THE UNIVERSITY OF TEXAS AT DALLAS

Physical Optics

EE 6317

Summer 2008
Assignment 1

Due Date:

June 4, 2008

Reading:

Eugene Hecht, *Optics*, Fourth Edition, Chapters 7 and 8

Reference Material:

Dr. Cantrell's notes on time-domain and frequency-domain analysis of transmission lines

Problems:

Please write your answers to the following problems on engineering paper. No credit will be given for work handed in on other types of paper.

1. In your undergraduate electromagnetics course, it was shown that, for a uniform plane wave propagating in the $\pm z$ directions,
$$\frac{\partial E_x}{\partial z} = -\mu\frac{\partial H_y}{\partial t}, \quad \frac{\partial E_y}{\partial z} = \mu\frac{\partial H_x}{\partial t}, \quad \frac{\partial H_x}{\partial z} = \epsilon\frac{\partial E_y}{\partial t}, \quad \text{and} \quad \frac{\partial H_y}{\partial z} = -\epsilon\frac{\partial E_x}{\partial t} \; .$$
It was also shown that E_x, E_y, H_x, and H_y all satisfy the homogeneous one-dimensional wave equation,
$$\left(\frac{\partial^2}{\partial z^2} - \frac{1}{v^2}\frac{\partial^2}{\partial t^2}\right) E_x = 0,$$
(and similarly for the other components of **E** and **H**), where
$$v = \frac{1}{\sqrt{\mu\epsilon}}.$$
It was also shown that the general solution of the one-dimensional wave equation
$$\left(\frac{\partial^2}{\partial z^2} - \frac{1}{v^2}\frac{\partial^2}{\partial t^2}\right) u = 0$$

is

$$u = f(z - vt) + g(z + vt),$$

where f and g are arbitrary twice-differentiable functions, and that $f(z - vt)$ represents a wave traveling in the $+z$ direction, while $g(z + vt)$ represents a wave traveling in the $-z$ direction.

Use these equations to prove the following facts for a uniform plane propagating in the $\pm z$ directions:

(a) For a wave propagating in the $+z$ direction, the ratio of E_x to H_y is

$$\frac{E_x}{H_y} = \eta$$

where

$$\eta = \sqrt{\frac{\mu}{\epsilon}}$$

has the units of impedance (ohms).

(b) For a wave propagating in the $+z$ direction, the ratio of E_y to H_x is

$$\frac{E_y}{H_x} = -\eta.$$

(c) For a wave propagating in the $-z$ direction, the ratio of E_x to H_y is

$$\frac{E_x}{H_y} = -\eta.$$

(d) For a wave propagating in the $-z$ direction, the ratio of E_y to H_x is

$$\frac{E_y}{H_x} = \eta.$$

(e) For this example,

$$\mathbf{H} = \frac{1}{\eta}\hat{\mathbf{k}} \times \mathbf{E}$$

where $\hat{\mathbf{k}}$ is a *unit* vector that points in the direction of propagation.

2. For the electromagnetic plane wave that has the electric field vector

$$\mathbf{E}(x, y, z, t) = \mathbf{E}_0 \cos(\omega t - \mathbf{k} \cdot \mathbf{r}),$$

find $\mathbf{H}$ and show that the *time-averaged* Poynting vector $\overline{\mathbf{S}} = \overline{\mathbf{E} \times \mathbf{H}}$ is

$$\overline{\mathbf{S}}(x, y, z, t) = \frac{\mathbf{E}_0^2}{2\eta}\hat{\mathbf{k}}$$

where η is the wave impedance of the medium in which the wave propagates.