COVERAGE ESTIMATION FOR MOBILE CELLULAR NETWORKS FROM SIGNAL STRENGTH MEASUREMENTS

by

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PREFACE

This dissertation was produced in accordance with guidelines which permit the inclusion as part of the dissertation the text of an original paper, or papers, submitted for publication. The dissertation must still conform to all other requirements explained in the "Guide for Preparation of Master's Theses, Doctoral Dissertations, and Doctor of Chemistry Practica Reports at The University of Texas at Dallas." It must include a comprehensive abstract, a full introduction and literature review, and a final overall conclusion. Additional material (procedural and design data as well as descriptions of equipment) must be provided in sufficient detail to allow a clear and precise judgement to be made of the importance and originality of the research reported.

It is acceptable for this dissertation to include as chapters authentic copies of papers already published, provided these meet type size, margin, and legibility requirements. In such cases, connecting texts which provide logical bridges between different manuscripts are mandatory. Where the student is not the sole author of a manuscript, the student is required to make an explicit statement in the introductory material to that manuscript describing the student's contribution to the work and acknowledging the contribution of the other author(s). The signatures of the Supervising Committee which precede all other material in the dissertation attest to the accuracy of this statement.

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In recent years with the rapid growth and need for high quality and high capacity cellular networks, estimating coverage accurately has become extremely important. The generic RF propagation prediction algorithms based on computer databases or empirical results give only approximate coverage, and are not suitable for detailed network design. To more accurately design the coverage of modern cellular networks, signal strength measurements must be taken in the service area using a test transmitter. Taking signal strength measurements is an expensive and a time consuming task. It is well known that the accuracy of coverage estimation increases with the distance of the signal strength measurement drive route. Given today's demand for high quality wireless networks, it is more important than ever before to understand how the accuracy of coverage estimates depends on the amount of drive testing.

This dissertation analyzes the dependence of the coverage estimate on the number of measurements, presents a technique to improve the coverage estimation when only a limited number of signal strength measurements are available, and also presents a technique to design networks for any required reliability depending on the distance of signal strength measurement drive route. This dissertation addresses the issue of estimating coverage from a finite number of signal strength measurements. First, the accuracy of the cell radius estimate on the number of

measurements for different propagation characteristics is shown. Secondly, the signal strength measurement drive route distance needed to obtain a desired reliability of coverage estimate is established. Thirdly, it is shown that cell radius is not very sensitive to the propagation slope and that in cases with limited signal strength measurements better results are obtained using a fixed propagation slope. Finally a new margin called the “effective fade margin” is proposed which, depending on the desired reliability, compensates for the log normal shadowing and the insufficiency of the signal strength measurement drive tests.

Chapter 1
Introduction

This chapter begins with a brief discussion on the evolution of mobile communications networks, followed by introduction to cellular concept. Then we discuss the concepts of frequency reuse, handoff and trunking. This is followed by a discussion on interference and system capacity where we calculate the co-channel interference using a geometric model. Then we discuss techniques to increase the system capacity. Finally we discuss the current and future architectures for wireless networks.

1.1 Evolution of Mobile Communications

The objective of early land to mobile radio systems was to achieve a large coverage area by using a single, high powered transmitter with an antenna mounted on a tall tower[1]. This was done by selecting several channels from a specific frequency allocation for use in autonomous geographic zones. The communications coverage area of each zone was usually planned to be as large as possible. The number of channels that could be obtained from the allocated spectrum was limited. There was generally no in system interference as the same frequencies were reused in the next service area which used to be several hundred miles away.

Some of the drawbacks of the early land to mobile systems were limited service capability, high blocking probabilities, inefficient frequency spectrum utilization. If there were two contiguous service areas then call had to be terminated and reinitiated in the next service area. There was no concept of handoff. In addition to this, the regulatory agencies could not make spectrum allocations in proportion to the increasing demand for mobile services, thus making it imperative to restructure the mobile radio system to achieve high capacity with limited radio spectrum, while at the same time covering very large areas.

Large scale integrated circuit technology reduced the size of mobile transceivers to one that could easily fit into the standard automobile. Another factor was the reduction in price of the mobile telephone unit. Technology, feasibility and service affordability caused the transition from early land to mobile systems to the cellular systems.

Mobile communications is currently at its fastest growth period in history, due to enabling technologies which permit wide spread deployment. Historically, growth in the mobile communications field has come slowly, and has been linked to technological advancements. The ability to provide wireless communications to an entire population was first conceived when Bell Laboratories developed the cellular concept in the 1960s and 1970s. The tremendous growth in the mobile communications is primarily due to development of highly reliable, miniature solid state devices and the development of the cellular concept. The future growth of consumer-based mobile and portable communication systems will depend on radio spectrum allocations, regulatory decisions, adoption of common standards, consumer needs and technology advances in the signal processing, access, and integration of voice and data networks.

Examples of Mobile Radio Systems

Some common examples of other mobile radio communication systems are garage openers, remote controllers for home entertainment equipment, cordless telephones, hand-held walkie-talkies, pagers and wireless LANs. We will discuss briefly the paging systems and the cordless telephone system below:

Paging systems are communication systems that send brief messages to a subscriber. The message may be either a numeric message, an alphanumeric message, or a voice message depending on the service available. In modern paging systems, news headlines, stock quotations, faxes and emails may be sent. A message is sent to a paging subscriber via the paging system access number or through the Internet. The paging system then transmits the page throughout the service area using base stations which broadcast the page on a radio carrier. The paging systems usually have large transmitter powers and use low data rates to achieve maximum coverage from each base station with minimum signal to noise requirement. Recent paging systems allow two way paging in which the subscribers can send a pre-defined set of messages.

Cordless telephone systems are full duplex communication systems that use a radio to connect a portable handset to a dedicated base station, which is then connected to a dedicated telephone line with a specific telephone number on the public switched telephone network (PSTN). The portable unit communicates only to the dedicated base unit and only over distances of a few tens of meters. Cordless telephone systems provide the user with limited range and mobility.

1.2 The Cellular Concept

The cellular concept[2] was a major breakthrough in solving the problem of spectral congestion and user capacity. It offered high capacity with a limited spectrum allocation without any major technological changes. The cellular concept is a system level idea in which a single, high power transmitter (large cell) is replaced with many low power transmitters (small cells). The area serviced by a transmitter is called a cell. Each small powered transmitter, also called a base station provides coverage to only a small portion of the service area. Base stations close to one another are assigned different groups of channels so that all the available channels are assigned to a relatively small number of neighboring base stations. Neighboring base stations are assigned different groups of channels so that the interference between base stations is minimized. By symmetrically spacing base stations and their channel groups throughout a service area, the available channels are distributed throughout the geographic region and may be reused as many times as necessary, so long as the interference between co-channel stations is kept below acceptable levels.

As the demand for service increases, the number of base stations may be increased, thereby providing additional capacity with no increase in radio spectrum. This fundamental principle is the foundation for modern mobile communication systems, since it enables a fixed number of channels to serve an arbitrarily large number of subscribers by reusing the channels throughout the region. The concepts of frequency reuse, handoff and system capacity are explained below.

1. 3 Frequency Reuse

The base station antennas are designed to achieve the desired coverage within the particular cell. By limiting the coverage area to within the boundaries of a cell, the same group of channels may be used to cover different cells that are separated from one another by distances large enough to keep interference levels within tolerable limits. The design process of selecting and allocating channel groups for all the cellular base stations within a system is called frequency reuse or frequency planning.

In Fig. 1.1 the cells labeled with the same letter use the same group of channels. The frequency reuse plan is overlaid upon a map to indicate where different frequency channels are used. The hexagonal cell shape shown is conceptual and is a simplistic model of the coverage for each base station. The hexagon has been universally adopted since the hexagon permits easy and manageable analysis of a cellular system. Also considering geometric shapes which cover an entire region without overlap and with equal area, hexagon has the largest area considering the distance between the center of a polygon and its farthest perimeter points. The actual footprint is determined by the contour in which a given transmitter serves the mobiles successfully.

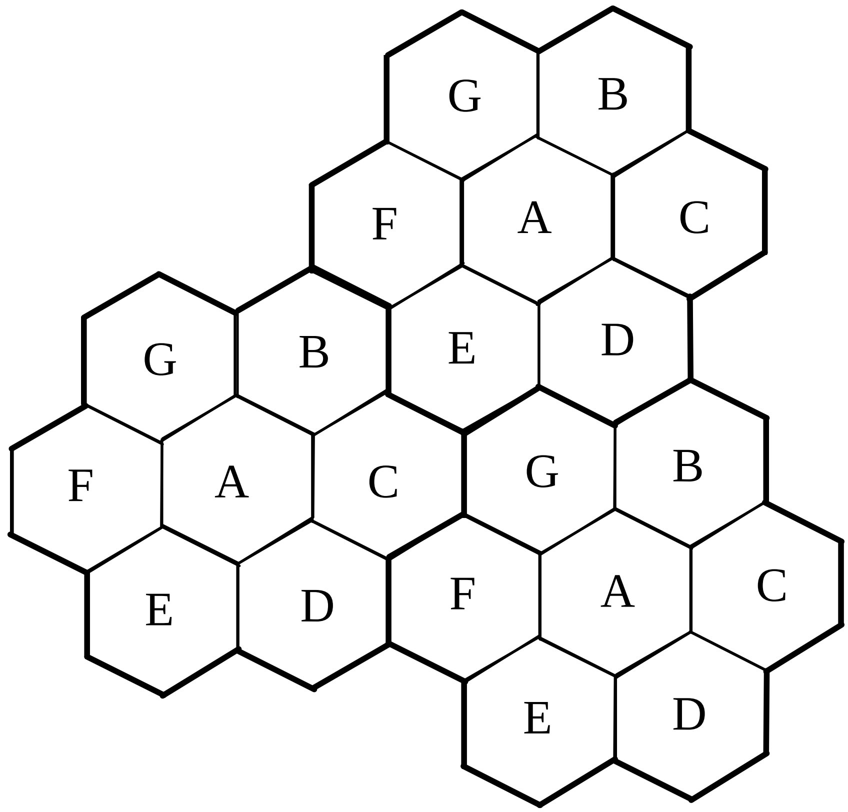

Figure 1.1 Diagram of cellular frequency reuse. Cell with the same letter use the same set of frequencies. The reuse shown here is for a cluster size of N = 7.

