

Vector operators in the major coordinate systems

The gradient of a scalar field ψ is

$$\nabla\psi = \frac{\partial\psi}{\partial x}\hat{\mathbf{x}} + \frac{\partial\psi}{\partial y}\hat{\mathbf{y}} + \frac{\partial\psi}{\partial z}\hat{\mathbf{z}} \quad (\text{V-1})$$

in Cartesian coordinates,

$$\nabla\psi = \frac{\partial\psi}{\partial r}\hat{\mathbf{r}} + \frac{1}{r}\frac{\partial\psi}{\partial\phi}\hat{\phi} + \frac{\partial\psi}{\partial z}\hat{\mathbf{z}} \quad (\text{V-2})$$

in circular cylindrical coordinates, where $r = \sqrt{x^2 + y^2}$, and

$$\nabla\psi = \frac{\partial\psi}{\partial r}\hat{\mathbf{r}} + \frac{1}{r}\frac{\partial\psi}{\partial\theta}\hat{\theta} + \frac{1}{r\sin\theta}\frac{\partial\psi}{\partial\phi}\hat{\phi} \quad (\text{V-3})$$

in spherical polar coordinates, where $r = \sqrt{x^2 + y^2 + z^2}$.

The divergence $\nabla \cdot \mathbf{F}$ of a vector field \mathbf{F} is

$$\nabla \cdot \mathbf{F} = \frac{\partial F_x}{\partial x} + \frac{\partial F_y}{\partial y} + \frac{\partial F_z}{\partial z} \quad (\text{V-4})$$

in Cartesian coordinates,

$$\nabla \cdot \mathbf{F} = \frac{1}{r}\frac{\partial(rF_r)}{\partial r} + \frac{1}{r}\frac{\partial F_\phi}{\partial\phi} + \frac{\partial F_z}{\partial z} \quad (\text{V-5})$$

in circular cylindrical coordinates, where $r = \sqrt{x^2 + y^2}$, and

$$\nabla \cdot \mathbf{F} = \frac{1}{r^2}\frac{\partial(r^2F_r)}{\partial r} + \frac{1}{r\sin\theta}\frac{\partial(\sin\theta F_\theta)}{\partial\theta} + \frac{1}{r\sin\theta}\frac{\partial F_\phi}{\partial\phi} \quad (\text{V-6})$$

in spherical polar coordinates, where $r = \sqrt{x^2 + y^2 + z^2}$.

The curl $\nabla \times \mathbf{F}$ of a vector field \mathbf{F} is

$$\nabla \times \mathbf{F} = \left(\frac{\partial F_z}{\partial y} - \frac{\partial F_y}{\partial z}\right)\hat{\mathbf{x}} + \left(\frac{\partial F_x}{\partial z} - \frac{\partial F_z}{\partial x}\right)\hat{\mathbf{y}} + \left(\frac{\partial F_y}{\partial x} - \frac{\partial F_x}{\partial y}\right)\hat{\mathbf{z}} \quad (\text{V-7})$$

in Cartesian coordinates,

$$\nabla \times \mathbf{F} = \left(\frac{1}{r}\frac{\partial F_z}{\partial\phi} - \frac{\partial F_\phi}{\partial z}\right)\hat{\mathbf{r}} + \left(\frac{\partial F_r}{\partial z} - \frac{\partial F_z}{\partial r}\right)\hat{\phi} + \frac{1}{r}\left(\frac{\partial(rF_\phi)}{\partial r} - \frac{\partial F_r}{\partial\phi}\right)\hat{\mathbf{z}} \quad (\text{V-8})$$

in circular cylindrical coordinates, where $r = \sqrt{x^2 + y^2}$, and

$$\nabla \times \mathbf{F} = \frac{1}{r\sin\theta}\left(\frac{\partial(\sin\theta F_\phi)}{\partial\theta} - \frac{\partial F_\theta}{\partial\phi}\right)\hat{\mathbf{r}} + \frac{1}{r}\left(\frac{1}{\sin\theta}\frac{\partial F_r}{\partial\phi} - \frac{\partial(rF_\phi)}{\partial r}\right)\hat{\theta} + \frac{1}{r}\left(\frac{\partial(rF_\theta)}{\partial r} - \frac{\partial F_r}{\partial\theta}\right)\hat{\phi} \quad (\text{V-9})$$

in spherical polar coordinates, where $r = \sqrt{x^2 + y^2 + z^2}$.

The Laplacian $\nabla^2\psi$ of a scalar field ψ is

$$\nabla^2\psi = \frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2} + \frac{\partial^2\psi}{\partial z^2} \quad (\text{V-10})$$

in Cartesian coordinates,

$$\nabla^2\psi = \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial\psi}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2\psi}{\partial\phi^2} + \frac{\partial^2\psi}{\partial z^2} \quad (\text{V-11})$$

in circular cylindrical coordinates, where $r = \sqrt{x^2 + y^2}$, and

$$\nabla^2\psi = \frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial\psi}{\partial r} \right) + \frac{1}{r^2 \sin\theta} \frac{\partial}{\partial\theta} \left(\sin\theta \frac{\partial\psi}{\partial\theta} \right) + \frac{1}{r^2 \sin^2\theta} \frac{\partial^2\psi}{\partial\phi^2} \quad (\text{V-12})$$

in spherical polar coordinates, where $r = \sqrt{x^2 + y^2 + z^2}$.

Poisson's equation is

$$\nabla^2\psi = -\frac{\rho}{\epsilon} \quad (\text{V-13})$$

where ψ is the electrostatic scalar potential, ρ is the volume charge density, and ϵ is the permittivity.

Laplace's equation is the homogeneous (charge density = 0) case of Poisson's equation:

$$\nabla^2\psi = 0 \quad (\text{V-14})$$

Commonly-used unit prefixes

Prefix	Meaning
f (femto)	10^{-15}
p (pico)	10^{-12}
n (nano)	10^{-9}
μ (micro)	10^{-6}
m (milli)	10^{-3}
c (centi)	10^{-2}
K (Kilo)	10^3
M (Mega)	10^6
G (Giga)	10^9
T (Tera)	10^{12}
P (Peta)	10^{15}