THE UNIVERSITY OF TEXAS AT DALLAS

Electromagnetic Engineering I

EE 4301

Spring 2008
Assignment 12

Due Date and Time:

April 14, 2008, at the beginning of class

Reading:

N. N. Rao, *Elements of Engineering Electromagnetics*, **Sixth Edition**, Chapter 10

Problems:

Please write your answers to the following problems on engineering paper. No credit will be given for work handed in on other types of paper.

1. In class we discussed Feynman's formula for the electric field of a moving charge q,

$$\mathbf{E}(\mathbf{r},t) = -\frac{q}{4\pi}\left[\frac{\mu_0}{R}\hat{R}\times(\mathbf{a}'\times\hat{R}) + \sqrt{\frac{\mu_0}{\epsilon_0}}\frac{1}{R^2}\left[\hat{R}\times(\mathbf{v}'\times\hat{R}) - 2(\hat{R}\cdot\mathbf{v}')\hat{R}\right] + \frac{\hat{R}}{\epsilon_0 R^2}\right]\Bigg|_{t'=t-R/c} \tag{1}$$

where the first line is the radiation field, the second line is the induction field, and the third line is the Coulomb (static) field. Note that the velocity $\mathbf{v}'(t')$ and the acceleration $\mathbf{a}'(t')$ are evaluated at the retarded time, $t' = t - R/c$.

(a) Suppose that the charge is at rest for times $t' < 0$. At $t' = 0$, the charge is subjected to an impulsive acceleration,

$$\mathbf{a}'(t') = \hat{\mathbf{z}}v_0\delta(t'). \tag{2}$$

Find the velocity $\mathbf{v}'(t')$ for $t' \leq 0$ and $t' > 0$.

(b) Using Eqs. (1) and (2), obtain a formula for $\mathbf{E}(\mathbf{r},t)$ at an angle of 90° to the z direction. [Hint: At 90° to the z direction, $\mathbf{v}'(t')\cdot\hat{R} = 0$ and $\mathbf{a}'(t')\cdot\hat{R} = 0$.]

(c) Using the result you obtained in part (b), obtain a formula for the ratio of the magnitude of the induction field to the magnitude of the Coulomb field for times such that $t - R/c > 0$. Your formula should contain the ratio v_0/c, where $c = 1/\sqrt{\mu_0\epsilon_0}$. What conclusion can you draw when $v_0/c \ll 1$, as is almost always the case in electrical engineering?

(d) Show from Eqs. (1) and (2) that an impulsive acceleration gives rise to a radiation field that is an impulse at retarded time $t - R/c = 0$. Sketch the magnitude of the z-component of the electric field, as a function of time, at a point in the $x - y$ plane that is at a distance R from the initial position of the charge.

(e) Sketch a few field lines of **E** for $t > 0$.

2. A point charge of 0.25 C has the coordinates

$$x = 0, \quad y = 0, \quad z = 10^{-6}\cos(6\pi \times 10^8 t)$$

(all in meters).

(a) Find the acceleration of the particle. (Please remember that acceleration is a vector with x, y and z components, and use correct units.)

(b) Show that a receiver at a distance $R = 25$ m is in the radiation zone. Assume that $\epsilon_r = 1$ and $\mu_r = 1$.

(c) For the receiver in part (b),

$$\hat{\mathbf{R}} = 0\hat{\mathbf{x}} - 0.6\hat{\mathbf{y}} + 0.8\hat{\mathbf{z}}.$$

Find the electric field vector at the receiver, taking into account the time required for an electromagnetic wave to travel from the charge to the receiver (*i.e.*, evaluate the electric field at the retarded time $t - R/v$).

(d) Find the magnetic field vector at the receiver, assuming that the receiver is in the radiation zone.

(e) Find the Poynting vector at the receiver.

3. A point P is 100 m away from an electromagnetic radiator in which the current is oscillating at a frequency of 3×10^8 Hz. Is P in the radiation zone? Justify your answer.

4. Consider a Hertzian dipole in which the current is oscillating at a frequency of 3×10^8 Hz. The height of the dipole is $h = 0.01$ m, and the maximum value of the current is 1 A. Evaluate the electric and magnetic fields at a distance of 100 m in a direction such that

$$\hat{R} = -\hat{\mathbf{x}}\sin\theta - \hat{\mathbf{z}}\cos\theta \tag{3}$$

where $\theta = \pi/4$ radians, and draw a diagram showing the Hertzian dipole antenna, **E**, and **H**. Also find the Poynting vector, and draw the Poynting vector on your sketch.