, **Sixth Edition**, Chapters 3 and 6
Professor Cantrell's notes on attenuation

Problems:

Please write your answers to the following problems on engineering paper. No credit will be given for work handed in on other types of paper.

1. At what frequency f_1 is the loss tangent $\sigma(\omega\epsilon)^{-1}$ equal to 1 for seawater ($\sigma = 4$ S/m, $\epsilon_r = 81$, $\mu = \mu_0$)? Below what frequency f_{100} is seawater a good conductor ($\sigma(\omega\epsilon)^{-1} \geq 100$)?

2. What is the value of the skin depth for seawater at a frequency of 1 MHz? At a frequency of 1 kHz?

3. In an optical fiber, the power received at a distance of 10 km from the transmitter is measured as 0.1 mW (which would usually be expressed as -10 dBm). The power transmitted is 1.0 mW (0 dBm). Find the attenuation coefficient in m^{-1} and in dB/km. [Hint: See page 1 of Professor Cantrell's notes on attenuation, but pay attention to the units!]

4. The attenuation coefficient of RG/58U coaxial cable is given in a manufacturer's data sheet as 4.6 dB/100 feet at a frequency of 100 MHz, and 17.5 dB/100 feet at a frequency of 1 GHz. Convert the manufacturer's data into values for the attenuation coeffcient α in units of m^{-1} at 100 MHz and 1 GHz. Also, for a propagation distance of 100 *meters*, calculate the ratio of the power received at 1 GHz to the power received at 100 MHz, assuming that equal power was transmitted at these two frequencies.

5. Fill in the following table: lambda-f-table.pdf

6. At what frequency is the skin depth equal to 0.1 μm in aluminum (conductivity $\sigma = 3.54 \times 10^7$ S/m, permeability $\mu = \mu_0$)?

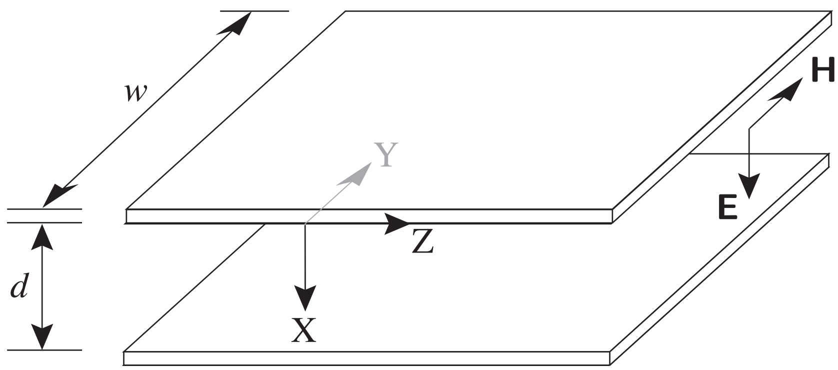

Figure 1: Fields between two perfectly conducting plates.

7. A plane wave with the electric field

$$\mathbf{E}(\mathbf{r}, t) = 1.5 \cos[2\pi \times 10^9 t - kz]\ \hat{\mathbf{x}} \quad \mathrm{V/m} \tag{1}$$

is propagating in the positive Z direction between infinite, perfectly conducting plates, as shown in Fig. 1. The medium between the PEC plates is a perfect dielectric with $\epsilon_r = 4.0$ and $\mu_r = 1.0$.

(a) Find the value of k.

(b) Find the magnetic field **H**. Please give correct units in your answer.

(c) Find the time-averaged magnitude and direction of the Poynting vector

$$\mathbf{S} = \mathbf{E} \times \mathbf{H}. \tag{2}$$

Please give correct units in your answer.

(d) Find the surface charge density, ρ_s, on the lower surface of the upper plate. Please give correct units in your answer.

(e) Find the surface current density, $\mathbf{J}_s$, on the lower surface of the upper plate. Please give correct units in your answer.

8. A plane wave that propagates in the positive Z direction and produces an upper-plate surface charge density

$$\rho_s(\mathbf{r}, t) = 8.0\epsilon_0 \cos[2\pi \times 10^9 t - 2\pi \times 6.67z] \quad \mathrm{C/m^2} \tag{3}$$

is confined between infinite, perfectly conducting plates, as shown in Fig. 1. The medium between the PEC plates is a perfect dielectric with $\epsilon_r = 4.0$ and $\mu_r = 1.0$. The units of the constant 8.0 in Eq. (3) are V/m.