1.4 Concept of Handoff

When a mobile moves into a different cell while a call is in progress, the mobile switching center (MSC) automatically transfers the call to a new channel belonging to the new base station. This handoff operation involves identifying a new base station, assigning a free channel in the new cell to the mobile to change the frequency and transfer the voice circuit to the new base station. Processing handoffs is an important task in any cellular radio system. The handoff process can be performed based on several criteria such as signal strength, bit error rate in digital systems or interference levels. For example, if the signal level is used to trigger the handoff, an optimum signal level at which to initiate handoff is specified which approximately corresponds to the boundary of the cell. Once particular signal level goes below the specified threshold the base station queries the received power from the mobile at the different neighboring base stations, and picks a base station which has a power higher than that seen in the serving base station by a specified margin. Also in deciding when to handoff, it is important to ensure that the drop in the measured signal level is not due to momentary fading and that the mobile is actually moving away from the serving base station. In order to ensure this, the base station monitors the signal level for a certain period of time before a hand-off is initiated. This running average measurement of signal strength should be optimized so that unnecessary handoffs are avoided, while ensuring that necessary handoffs are completed before a call is terminated due to poor signal level.

1.5 Concept of Trunking

Cellular systems depend on trunking to accommodate a large number of subscribers in a limited number of channels. The concept of trunking allows a large number of users to share a relatively small number of channels by providing access to each user, on demand, from a pool of available channels. In a trunked system, each user is assigned a channel on a per call basis, and upon termination of the call, the previously occupied channel is immediately returned to the pool of available channels. Trunking exploits the statistical behavior of users so that a fixed number of channels or circuits may accommodate a large number of users. The grade of service (GOS) is a measure of the ability of a user to access a trunked system during the busiest hour of call traffic. It is clear that there is a trade-off between the number of available channels and the likelihood of a particular user finding that no channels are available during the peak calling time. The number of channels required is determined based the number of subscribers, desired GOS, average call holding time and traffic distribution with time.

1.6 Interference and System Capacity

Interference is the limiting factor in the performance of cellular systems[4,5]. Sources of interference include another mobile in the same cell, a call in progress in a neighboring cell, other base stations operating in the same frequency band, or any non cellular system which inadvertently leaks energy into the cellular frequency band. Interference on voice channels causes background noise and results in poor voice quality. On control channels, interference leads to missed and blocked calls due to errors in the digital signaling. Interference is the major bottleneck in increasing capacity of a cellular network. The major types of system generated cellular interference are co-channel interference and adjacent channel interference. Here we will consider in-band interference only i.e., the interference caused by other cellular users.

Co-Channel Interference and System Capacity

Cells in a given coverage area that use the same set of frequencies are called co-channel cells and the interference between signals from these cells is so called co-channel interference. The co-channel interference cannot be reduced by increasing the carrier power of a transmitter, since an increase in carrier transmit power also increases the interference power to neighboring co-channel cells, thus keeping the carrier to interference ratio constant. If we assume base stations with omni antenna systems with no power control, the only way to reduce the co-channel interference is to increase the physical distance between the co-channel cells to provide sufficient isolation due to propagation loss.

In a cellular system, where the size of each cell is the same, co-channel interference is independent of the transmitted power. The co-channel interference is only a function of the ratio of radius of the cell R, and the distance to the center of the nearest co-channel cell D. Increasing the ratio of D/R, the spatial separation between co-channel cells relative to the coverage distance of a cell increases the carrier to the interference ratio. The parameter Q, called the co-channel reuse ratio is related to the cluster size N for hexagonal geometry by

$$Q = \frac{D}{R} = \sqrt{3N}$$

(1.1)

A small value of Q provides larger capacity since the cluster size is small but increases the interference, whereas a large value of Q provides lesser capacity but reduces the co-channel interference.

Cluster Size	Co-channel Reuse ratio (Q)
7	4.58
12	6
13	6.24

Table 1.1 Co-channel Reuse Ratio for some values of N

Let i_o be the number of co-channel interfering cells. Then, the carrier to interference ratio *C/I* for a mobile receiver which monitors a forward channel can be expressed as

$$\frac{C}{I} = \frac{C}{\sum_{i=1}^{i_o} I_i}$$

(1.2)

where C is the desired carrier power from the desired base station and I_i is the interference power caused by the i th interfering co-channel cell base station. If the signal levels of co-channel cells are known, then the *C/I* ratio for the forward link can be found using (1.2).

Propagation measurements in a mobile radio channel indicate that the average received signal strength at any point decays as a power law of the distance of separation between a transmitter and receiver. The average power P_r at a distance d from the transmitting antenna is approximated by

$$P_r = P_o \left(\frac{d}{d_o} \right)^{-n}$$

(1.3)

where P_o is the power received at a close in reference point in the far field region of the antenna at a small distance d_o from the transmitting antenna, and n is the path loss exponent. Consider the forward link where the mobile is at a distance R from the serving base station and where the interference is due to co-channel base stations. If D_i is the distance of the i th interferer from the mobile, the received power at a mobile due to the i th interfering cell will be proportional to $(D_i)^{-n}$. The path loss exponent typically ranges between 2 and 4 in frequency ranges being used for cellular systems.

When the transmit power of each base station is equal and the path loss exponent is the same throughout the coverage area, C/I for a mobile can be approximated as

$$\frac{C}{I} = \frac{R^{-n}}{\sum_{i=1}^{i_o} (D_i)^{-n}}$$

(1.4)

Considering only the first tier of interfering cells, if all the interfering base stations are equidistant from the desired base station and if this distance is equal to the distance D between cell centers, then equation (1.4) simplifies to

$$\frac{C}{I} = \frac{(D/R)^n}{i_o} = \frac{(\sqrt{3N})^n}{i_o}$$

(1.5)

Equation (1.5) gives the relation between the C/I to the cluster size N, which in turn determines the overall capacity of the system. For AMPS systems, typically a C/I of 17 dB is needed to provide an acceptable voice quality. Assuming a path loss exponent of 4, a minimum cluster size of 7 is required to meet a *C/I* requirement of 18 dB. This result is based on geometry and does not take into account the shadowing characteristics of the propagation environment and is thus optimistic.

In Figure 1.2 it can be seen that for an $N = 7$ cell cluster, with the mobile unit at the cell boundary, the mobile is a distance *D-R* from the two nearest co-channel interfering cells and approximately *D + R/2* , *D*, *D-R/2*, and *D+R* from the other interfering cells in the first tier.

Using equation (1.5) and assuming *n* equals 4, the signal to interference ratio for the worst case can be closely approximated as

$$\frac{C}{I} = \frac{R^{-4}}{2(D-R)^{-4} + (D-R/2)^{-4} + (D+R/2)^{-4} + (D+R)^{-4} + D^{-4}} \tag{1.6}$$

This expression can be used to calculate the carrier to interference ratio geometrically for different cluster sizes.

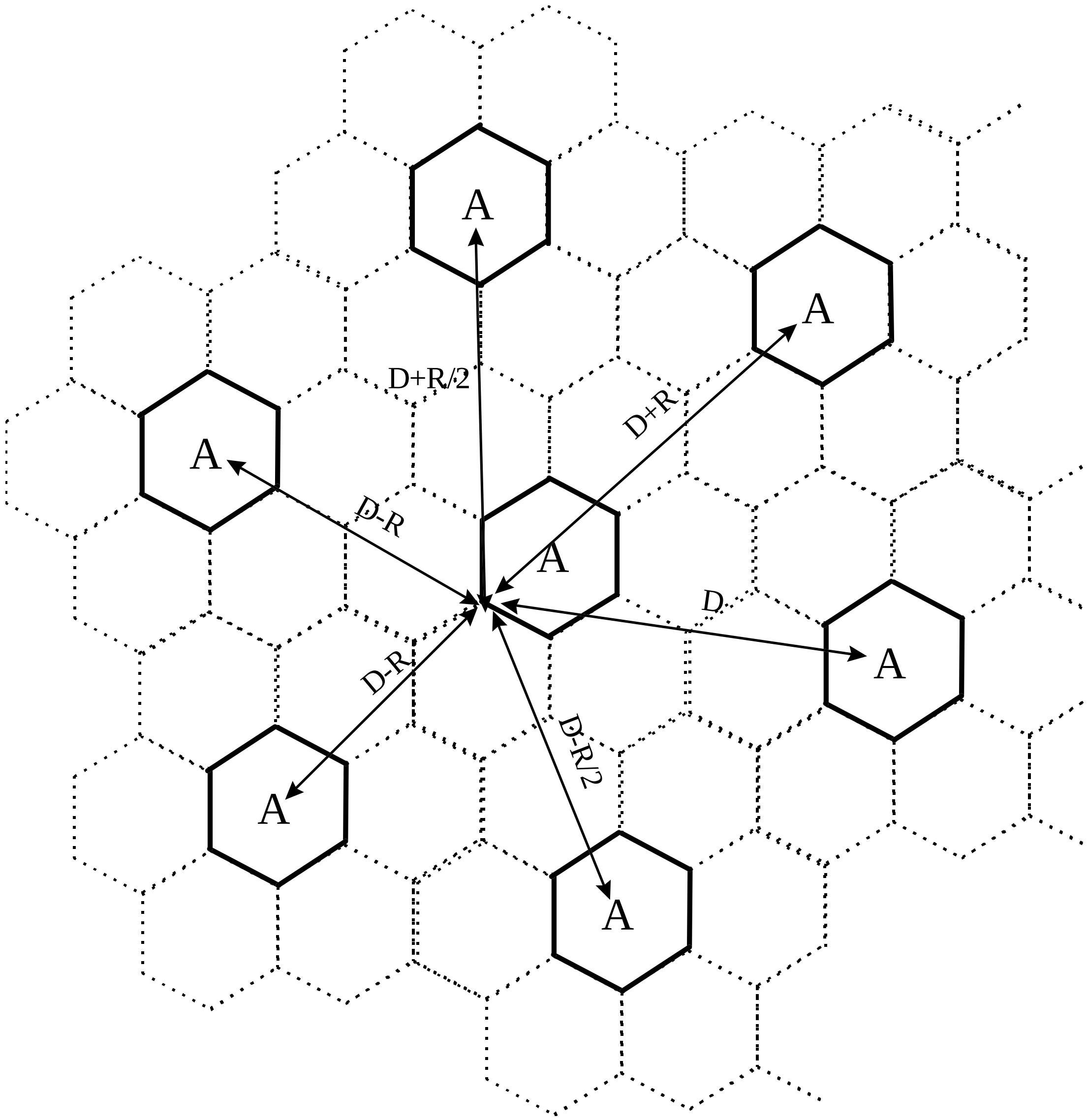

Figure 1.2 The diagram shows the first tier of cells for a cluster size of N=7. Also shown are the distances from a mobile at the edge of a cell to the base stations in co-channel cells.

1.7 Increasing the System Capacity

Different techniques such as cell splitting, sectoring, cell tiering, power control and smart antennas can be used to increase the system capacity. We will briefly discuss some of these techniques.

Cell splitting

Cell splitting is the process of subdividing a congested cell into smaller cells, each with its own base station and a corresponding reduction in antenna height and transmitter power. Cell splitting increases the capacity of a cellular system since it increases the number of times that channels are reused. By making these the new cells to have smaller radius than the original cells and by installing these smaller cells between the existing cells, capacity increases due to the additional number of channels per unit area. Cell splitting achieves capacity improvement by essentially re-scaling the system. By decreasing the radius *R* and keeping the co-channel reuse ratio *D/R* unchanged, cell splitting increases the number of channels per unit area.

