

# OPTICAL AMPLIFIERS

Notes prepared for EE 6310

by

Professor Cyrus D. Cantrell

August–December 2003

## AMPLIFIERS FOR OPTICAL COMMUNICATIONS

- Configurations

  - Booster amplifier following transmitter

  - In-line amplifier

    - Functions as a non-regenerative repeater

  - Preamplifier before receiver

- Technologies

  - Semiconductor optical amplifiers (SOAs)

  - Rare-earth-doped fiber amplifiers (EDFAs, EDFFAs, PDFAs, etc.)

- Classifications

  - Narrowband (**Fabry-Pérot amplifiers** such as SOAs)

  - Broadband (**traveling-wave amplifiers** such as EDFAs)

## GAIN OF A TWO-ENERGY-LEVEL MEDIUM

- For a lossless traveling-wave amplifier (the simplest case!),

$$\frac{dP}{dz} = gP \quad \text{where } P = \text{power, } g = \text{gain coefficient}$$

- If all atoms or ions have the same difference of energy,  $\hbar \omega_0$ , between excited and ground states, then the medium is **homogeneously broadened** and the frequency dependence of the gain coefficient is

$$g(\omega) = \frac{g_p}{1 + (\omega - \omega_0)^2 T_2^2 + P/P_s}$$

where  $P_s =$  **saturation power**,  $T_2 =$  **dipole relaxation time**,  
 $g_p =$  peak gain coefficient

At powers well below the saturation power,

$$P \ll P_s \Rightarrow g(\omega) \approx \frac{g_p}{1 + (\omega - \omega_0)^2 T_2^2}$$

**AMPLIFICATION FACTOR**

- For a traveling-wave amplifier with loss and with imperfect mode confinement,

$$\frac{dP}{dz} = (g - \alpha)P$$

where  $\alpha$  = attenuation coefficient and  $g$  = confinement factor

- The **amplification factor** is the ratio of power out to power in,

$$G = \frac{P_{\text{out}}}{P_{\text{in}}}$$

At powers well below the saturation power,

$$G = e^{(g - \alpha)L}$$

where  $L$  = length of active medium

## GAIN NARROWING

- Single-pass amplification factor:

$$G(\omega) = e^{(g(\omega) - \alpha)L}$$

$\alpha$  = confinement factor,  $g$  = small-signal gain per unit length,  
 $\alpha$  = attenuation coefficient,  $L$  = length of active region

$G(\omega)$  may be much narrower than  $g(\omega)$ , depending on  $\alpha$  and  $L$

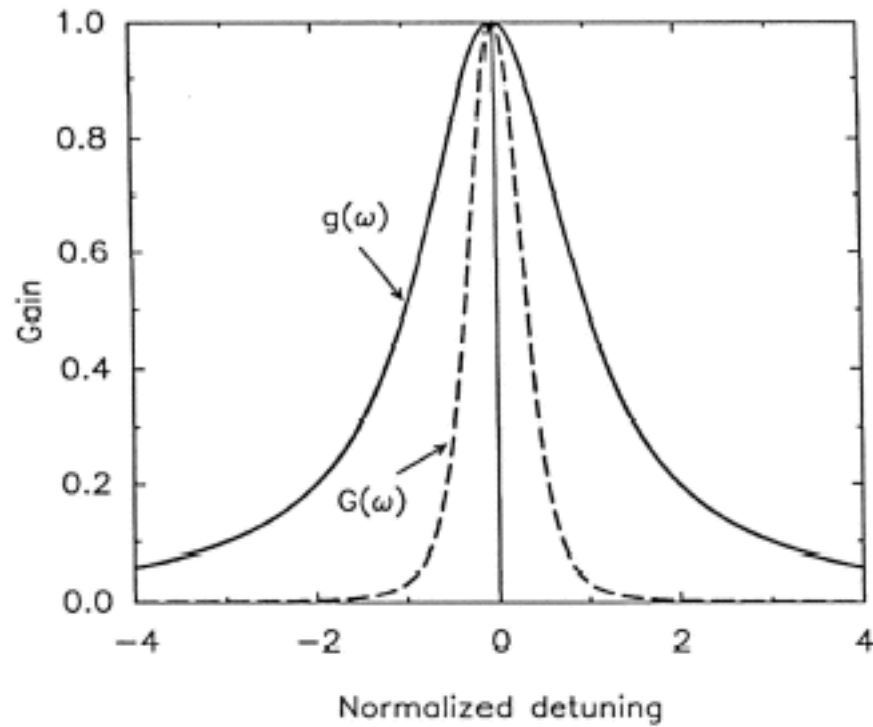
The half-width at half-maximum of  $G$  is such that

$$e^{(g(\omega_0 + \Delta\omega) - \alpha)L} = G(\omega_0 + \Delta\omega) = \frac{1}{2}G(\omega_0) = e^{-\ln 2 + (g(\omega_0) - \alpha)L}$$

$$\Rightarrow \frac{g(\omega_0) - g(\omega_0 + \Delta\omega)}{g(\omega_0)} = \frac{\ln 2}{Lg(\omega_0)}$$