(a) Find the electric field **E** between the plates. Please give correct units in your answer.

(b) Find the magnetic field **H** between the plates. Please give correct units in your answer.

(c) Find the surface current density, $\mathbf{J}_s$, on the lower surface of the upper plate. Please give correct units in your answer.

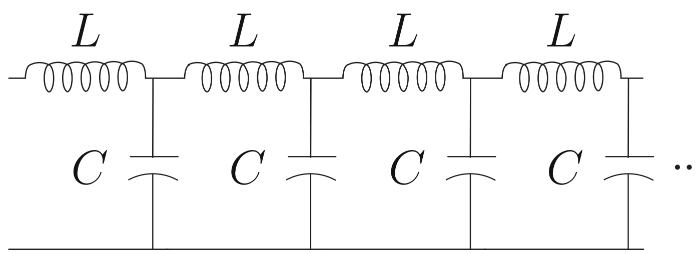

Figure 2: Lumped model of an infinite transmission line.

9. A plane wave propagating in the Z direction is incident upon a sheet of polarizing material. The sheet lies in the $X-Y$ plane. The **E** field of the plane wave is also in the $X-Y$ plane, but is inclined at an angle θ with respect to the X axis. The polarizing material is designed in such a way that if a plane wave's **E** field is along the X axis, then the E_x and H_y components of the wave wave go through the sheet with no loss and no reflection. However, if a wave's **E** field is along the Y axis, then the sheet completely blocks both the E_y and H_x components of the wave, so that no power is transmitted through the sheet.

   Show that, if $\mathbf{S}_0$ is the Poynting vector of the plane wave before it gets to the sheet, and $\mathbf{S}_1$ is the Poynting vector of the wave (if any) that is transmitted through the sheet,

$$|\mathbf{S}_1| = \cos^2(\theta)|\mathbf{S}_0|. \tag{4}$$

   This relation is known in optics as Malus' law. You can observe it qualitatively with a sheet of Polaroid film or Polaroid sunglasses.

10. A message with a logical length of 2.5×10^6 bits is transmitted at a rate of 2.5×10^9 bits/second.

   (a) How long does it take to transmit the message?

   (b) Assuming that the transmitted message is propagated on an optical fiber with a relative dielectric permittivity $\epsilon_r = 2.25$ and a relative magnetic permittivity $\mu_r = 1.00$, what is the phase velocity of light in the fiber?

   (c) Assuming that the fiber is 1000 km long, what is the end-to-end propagation time for the first bit of the message?

   (d) What is the physical length of the message on the fiber (in m)?

11. We briefly discussed in class a lumped model of a transmission line, as shown in Fig. 2, where

$$L = L'\Delta z, \qquad C = C'\Delta z, \tag{5}$$

   L', C' are the inductance per unit length and capacitance per unit length, and Δz is the length of a short segment of a very long (theoretically infinite) transmission line.

   Since the line is modeled as being infinitely long, it contains infinitely many $L-C$ sections. Therefore its impedance, Z_0, does not change if another $L-C$ section is added to the line, as shown in Fig. 3.

   (a) Turn Fig. 3 into a circuit equation, and solve it for Z_0 at a given segment length Δz, assuming that the input to the circuit is a sinusoidal signal at a given frequency ω.

   (b) Show from your equation for Z_0 at a frequency ω and a segment length Δz that

$$\lim_{\Delta z \to 0} Z_0 = \sqrt{\frac{L'}{C'}}, \tag{6}$$

   independent of the frequency ω, thereby establishing that the lumped model of a transmission line correctly reproduces the impedance of a transmission line as defined in lecture (in the limit in which the segment length in the lumped model becomes very small).

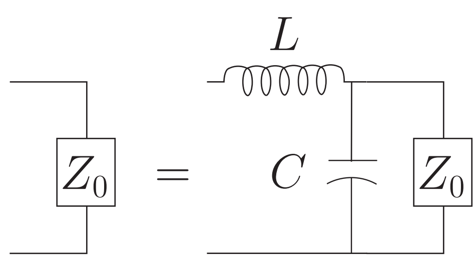

Figure 3: Calculation of the impedance of an infinite transmission line in a lumped model.