Sectoring

Sectoring is another way to increase capacity. In this approach, capacity improvement is achieved by reducing the number of cells in a cluster and thus increasing the frequency reuse. The co-channel interference in a cellular system may be decreased by replacing a single omni directional antenna at the base station by several directional antennas, each radiating within a specified sector. By using directional antennas, a given cell will receive interference and transmit with only a fraction of the available co-channel cells. The technique for decreasing co-channel interference and thus increasing system capacity by using directional antennas is called sectoring.

A cell is usually sectored into three sectored or six sectored configurations. When sectoring is employed, the channels used in a particular cell are broken down into sectored groups and are used only within a particular sector, as shown in the figure. Assuming a 7 cell reuse of the case of 120 degree sectors, the number of interferers in the first tier is reduced from 6 to 2. This is because only 2 of the 6 co-channel cells receive interference with a particular sectored channel group. The resulting *C/I* for this case can be found using equation (1.4) to be 24.2 dB, which is a significant improvement over the omni-directional case, where the worst case *C/I* was 17 dB.

Cell Tiering

Dividing the cell into two tiers and reusing frequencies intelligently such that the frequencies used in the outer tier in the serving cell are used in the inner tier in the co-channel cells increases the C/I ratio. The channel set allocated to each site is divided into two groups. The maximum transmit power for the inner tier channels is set so that signal level hits the threshold at the tier boundary. Handoff process takes place at the tier boundary.

The reduction in interference offered by sectoring and tiering enable planners to reduce the cluster size *N*, and provides an additional degree of freedom in assigning channels. The penalty for improved *C/I* is an increased number of antennas, a decrease in the trunking efficiency due to channel sectoring at the base station and an increase in processing due to handoff activity between sectors and tiers.

Power Control

Controlling the transmit power of the mobile and base station reduces the system interference and thus can be used to reduce the cluster size if implemented properly. In practical cellular radio and personal communication systems the power levels transmitted by every subscriber unit are under constant control by the serving base stations. This is done to ensure that each mobile transmits the smallest power necessary to maintain a good quality link on the reverse channel. Power control not only helps prolong battery life for the subscriber unit, but also dramatically reduces the reverse channel *C/I* in the system.

Further improvements in *C/I* is achieved by down-tilting the antenna such that the radiation pattern is the vertical plane has a notch pointing at the nearest co-channel cell. Also other techniques such as use of smart antennas, and co-channel interference cancellation algorithms can be used to improve the *C/I*. In practical high capacity systems all the above mentioned techniques are used together in dense urban environments to achieve high system capacity.

1.8 Architecture and development of Wireless Networks

The early version of analog cellular networks are called as first generation networks. The current digital cellular networks are called second generation networks and the future cellular networks under development are called third generation networks.

First Generation Wireless Networks

All first generation cellular networks are based on analog technology and use FM modulation. An example of the first generation cellular telephone system is Advanced Mobile Phone Services (AMPS)[3].

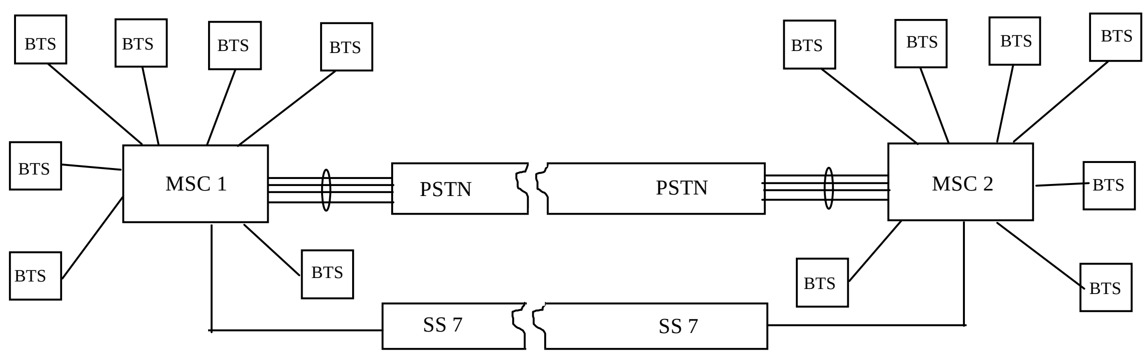

Figure 1.3 Block diagram of the first generation cellular network.

The block diagram of a first generation cellular radio network architecture is shown in figure 1.3, which includes the mobile terminals, the base station and the mobile switching center (MSC). In The control for entire system resides in the MSC, which maintains all mobile related information and controls each mobile hand-off. The MSC also performs all, of the network management functions, such as call handling and processing, billing, and fraud detection within the market. The MSC is interconnected with the public switched telephone network (PSTN) via land-line trunked lines (trunks) and a tandem switch. MSC’s also are connected with other

MSCs via dedicated signaling channels for exchange of location, validation, and call signaling information.

PSTN is a separate network from the SS7 signaling network. In modern cellular telephone systems, long distance voice traffic is carried on the PSTN, but the signaling information used to provide call set-up and to information used to provide call set-up and inform MSCs about a particular user is carried on the SS7 network.

Network protocol standard like IS-41 allows different cellular systems to automatically accommodate subscribers who roam into their coverage region. IS-41 allows MSCs of different service providers to pass information about their subscribers to other MSCs on demand. IS-41 relies on autonomous registration. The mobile accomplishes autonomous registration by periodically keying up and transmitting its identity information, which allows the MSC to constantly update its subscriber list. The MSC can distinguish home users from roaming users based on the mobile identification number (MIN) of each active user, and maintains a real time user list of home location register (HLR) and visitor location register (VLR). IS-41 allows the MSCs of neighboring systems to automatically handle the registration and location validation of roamers so that users no longer need to manually register as they travel. The visited system creates VLR record for each new roamer and notifies the home system via IS-41 so it can update its own HLR.

Second Generation Wireless Networks

Second generation wireless systems employ digital modulation and advanced call processing capabilities. Examples of second generation wireless systems include the Global System for Mobile (GSM), the IS-54 TDMA and the IS-95 CDMA TIA digital standards.

Second generation wireless networks have introduced new network architectures that have reduced the computational burden of the MSC. The GSM has introduced the concept of a base station controller (BSC) which is inserted between the several base stations and the MSC. This architectural change has allowed the data interface between the base station controller and the MSC to be standardized, thereby allowing carriers to use different manufacturers for MSC and BSC components.

All second generation systems use digital voice coding and digital modulation. Second generation systems also provide dedicated voice and signaling trunks between MSCs, and between each MSC and PSTN. The second generation systems in addition to voice also provide paging, and other data services. The network controlling structure is more distributed in the second generation wireless systems, since mobile stations assume greater control functions. In second generation wireless networks, the handoff process can use mobile-assisted handoff (MAHO) in which the mobile sends the reading of receive signal strength or bit error rate based on the forward link. The mobile units in these networks perform several other functions not performed by first generation subscriber units, such as received power reporting, adjacent base station scanning, data encoding, and encryption.

Third Generation Wireless Networks

Third generation wireless systems will evolve from mature second generation systems. The aim of third generation wireless networks is to provide a single system that can meet a wide range of applications and provide universal access. The third generation networks will carry many types of information such as voice , data and video and serve both stationary and fixed users. Some of the systems proposed for the third generation systems are CDMA2000 which is backward compatible to systems based on IS 95 and WCDMA which is backward compatible to GSM systems.

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Chapter 2

Coverage Estimation and Verification

In this chapter we start with the discussion on the importance of coverage estimation in modern cellular network and the reason why the empirical and computer based prediction algorithms are not sufficient for RF coverage estimation in modern cellular networks and discuss the importance of obtaining signal strength measurements in the service area. This is followed by traditional concept of cell edge reliability, cell area reliability and shadow fade margin. Then, the relationship between the cell edge reliability, cell area reliability and cell radius is shown. We then discuss the available techniques for coverage verification and the importance of using the same measure for coverage estimation and coverage verification. Finally the importance of effective cell radius of a cell a coverage metric is discussed.

2.1 Evolution in Coverage Estimation Techniques

The primary objective of the first generation land to mobile systems was to provide coverage to the required area using a single tall tower. All the traffic was supported with a single base station. The land to mobile systems were available only in the large cities and the typical radius of coverage was about 40 miles. Same channels were reused in other cities since the co-channel interference was low as the cities were geographically apart. Since the land to mobile systems were coverage limited, stronger signal was better. For the land to mobile systems, the coverage estimation was mostly based on empirical propagation models[1]. The empirical models are developed for different morphologies for some typical cities, and are limited to the ability of the engineer to classify the morphology and only provide an approximate estimate of coverage. Since only one base station was used, the prediction was used to determine the approximate boundary where the subscribers could use the service.

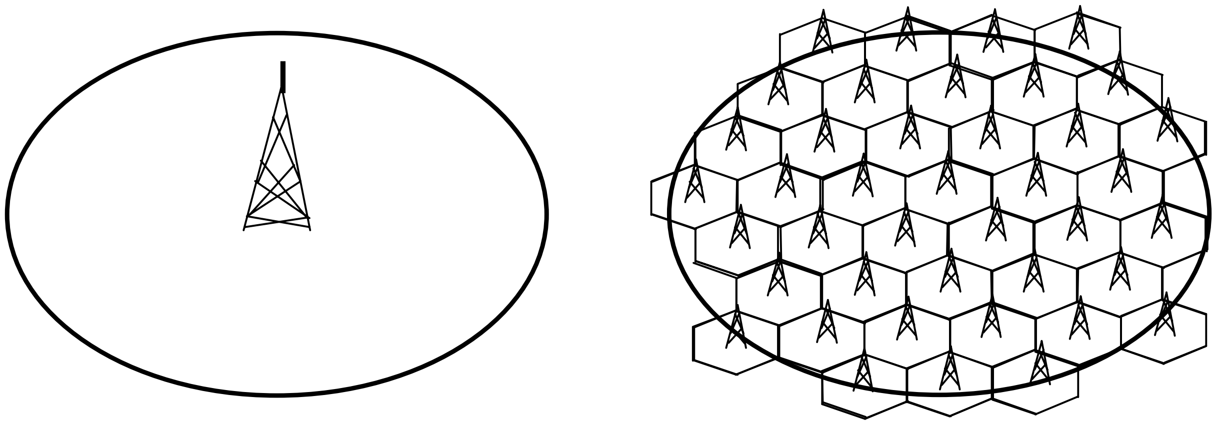

Figure 2.1 Illustration showing the importance of accurate coverage estimation in cellular networks as compared to early land to mobile systems.