If  $Lg(\omega_0) \gg 1$  (as it must be for laser action), then a very small relative change in  $g$  changes  $G$  by a factor of 2

# ILLUSTRATION OF GAIN NARROWING



## USABLE AMPLIFIER BANDWIDTH (1)

- The gain bandwidth (HWHM) is

$$\Delta \omega_g = \frac{1}{T_2}$$

where  $T_2$  is the dipole relaxation time

- The usable amplifier bandwidth,  $\Delta \omega_G$ , is

$$\Delta \omega_G \approx \Delta \omega_g \sqrt{\frac{\ln 2}{\ln G_0}}$$

where

$$G_0 = e^{(g(\omega) - \alpha)L}$$

Multiple channels can be amplified simultaneously if they all lie within the amplifier bandwidth

## USABLE AMPLIFIER BANDWIDTH (2)

- Derivation of

$$\Delta \omega_G \approx \Delta \omega_g \sqrt{\frac{\ln 2}{\ln G_0}} :$$

Parabolic approximation to  $g$  near its peak at  $\omega = \omega_0$ :

$$g(\omega_0 + \Delta \omega_G) \approx g(\omega_0) \left[ 1 - \left( \frac{\Delta \omega_G}{\Delta \omega_g} \right)^2 \right]$$

Expression for the amplification factor at half-maximum:

$$\frac{G_0}{2} \approx \exp \left\{ L g(\omega_0) \left[ 1 - \left( \frac{\Delta \omega_G}{\Delta \omega_g} \right)^2 \right] \right\}$$

Rearrange to get

$$\frac{\Delta \omega_G}{\Delta \omega_g} \approx \sqrt{1 - \frac{\ln(G_0/2)}{\ln G_0}} = \sqrt{\frac{\ln G_0 - \ln(G_0/2)}{\ln G_0}} = \sqrt{\frac{\ln 2}{\ln G_0}}$$

## GAIN SATURATION

- On resonance ( $\omega = \omega_0$ ), the equation for the optical power in a traveling-wave amplifier is

$$\frac{dP}{dz} = \left( \frac{g(\omega_0)}{1 + P/P_s} - \alpha \right) P \approx \frac{g(\omega_0)}{1 + P/P_s} P$$

if the gain greatly exceeds the attenuation

An implicit equation for the resulting amplification factor is

$$G = G_0 \exp \left( -\frac{G - 1}{G} \frac{P(L)}{P_s} \right)$$

The **saturation output power** is the output power for which the amplification factor is reduced by 3 dB as a result of saturation:

$$P_{s,\text{out}} = \frac{G_0 \ln 2}{G_0 - 2} P_s$$

In practical amplifiers  $G_0 \gg 2$ , making

$$P_{s,\text{out}} \approx \ln 2 P_s$$

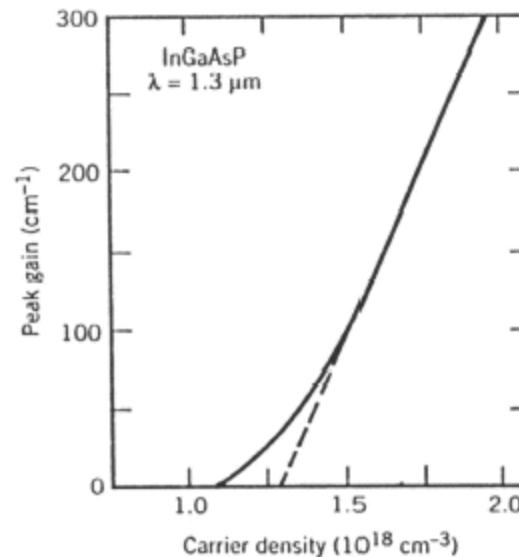
**PEAK GAIN COEFFICIENT IN A SEMICONDUCTOR LASER**

- Peak gain coefficient of a semiconductor medium:

$$g_p = g(N - N_T)$$

$N_T$  = value of carrier density that makes the medium transparent

- Peak gain coefficient of InGaAsP vs.  $N$ :



## FABRY-PEROT AMPLIFIER

- The amplification factor when significant facet reflections are present is not the same as the gain without reflections

Amplification factor with reflections:

$$G_{\text{FP}}(\omega) = \frac{(1 - R_1)(1 - R_2)G(\omega)}{(1 - G\sqrt{R_1 R_2})^2 + 4G\sqrt{R_1 R_2} \sin^2[(\omega - \omega_m)/2\Delta \omega_L]}$$

$\Delta \omega_L$  = longitudinal mode spacing,  $R_1, R_2$  = facet reflection coefficients

The denominator is periodic

- This produces Fabry-Pérot resonances with a spacing equal to the longitudinal mode spacing

## OPTICAL FIBER AMPLIFIER TECHNOLOGIES

- EDFAs

Erbium-doped fiber amplifiers

- Began as a university research project

Optically pumped at 980 nm or 1480 nm

Most commonly used in the C band (1530–1560 nm)

- EDFFAs

Erbium-doped fluoride fiber amplifiers

- PDFAs

Praseodymium-doped fiber amplifiers