The situation in modern cellular system is different. The modern cellular systems are being built to provide high quality of service and high capacity. The typical cell radius in urban areas is less than one kilometer. To minimize the interference (co-channel and adjacent channel interference) it is important to determine the coverage boundary accurately.

Estimating inaccurate coverage has severe impact on the network performance. Over estimating coverage results in areas with signal strengths weaker than the minimum required threshold. Under estimating coverage will create coverage overlap, which can result in interference. Thus, accurate estimation of coverage is essential for a good design.

As explained earlier, empirical formulas do not estimate the coverage to the accuracy needed for designing a modern cellular network. Several computer based prediction tools[2-4] model the physical phenomenon of RF propagation using terrain and clutter (land use) data. The accuracy of coverage estimation using these tools depends on the accuracy and resolution of the available data. Even when accurate and high resolution clutter data is available, the effect of the clutter on the propagation is different in different areas. For example, if the clutter data classifies an area as dense urban, and provides average building height, there is still an ambiguity about the density of the buildings in the area and also the propagation depends on the materials used to construct the buildings. Thus the propagation characteristics in an area classified as dense urban in one country can be very different from that in another country. Also the high resolution clutter data with heights, is extremely expensive to obtain as they have to be obtained by airal photography. Even though computer prediction tools may give better coverage estimates than empirical formulas, they alone are not good enough to be used to design a modern cellular network. We will refer to the methods of coverage estimation without using any signal strength measurements from the service area as untuned predictions.

From the discussion it is clear that untuned predictions will not provide accurate estimate of coverage for designing modern cellular networks. The common practice now in designing modern cellular networks is to measure the signal strength for a test transmitter in the service area and to tune the propagation model using the measured data. Using this technique, coverage can be more accurately and reliably estimated.

2.2 Concept of Log-Normal Shadowing, Shadowing Fade Margin and Cell Radius

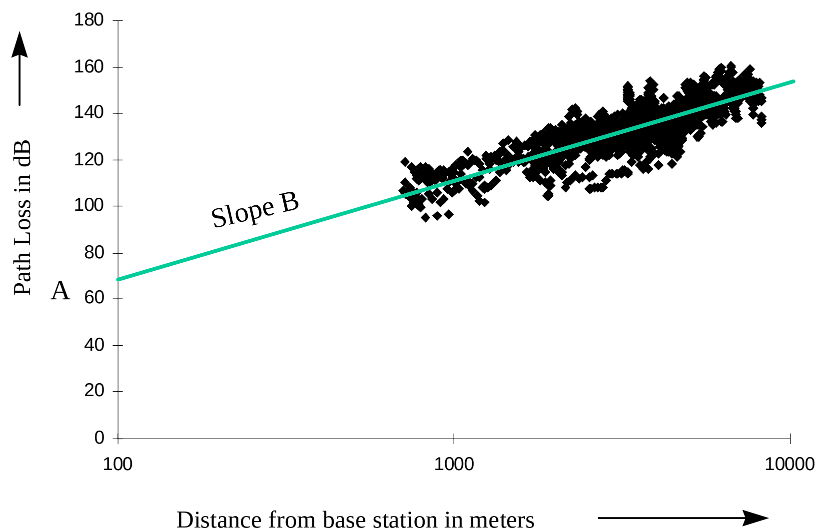

Figure 2.2. Plot of path loss in dB versus the distance of the mobile from base station in meters.

Propagation models based on measurements indicate that the average received signal power decreases logarithmically with distance for outdoor radio channels[5]. This is clearly seen in figure 2.2 which shows the path loss in dB versus distance plotted on a logarithmic scale for measurements at 850 MHz. The average large-scale path loss for an arbitrary transmit - receive (T-R) separation can be expressed as a function of distance.

$$\overline{PL(d)} = A + B\log\left(\frac{d}{do}\right) \tag{2.1}$$

$\overline{PL(d)}$ denotes the average path loss at a distance *d* from the transmitter and d_o is the close-in reference distance which is determined from measurements close to the transmitter. When plotted on a log-log scale, the modeled path loss is a straight line with an intercept *A* with a slope equal to *B* dB per decade. The values of A and B depend on several factors including the propagation environment, frequency of operation and transmitter height.

In macro cellular systems where the cell coverage is large, reference distances of 1 Km are commonly used. The reference distance should be in the far field of the antenna so that near-field effects do not alter the reference path loss.

The model in equation (2.1) gives only the average path loss and does not consider the fact that the surrounding environmental clutter may be different at different locations at the same distance from the transmitter. Measurements have indicated that at any value of *d*, the path loss *PL(d)* at a particular location is random and log-normally (normal in dB) distributed about the mean distance dependent value. That is

$$PL(d) = \overline{PL(d)} + X = A + B\log\left(\frac{d}{do}\right) + X \tag{2.2}$$

where *X* is a zero mean Gaussian distributed random variable with standard deviation σ (dB). The probability density function of *X* is given by,

$$P(X) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(\frac{-x^2}{2\sigma^2}\right).$$

(2.3)

This phenomenon is referred to as log normal shadowing. The value of *A*, *B* and σ can computed from measured data using linear regression such that the difference between the measured and estimated path losses is minimized in a mean square error sense over all the measurements. Also the close in reference is usually taken as 1 km.

The average received power $\overline{P_R(d)}$ (in dBm) at any distance d from the transmitter with a transmit power P_T (in dBm) is given by

$$\overline{P_R(d)} = P_T - \overline{PL(d)}.$$

(2.4)

The received power $P_R(d)$ (in dBm) at any distance *d* from the transmitter with transmit power P_T (in dBm) is given by

$$P_R(d) = P_T - PL(d).$$

(2.5)

Thus, the effective distribution of the received signal strength can be described by a normal distribution with mean of $\overline{P_R(d)}$ and variance σ^2.

$$N\left(\overline{P_R(d)}, \sigma^2\right)$$

(2.6)

Since *PL(d)* is a random variable with a normal distribution in dB about the distance dependent mean, so is $P_R(d)$, and the *Q* function or error function (*erf*) may be used to determine the probability that the received signal level will exceed (or fall below) a particular level. The *Q* function is defined as

$$Q(z) = \frac{1}{\sqrt{2\pi}} \int_{z}^{\infty} e^{-\frac{x^2}{2}} dx = \frac{1}{2}\left[1 - erf\left(\frac{z}{\sqrt{2}}\right)\right]$$

(2.7)

where

$$Q(z) = 1 - Q(-z)$$

(2.8)

The probability that the received signal level will exceed a certain value P_{MIN} can be calculated from the cumulative density function as

$$\Pr\left[P_R(d) > P_{MIN}\right] = Q\left(\frac{P_{MIN} - \overline{P_r(d)}}{\sigma}\right)$$

(2.9)

Similarly, the probability that the received signal level will be below P_{MIN} is given by

$$\Pr\left[P_R(d) < P_{MIN}\right] = Q\left(\frac{\overline{P_r(d)} - P_{MIN}}{\sigma}\right)$$

(2.10)

Using a fade margin for the lognormal shadowing, systems can be designed for any coverage reliability. The margin needed to design the system for a desired reliability can be easily calculated. Cellular networks are designed for certain required service reliability. A shadow fade margin FM_σ, that ensures the desired cell edge reliability, $F(z)$, can be approximated as

$$FM_\sigma = z \cdot \sigma$$

(2.11)

where $F(z) = 1 - Q(z)$

(2.12)

To design a system for a cell edge reliability of $F(z)$, the mean received signal strength $\overline{P_R(R)}$ at the cell edge at a distance R from the base station has to be

$$\overline{P_R(R)} = P_{MIN} + FM_\sigma$$

(2.13)

Thus from (2.1), (2.4) and (2.13) the cell radius R using a 1 km intercept for a reliability of $F(z)$ is given by

$$R = 10^{\frac{(P_T - P_{MIN} - FM_\sigma - A)}{B}}$$

(2.14)

2.3 Determination of Area Reliability.

The relationship between the reliability of coverage over a circular area and the reliability of coverage on the perimeter of the circle was first established by D. O. Reudink[6]. The main finding of this study was that cell area reliability and cell edge reliability obey the simple relationship. Due to random effects of shadowing, some locations within a coverage area will be below a particular desired received signal threshold. It is useful to compute how the boundary coverage relates to the percent of area covered within the boundary. For a circular coverage area having radius R from a base station, let there be some desired received signal threshold P_{MIN}. We are interested in computing F_u, the fraction of useful service area (i.e. the percentage of area with a received signal that is equal to or greater than P_{MIN}) given a known likelihood of coverage at the cell boundary. Letting $d = r$ represent the radial distance from the transmitter, it can be shown that if $\Pr[P_R(r) > P_{MIN}]$ is the probability that the random received signal at $d = r$ exceeds the threshold P_{MIN} within an incremental area dA, then F_u can be found by [6]

$$F_u = \frac{1}{\pi R^2} \int \Pr\left[P_R(r) > P_{MIN}\right] dA = \frac{1}{\pi R^2} \int_0^{2\pi} \int_0^R \Pr\left[P_R(r) > P_{MIN}\right] r\,dr\,d\theta$$

(2.15)

Using (2.9), $\Pr[P_R(r) > P_{MIN}]$ is given by

$$\Pr\left[P_R(r) > P_{MIN}\right] = Q\left[\frac{P_{MIN} - \overline{P_R(r)}}{\sigma}\right] = \frac{1}{2} - \frac{1}{2} erf\left[\frac{P_{MIN} - \overline{P_R(r)}}{\sigma\sqrt{2}}\right]$$

$$= \frac{1}{2} - \frac{1}{2} erf\left(\frac{P_{MIN} - [P_t - (\overline{PL}(d_o) + 10n \log(r/d_o))}{\sigma\sqrt{2}} \right)$$

(2.16)

In order to determine the path loss as referenced to the cell boundary ($r = R$), it is clear that

$$\overline{PL(r)} = B \log\left(\frac{R}{d_o}\right) + B \log\left(\frac{r}{R}\right) + A$$

(2.17)

and equation (2.16) may be expressed as

$$\Pr\left[P_R(r) > P_{MIN}\right] = \frac{1}{2} - \frac{1}{2} erf\left(\frac{P_{MIN} - [P_t - (A + B\log(R/d_o) + B\log(r/R))}{\sigma\sqrt{2}} \right)$$

(2.18)

If we let $a = (P_{MIN} - P_t + A + B\log(R/d_o))/\sigma\sqrt{2}$ and $b = (B \log e)/\sigma\sqrt{2}$, then,

$$F_u = \frac{1}{2} - \frac{1}{R^2} \int_0^R r \cdot erf\left[a + b \ln \frac{r}{R}\right] dr$$

(2.19)

By substituting $t = a + b\log(r/R)$ in equation (2.19), it can be shown that

$$F_u = \frac{1}{2}\left(1 - erf(a) + \exp\left(\frac{1-2ab}{b^2}\right)\left[1 - erf\left(\frac{1-ab}{b}\right)\right]\right)$$

(2.20)

By choosing the signal level such that $\overline{P_r(R)} = P_{MIN}$ (i.e. a = 0), F_u can be shown to be

$$F_u = \frac{1}{2}\left[1 + \exp\left(\frac{1}{b^2}\right)\left(1 - erf\left(\frac{1}{b}\right)\right)\right]$$

(2.21)

Equation (2.20) may be evaluated for a large number of values of σ and B, as shown in figure 2.3. Thus given the exact knowledge of σ and B, the cell area reliability can be exactly computed for any cell edge reliability. For example if B = 40 and σ = 8 dB, and if the boundary is to have 75% boundary coverage (75% of the time the signal is to exceed the threshold at the boundary), then the area coverage is equal to 94%.

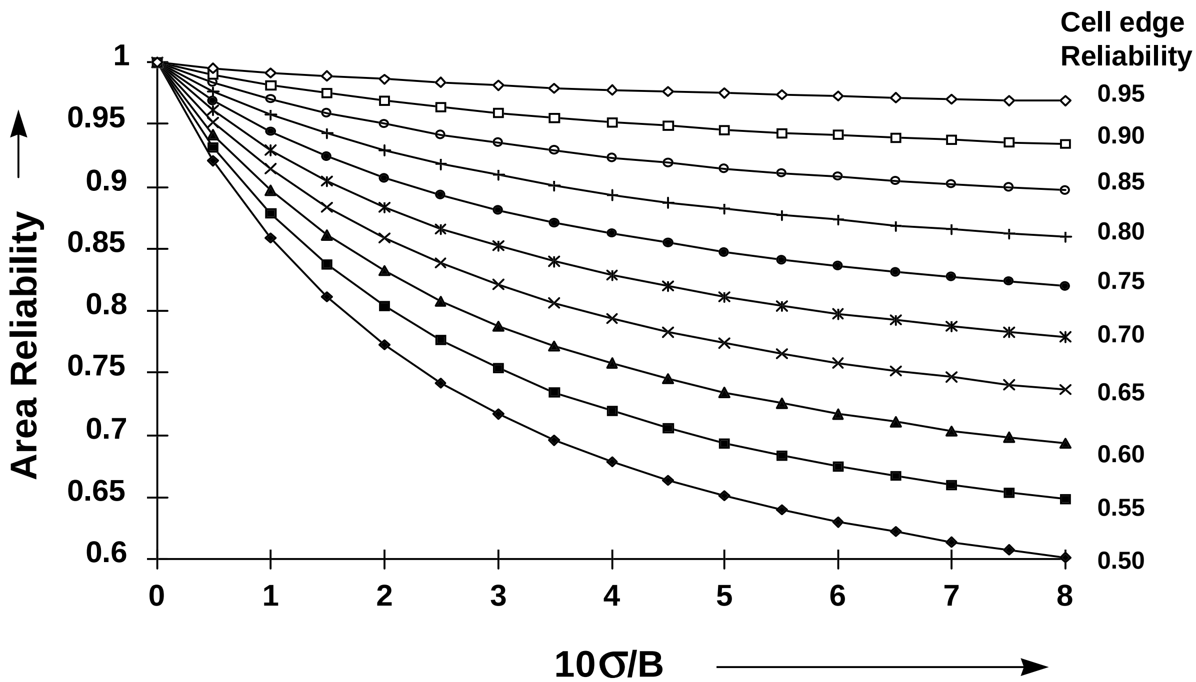

Figure 2.3. Family of curves relating fraction of total area with signal above threshold, as a function of probability of signal above threshold on the cell boundary.

As was seen from (2.14) the cell radius depends on the desired cell edge reliability for a fixed transmit power and propagation environment. Increasing the cell edge reliability decreases the cell radius. From the relation (2.20), increasing the cell area reliability reduces the cell radius. The relation between the cell radius and area reliability for a fixed transmit power[] is given by (2.22)

$$F_u = Q(c + d \ln R) + \frac{e^{\frac{-2c}{d} + \frac{2}{d^2}}}{R^2}\left[1 - Q\left(c + d \ln R - \frac{2}{d}\right)\right] \tag{2.22}$$

where $c = \dfrac{P_{MIN} - P_T - A}{\sigma}$ and $d = \dfrac{B \log_{10} e}{\sigma}$

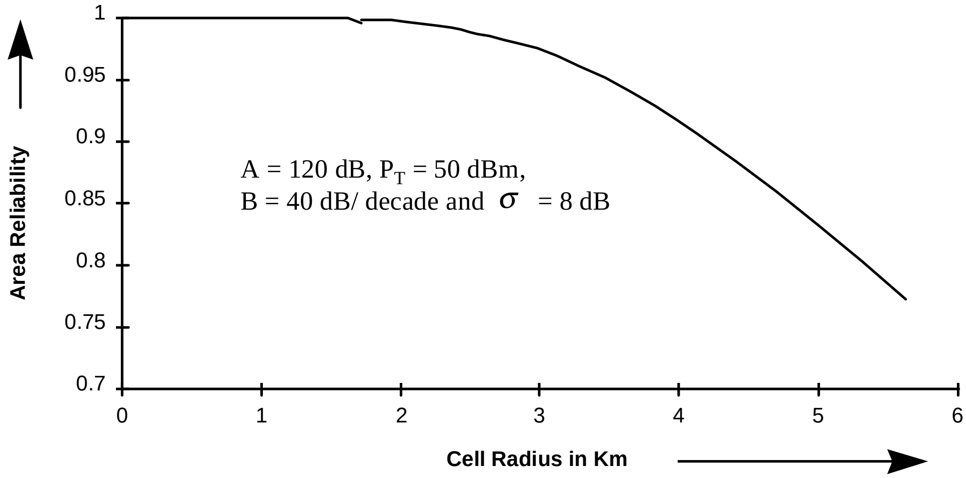

Figure 2.4. This graph shows the relationship between the cell area reliability and cell radius for A = 120 dB, P_T = 50 dBm, B = 40 dB/decade and σ = 8 dB.

Figure 2.4 shows the relation between the area reliability and cell radius assuming fixed transmit power. Since the maximum transmit power of the base station and the mobile is fixed, the base station count increases exponentially with increase in coverage reliability. The trade off between the desired reliability and the cost needs to be carefully evaluated by the cellular operators.

2.4 Macro cellular coverage verification

The results of computer based prediction tools, empirical models and even tuned predictions with drive test data cannot determine the exact actual coverage. This is due to the fact that the propagation characteristics in drive test area used for measurements in the pre-build stage may be different from that in other areas, the tower heights , antennas patterns and locations of the sites may also be different from that of the pre-build drive test. In cases where no pre-build drive tests are done, the accuracy of the coverage estimation is entirely dependent on the accuracy of the databases. To exactly determine the actual service area and reliability, a post build coverage verification needs to be done. This is also done if the service provider would like to ascertain that they have been delivered the network that they had agreed to buy. Also the process of design and verification should be the same. If a cellular system is designed using an empirical model in the pre-build stage, then the same model must be used at the verification stage. Different measures for the design and verification cannot be used to validate a design.

There are different approaches to coverage verification. One of the earliest approaches was proposed by David Talley[7] in 1965. In this approach, he recommends that the initial coverage prediction be done using the Boese report. (37 dbu) carey contours. The paper recommended that the 37 dbu field strength contour distance be derived from the appropriate curve (extracted from Boese Report) for the transmitting antenna height above average terrain within a distance of 2 to 10 miles from the base station. Also the height of the center of the antenna above the average ground elevation for each radial direction has to be resolved by subtracting the average terrain elevation from the height of the center of the antenna. The point where the selected curve crosses the 37 dbu line shows the distance in miles corresponding to a field strength of 37 dbu received from a base station of 250 watts effective radiated power (ERP).

The paper recommended that for coverage verification, measurements be taken by driving 8 radial routes away from the base station. Then the measurements should be compared to the 37 dbu field strength contour limits that were formerly prepared. The paper states that there should be substantive agreement between the Boese computed primary coverage area and the comparative results of the field radio survey tests.

Area reliability and Estimates of Proportion[8,9]

A method for coverage verification based on estimates of proportion is given in the Telecommunications Industry Association (TIA) report on Technology independent methodology for the modeling, simulation and empirical verification of wireless communications system performance in noise and interference limited systems operating on frequencies between 30 and 1500 MHz . The technique is briefly discussed below.

The service area is divided by a grid pattern to produce a large number of uniformly sized tiles, or test tiles. In one method, within each test tile a test location is randomly selected. At each of these locations a series of sequential measurements is made. This test location measurement, containing a number of sub-samples, constitutes the test sample for this location. Alternatively, the grid pattern is used to develop a test route that is uniformly distributed throughout the service area with an approximately equal distance traveled in each grid. This test route shall pass once through each test tile while collecting data. Thus a large number of test samples is collected and evenly distributed throughout the service area.

The service area reliability is determined by the percentage of the test locations that meet of exceed the threshold. Thus, service area reliability (%) = $\frac{T_p}{T_t} \cdot 100\%$, where T_p is the total of tests passed and T_i is the total number of tests. The estimate of proportions is used to determine with a high degree of confidence that sufficient test grids have been developed to accurately determine the Area Reliability.

The number of test locations T_l is determined using,

$$T_l = \frac{Z^2 pq}{e^2} \tag{2.23}$$

where Z is the standard deviate unit (corresponding to the confidence level), p is the true service area reliability (decimal), $q = 1-p$, r is the service are reliability criterion (decimal) and e is the sampling error allowance (decimal). This is subject to a limit such that $T \geq 100$. The requirement is that T_l be the larger of the two values. The values for the standard deviate are available in most statistical books.

To ensure that the desired coverage is met, the signal strength must meet or exceed the minimum required threshold. This necessitates a slight “over design” of the system by e % to provide the statistical margins for passing the conformance test as defined. For this test configuration, Z has one tail and e is the decimal percentage of over design. Also a confidence of 99% should be used unless this choice forces the size of the test grids for the desired service area to become too small, i.e., < 100λ.

Area Reliability and Reudink’s Approach

Another approach has been proposed [10] for coverage verification using area reliability with Reudink’s relations. In this approach, linear regression is applied to the signal strength measurements to determine the intercept A, slope B and standard deviation (sigma). Using (2.14), determine the cell radius for required reliability. Use the radius and the propagation parameters to estimate the reliability of coverage F_u, over the cell area (Reudink ’s expression) using (2.20). This technique is similar to the design technique where the cell radius is determined from the signal strength measurements and thus uses the same measure for design and verification.

A comparison of determining area reliability by estimates of proportions with Reudink ‘s approach is discussed in [10]. A brief summary of the comparison is presented below.

The path loss models used in both the approaches are compared. It is known that the outages increase with distance from the base station. The Reudink’s method assumes a linear path loss, whereas the Estimate of Proportions method assumes that there is no deterministic path loss. This is because, it assumes that the outages are completely random. Therefore the authors conclude that the Reudink's path loss model is a more valid assumption. In estimating the cell radius using the Estimates of Proportion approach, the radius is progressively reduced until the desired reliability is reached. The authors point out that this approach does not benefit from any measurements that fall outside the cell.

Also the paper compares the precision of both the techniques. The authors show that the ratio of the error in the coverage estimate by estimates of proportion ΔF_N to the error in the coverage estimate by Reudink's approach ΔF_u is given by

$$\frac{\Delta F_N}{\Delta F_u} \approx \frac{1}{(0.0695\sigma + 0.1754)\sqrt{F_u^3(1 - F_u)}} \tag{2.24}$$

where F_u is the true area reliability and σ is the standard deviation of the lognormal shadowing. This expression is valid provided 6< σ < 10 and F_u > 90% and N > 100, where N is the number of measurements taken. Comparing these approaches for sigma of 8 dB and area reliability of 90% gives $\Delta F_N / \Delta F_u$ = 5.07. Finally the authors conclude that the Reudink's approach requires fewer measurements to achieve the same area reliability accuracy as the Estimates of Proportion technique. Also Reudink's technique requires an order of magnitude fewer measurements to achieve the same accuracy as the Estimate of Proportions method.

Coverage Validation using Cell Radius

Recently it has been shown that cell radius is a better RF validation criterion than area reliability[11]. The cell radius is obtained by applying linear regression to the measured signal strength data as explained earlier in this chapter.

A simulation was used to determine the standard deviations of the cell radius estimate and the area reliability estimate as a function of the number of independent signal strength samples. The inaccuracy, ΔR, of the estimate of the cell radius at a 95% confidence level was determined. The inaccuracy was measured from simulations and ΔR is determined such that

$$P\left(R - \Delta R \le \hat{R} \le R + \Delta R\right) = 95\% \tag{2.25}$$

Similarly, the inaccuracy of the area reliability estimate, $\hat{F}_u$, was estimated from the simulation and determining $\Delta\hat{F}_u$ such that

$$P\left(\hat{F}_u \le F_u + \Delta F_u\right) = 95\% \tag{2.26}$$

The inaccuracies of both of the estimate of cell radius $\hat{R}$ and the estimate of the area reliability $\hat{F}_u$ were determined empirically through the simulation analysis to be

$$\delta_R = \frac{\Delta R}{R} \approx \frac{3.821\hat{\sigma} + 4.619}{N} \tag{2.27}$$

$$\delta_F = \frac{\Delta F_u}{F_u} \approx \frac{(0.1143\hat{\sigma} + 0.2886)\hat{F}_u(1 - \hat{F}_u)}{\sqrt{N}} \tag{2.28}$$

where N was the number of independent samples in the regression, $\hat{\sigma}$ was the estimated standard deviation of lognormal fading in the cell and $\hat{F}_u$ was the estimated area reliability. The equations (2.27) and (2.28) are completely general expressions, provided $6 \le \sigma \le 10$, $F_u \ge 90\%$ and $N \ge 100$.

From (2.27) and (2.28) it is clear that that for a given number of signal strength samples, N, the area reliability is much more precise (by one or two orders of magnitude) than the estimate of the cell radius. Specifically it takes fifty times as many signal strength samples to estimate the cell radius R, than to estimate the area reliability F_u. For a real drive test data, the true cell radius is unknown and must be statistically estimated. As long as the radius estimate is sufficiently precise, so is the area reliability estimate. Thus the most important conclusion is that the precision of the estimate of the cell radius is the limiting factor in determining the quality of RF coverage, and not the precision of the area reliability estimate.

2.5 Conclusion

This chapter has shown the importance of using signal strength measurements and determining the cell radius for RF coverage estimation and verification. The relations between cell radius, cell edge reliability, cell area reliability and shadow fade margin were explained. Different coverage verification techniques were discussed.

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Chapter 3
Research

Even though much work has been done in the field of propagation prediction [1-7] and several empirical results have been published, not much research has been done in understanding the effects of finite sampling on signal strength estimation. As explained in the chapter on coverage estimation the phenomenological models or the generic empirical models are not suitable to design a modern cellular network. Also from the discussions in chapter on coverage estimation it follows that propagation models must be derived from the measurements taken in the service area to design a cellular network with a high quality of service.

Typically a test transmitter is placed at the proposed location in the service area connected to an omni antenna on an existing tower or building or a crane. A continuous wave signal at the required frequency is fed to the transmitter and signal strength measurements are taken along with their corresponding locations. One drawback to this is that taking signal strength measurements is an expensive and time consuming task.

At the current time, not much research has been done tin determine the number of strength measurements needed for a desired accuracy or reliability of coverage estimation. This is extremely important as it can be used to determine the minimum required drive distance for a certain accuracy.

Recently a Telecommunications Industry Association (TIA) report on a technology independent methodology for the modeling, simulation and empirical verification of wireless communications system performance in noise and interference limited systems operating on frequencies between 30 and 1500 MHz was released [8]. The following is the summary from the section of reliability prediction from the report. The prediction of mean signal strength at a given location could vary from the measured signal for many reasons, including the following:

1) the prediction algorithm is not adequate, 2) imperfect terrain database / land cover database imperfections, 3) predictions may be made at location where measurements are not taken. Because of these errors, the signal at any one location can vary from that predicted by the model. The report recommended that a 1 dB margin be added to compensate for these “uncertainty” effects.

Even though the TIA report recommends a 1 dB margin for uncertainty effects, it does not show the relation between the margin and the number of signal strength measurements or the theoretical justification for the proposed margin. Determining the margin needed for a desired reliability depending on the distance of the signal strength measurement drive route will be very useful.

Also no analysis has been published showing the minimum required signal strength measurement drive route distance depending on the morphology and lognormal shadowing conditions to minimize the margin for a desired reliability. Determining a margin for required reliability of coverage estimation depending on the distance of the signal strength measurement drive route would be extremely useful as it allows the resources needed for drive testing to be traded against the additional cells (margin) needed to cover the area of interest.

It has long been noticed by RF engineers that when linear regression is applied to a small number of signal strength measurements, a very wide range of path loss slopes are obtained (20 to 60 dB/decade). Yet, if larger data sets are used the slope is close to that recommended by Hata[2]. It would be useful to analyze this. Hence we determine the sensitivity of the cell radius to the propagation parameters and find a more accurate method of estimating coverage when only a few signal strength measurements are available.

This dissertation investigates the above mentioned issues of estimating coverage from signal strength measurements.

Manuscript 1 investigates an approach of improving the accuracy of cell radius estimation when only a limited number of signal strength measurements are present. Our analysis in manuscript 1 have indicated that the cell radius estimation is very sensitive to small errors in the intercept of the propagation model. Even a limited number of signal strength measurements provides valuable information about the intercept of the propagation model and thus about RF coverage. In contrast, the cell radius estimate is not very sensitive to small errors in the path loss exponent. However, estimating the slope from less than 150 uniformly sampled signal strength measurements produces large errors in slope estimate and also in the estimated RF coverage. Thus for situations when the number of signal strength measurements is less than 150 samples it is better to predict the slope via Hata’s equations than to estimate it from the data.

In manuscript 2 we propose a new margin called the “effective fade margin” to compensate for an insufficient distance of the signal strength measurement drive route depending on the desired reliability and the distance of the uniform drive route. We also investigate the factors affect the effective fade margin. The analysis in manuscript 2 suggest that the margin needed in the link budget for desired reliability depends on the distance of signal strength measurement drive route, the lognormal standard deviation, morphology and the variation of clutter in a given cell. The simulation results indicate that to minimize the margin, at least a distance equivalent to 8 cell radii are needed. Also the results indicate that the effect of variation of clutter in any given cell has no impact on the required margin if the drive route distance is more than 8 cell radii.

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Manuscript 1

Coverage Prediction for Cellular Networks from Limited Signal Strength Measurements *

Kanagalu Manoj, Pete Bernardin and Lakshman Tamil

ABSTRACT

The quality of coverage of any wireless network design depends on the accuracy of the propagation model. For accurate designs, the propagation models are estimated from signal strength measurements taken in the service area. Even though it is known that modeling error is introduced when only a few signal strength measurements are processed, this is not completely understood. In this paper, we investigate the impact of a limited number of signal strength measurements on the accuracy of coverage prediction and estimation of propagation parameters. We find that when there are fewer than 150 independent signal strength measurement samples, which corresponds approximately to 3% of the cell area, better results are obtained by fixing the propagation slope and calculating the intercept via method of least-squares.

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I. INTRODUCTION

As more wireless networks are being deployed for higher quality of service, it is becoming increasingly important to accurately design the networks. Wireless networks are designed for both coverage and capacity requirements. Some common requirements for quality are desired cell edge bit error rate (BER), frame error rate (FER), the reliability (or outage probability) at the cell edge and the grade of service. The propagation characteristics of radio frequency signals in the service area has a major impact on the design. Propagation models are used to predict the signal strength at various locations for both coverage and interference criteria. Several investigators [1,2] have studied the radio frequency propagation and published empirical formulas for calculating the path loss at various frequencies for different morphologies. These results are limited to approximate path loss calculations and are subject to additional errors due to morphology classification. To obtain a more precise propagation model, actual signal strength measurements and their corresponding locations must be taken in the desired service area. In addition, the true signal strength reliability of the network depends on the accuracy of the propagation model, which in turn depends on the number of measurements. Obtaining these signal strength measurements can be a tedious task and in many areas, due to physical limitations, only a small number of independent measurements can be taken.

Most of the previous propagation work [1,2] has focused on predicting signal strength for the purpose of cell planning. Not much work has been done in determining the cell coverage from a limited number of measurements, though some work has been done in determining the number of independent signal strength measurements[3,4] needed to obtain an accurate propagation model. This research is focused on determining techniques to improve the accuracy of propagation modeling when only a few independent signal strength measurements are available in the service area.

PROPAGATION MODEL AND CELL RADUIS ESTIMATION

Mobile radio propagation characteristics have been studied by several investigators [1-3]. Based on their results, the propagation in the VHF/UHF band is characterized by Rayleigh fading, lognormal shadowing and path loss. The average received power decreases logarithmically with distance. The average large scale path loss $\overline{PL(d)}$ in decibels (dB) for a distance d can be expressed as

$$\overline{PL(d)} = A + B \cdot \log(d) \tag{1}$$

where A (the intercept) is the path loss at some reference distance in dB and B is the propagation slope [3]. The parameters A and B depend on the morphology and the height of the base station transmitter.

When the received signal is measured over the distance of a few tens of wavelengths, the received signal envelope shows rapid and deep fluctuations about the local mean with the movement of the mobile terminal. These fluctuations are caused by multipath propagation and movement of the mobile terminal. The signal envelope can be approximated by the Rayleigh distribution, and the received signal exhibits Rayleigh fading.

When measurements are averaged over more than 40 wavelengths, the path loss $PL(d)$ at a distance d , at different azimuth angles is random and log-normally distributed about the mean distance dependent value [2]. This lognormal shadowing is a relatively slow variation around the local mean and is due to the different environmental clutter found at different azimuth locations at the same distance from the base station. Thus, the path loss at a distance d can be expressed as

$$PL(d) = A + B \cdot \log(d) + X \tag{2}$$

where X is a zero mean Gaussian distributed random variable with standard deviation σ (dB) and the probability density function of X is given by,

$$P(X) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(\frac{-x^2}{2\sigma^2}\right) \tag{3}$$

The received power is given by

$$P_R(d) = P_T - PL(d) \tag{4}$$

where $P_R(d)$ is the received power (dBm) at any point at a distance *d* from the transmitter, P_T is the base station transmit power (dBm). Thus, the effective distribution of the actual signal strength can be described by

$$N\left(\overline{P_R(d)}, \sigma^2\right). \tag{5}$$

Using a fade margin for the lognormal shadowing, systems can be designed for any coverage reliability. The margin needed to design the system for a desired reliability can be easily calculated. If P_{MIN} is the minimum required signal strength, then the probability that the received signal, *x*, at a distance, *d*, will be greater than the minimum signal in terms of the mean signal is given by

$$P\left(x > P_{MIN}\right) = \frac{1}{\sigma\ \sqrt{2\pi}} \int_{P_{MIN}}^{\infty} e^{-\frac{(x-\overline{P_R(d)})^2}{2\sigma^2}} dx\,. \tag{6}$$

This can be expressed as

$$P(x > P_{MIN}) = Q\left(\frac{P_{MIN} - \overline{P_R(d)}}{\sigma}\right) \tag{7}$$

where

$$Q(z) = \frac{1}{\sqrt{2\pi}} \int_{z}^{\infty} e^{-\frac{x^2}{2}} dx\,. \tag{8}$$

Cellular networks are designed for certain required service reliability. A shadow fade margin FM_σ, that ensures the desired cell edge reliability, $F(z)$, can be approximated as

$$FM_\sigma = z \cdot \sigma \tag{9}$$

and

$$F(z) = 1 - Q(z) \tag{10}$$

To design a system for a cell edge reliability of $F(z)$, the mean received signal strength $\overline{P_R(R)}$ at the cell edge at a distance R from the base station has to be

$$\overline{P_R(R)} = P_{MIN} + FM_{\sigma} \tag{11}$$

Thus from eqns. (1), (4) and (11) the cell radius R for a reliability of $F(z)$ is given by

$$R = 10^{\frac{(P_T - P_{MIN} - FM_{\sigma} - A)}{B}} \tag{12}$$

III. SENSITIVITY OF CELL RADIUS TO THE ESTIMATES OF THE PROPAGATION PARAMETERS

In this section, we investigate the sensitivity of the cell radius to all the propagation parameters. Sensitivity analysis identifies the relative importance of the accuracy of each of the propagation parameters.

We define

$$S_{R_A} = \left|\frac{\partial R}{\partial A}\right| \cdot \frac{A}{R} \tag{13}$$

$$S_{R_B} = \left|\frac{\partial R}{\partial B}\right| \cdot \frac{B}{R} \tag{14}$$

$$S_{R_\sigma} = \left|\frac{\partial R}{\partial \sigma}\right| \cdot \frac{\sigma}{R} \tag{15}$$

Where S_{R_A} is the sensitivity of cell radius R to the estimate of the intercept A, S_{R_B} is the sensitivity of cell radius to the estimate of the propagation slope B, and S_{R_σ} is the sensitivity of the cell radius to estimate of the standard deviation. The sensitivity of R with respect to A, B and σ is given by

$$S_{R_A} = \frac{2.3 \cdot A}{B} \tag{16}$$

$$S_{R_B} = \frac{2.3\left(P_T - P_{MIN} - A - z \cdot \sigma\right)}{B} \tag{17}$$

$$S_{R_\sigma} = \frac{2.3 \cdot z \cdot \sigma}{B}. \tag{18}$$

Figure 1 shows an example where the sensitivity of cell radius to each of the propagation parameters is plotted for a varying propagation intercept. Figure 2 is an example where the sensitivity of the cell radius to each of the propagation parameters is plotted for a varying propagation slope. The figures are plotted for values of P_{MIN} = -95 dBm, P_T = 50 dBm, σ = 8 dB and z = 0.675 (i.e., 75% cell edge reliability).

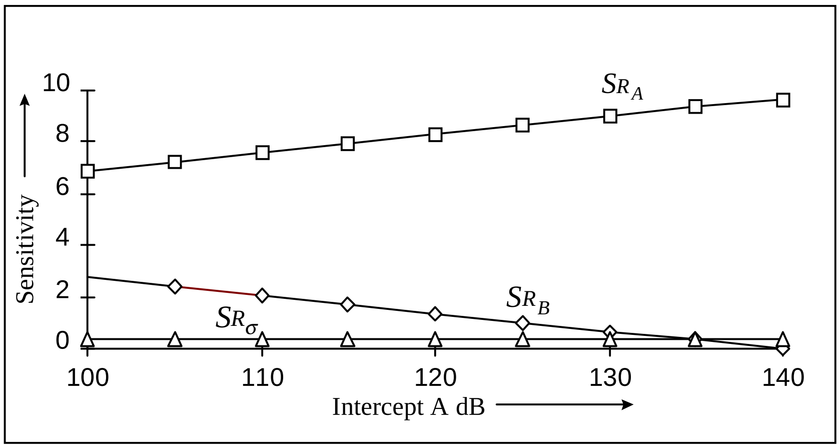

Figure 1. Sensitivity of cell radius to intercept A, slope B and standard deviation σ as a function of changing intercept A .

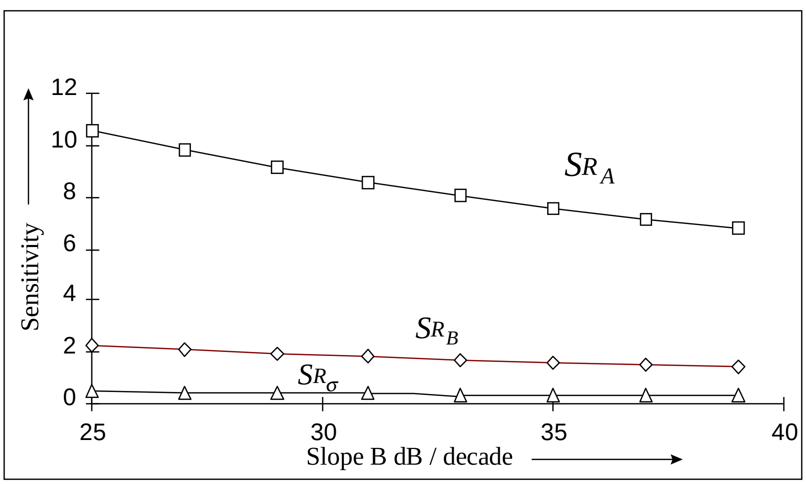

Figure 2. Sensitivity of cell radius to intercept A, slope B and standard deviation σ, as a function of slope.

Based on the analysis done by Hata[1], the slope is the same for all the morphologies but the intercept changes with morphologies. We use this to show the sensitivity of cell radius to the different propagation parameters. Depending on the actual terrain, building density etc., the average value of *A* for dense urban morphologies at 800 MHz is 130 dB and for rural morphologies is 108 dB. The propagation slope varies as a function of the tower height [1]. A propagation slope of 31 dB/decade corresponds to a tower height of 100 meters and 38 dB/decade corresponds to a height of 10 meters. From Figures 1 and 2 we find that the cell radius is very sensitive to changing intercept, and not very sensitive to the slope and standard deviation. The cell radius is least sensitive to the propagation slope in dense urban morphologies. These figures also indicate that the propagation intercept is the most important parameter and must be accurately determined before the cell radius can be accurately estimated. Minor variations to the propagation slope do not have much impact on the estimate of the cell radius.

VI. EFFECT OF FINITE SAMPLING ON THE ESTIMATION OF PROPAGATION PARAMETERS

In this section, we determine the effect of finite sampling on the estimation of the propagation parameters. The accuracy of the propagation parameter estimates is determined by the variance of the relative error between the actual propagation parameters and the parameters that are estimated from a finite number of signal strength measurements.

This analysis was done by simulating in a single radial direction. However, it is assumed that the simulated measurements result from a uniform area sampling of the cell. Hence, the samples along the single radial represent the composite path loss fluctuations of radials in all azimuthal directions. Since the regression is actually evaluated across all azimuth angles simultaneously, an uncorrelated Gaussian shadow fading model is chosen. Using the true A, B and σ, the received power at N uniformly distributed points in the cell is calculated. The intercept $\hat{A}$, slope $\hat{B}$ and standard deviation $\hat{\sigma}$, are estimated from the simulated samples via linear regression.

The sensitivity analysis of the previous section has indicated that the cell radius estimate is not very sensitive to the propagation slope, so another simulation model is generated where it is assumed that the slope B, is known to be within some fixed tolerance. The other two propagation parameters A and σ are estimated via least squares as before.

We define the relative errors e_R, e_A, e_B and e_σ as

$$e_A = \frac{A - \hat{A}}{A} \tag{19}$$

$$e_B = \frac{B - \hat{B}}{B} \tag{20}$$

$$e_\sigma = \frac{\sigma - \hat{\sigma}}{\sigma} \quad . \tag{21}$$

These errors are determined via Monte Carlo simulation for different values of σ and N. The analysis shows that e_A, e_B and e_σ are zero-mean, Gaussian random variables, with variances, σ_{eA}^2, σ_{eB}^2 and $\sigma_{e\sigma}^2$. The experiment is performed for different standard deviations and different numbers of simulated samples. We are interested in determining the inaccuracy ΔA such that

$$P(A - \Delta A \le \hat{A} \le A + \Delta A) = 95\% \tag{22}$$

The corresponding two sided normalized intercept error, δ_A is

$$\delta_A = \frac{\Delta A}{A} = 1.96\sigma_{eA} \tag{23}$$

where δ_A is a dimensionless percentage of intercept A. The error, δ_A, is plotted as function of the number of independent samples.

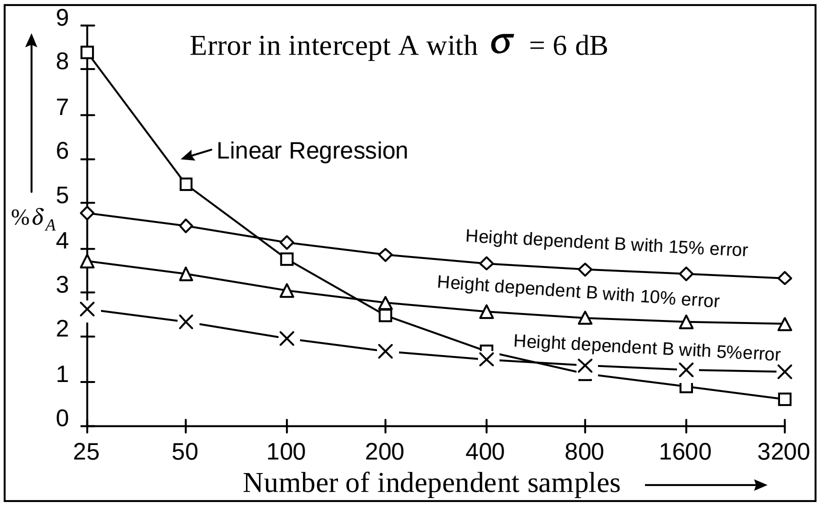

Figure 3. Comparison of the error in the propagation intercept A, δ_A, determined by linear regression and by assuming a slope (with 5%, 10% and 15% error) and determining the intercept by least squares versus the number of independent samples for a standard deviation of 6 dB.

The Figures 3-5 show that even when the assumed propagation slope has 15 % error, if less than about 150 independent samples are processed, it is better to estimate the intercept only (from the data), rather than both the slope and intercept.

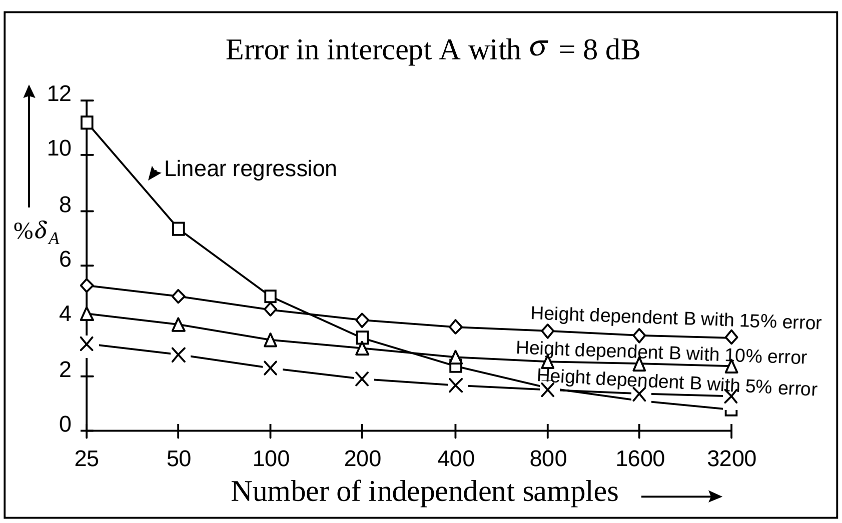

Figure 4. Comparison of the error in the propagation intercept A, δ_A, determined by linear regression and by assuming a slope (with 5%, 10% and 15% error) and determining the intercept by least squares versus the number of independent samples for a standard deviation of 8 dB.

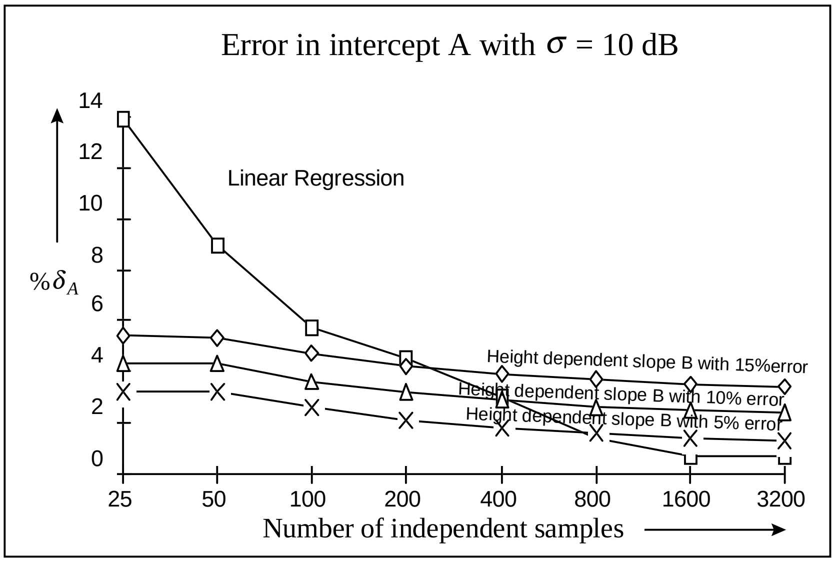

Figure 5. Comparison of the error in the propagation intercept A, δ_A, determined by linear regression and by assuming a slope (with 5%, 10% and 15% error) and determining the intercept by least squares versus the number of independent samples for a standard deviation of 10 dB.

EFFECT OF FINITE SAMPLING ON CELL RADUIS ESTIMATION

In this section we compare the accuracy of cell radius, estimated by assuming a height dependent slope, with the accuracy of estimating the cell radius by linear regression when a limited number of signal strength measurements are available.

The estimated cell radius is calculated from the equation below (see also (12)):

$$\hat{R} = 10^{\frac{P_T - P_{MIN} - \hat{A} - z \cdot \hat{\sigma}}{\hat{B}}} \quad (24)$$

and the error in radius estimation e_R is given by

$$e_R = \frac{R - \hat{R}}{R} \quad (25)$$

The analysis shows that the relative error e_R is a zero mean Gaussian random variable, its variance σ_e^2 depends on N and the lognormal shadowing σ. We are interested in determining the inaccuracy ΔR such that

$$P(R - \Delta R \le \hat{R} \le R + \Delta R) = 95\% \quad (26)$$

The corresponding two sided normalized radius inaccuracy δ_R is

$$\delta_R = \frac{\Delta R}{R} = 1.96\sigma_e \quad (27)$$

where δ_R is a dimensionless percentage of cell radius R.

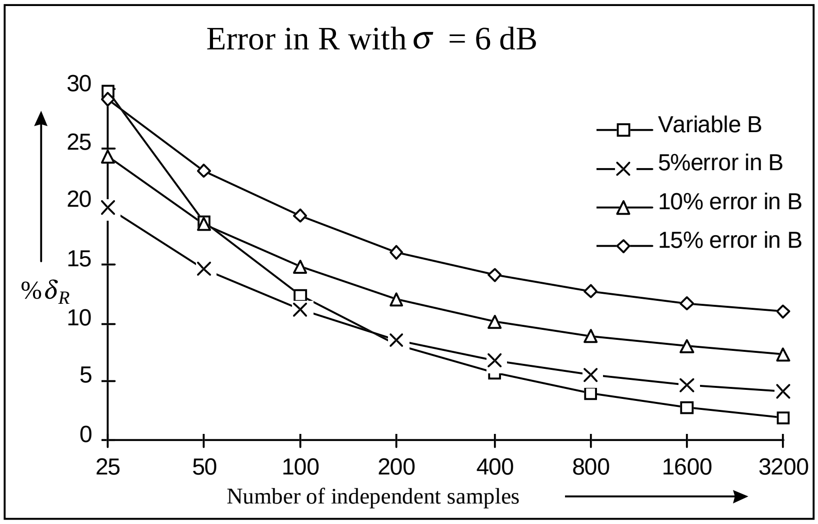

Figure 6. Comparison of the error in cell radius estimation assuming a propagation slope via least squares and the error in cell radius estimation via linear regression for a standard deviation of 6 dB. Assumed propagation slope having 5%, 10% and 15% error about the true slope are shown.

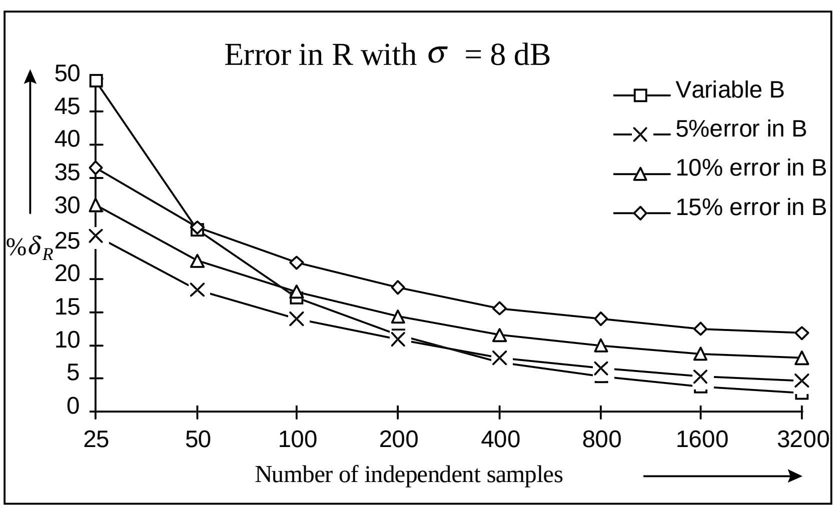

Figure 7. Comparison of the error in cell radius estimation assuming a propagation slope via least squares and the error in cell radius estimation via linear regression for a standard deviation of 8 dB. Assumed propagation slope having 5%, 10% and 15% error about the true slope are shown.

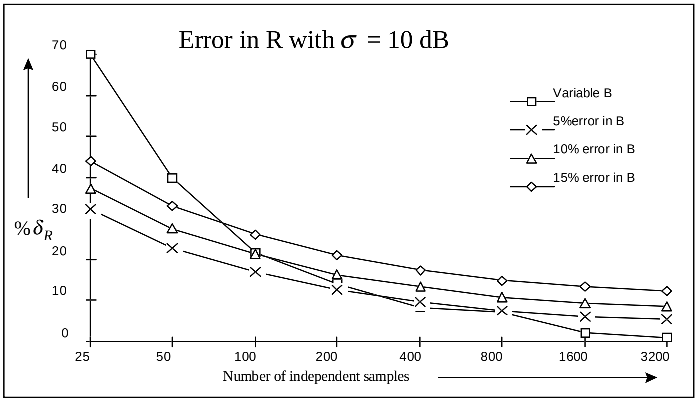

Figure 8. Comparison of the error in cell radius estimation assuming a propagation slope via least squares and the error in cell radius estimation via linear regression for a standard deviation of 10 dB. Assumed propagation slope having 5%, 10% and 15% error about the true slope are shown.

From figures 6-8, several conclusions can be made. When less than about 150 samples are available, more accurate coverage estimation can be obtained by predicting the path loss slope via Hata's method and tuning the intercept via regression. These results become even more significant when the standard deviation of the lognormal shadowing is high.

VI. CONCLUSION

We have presented a sensitivity analysis that demonstrates how the propagation model parameters are affected when they are estimated from a limited number of signal strength samples.

Cell radius estimation is very sensitive to small errors in the intercept of the propagation model. Even a limited number of signal strength samples provides valuable information about the intercept of the propagation model, and thus also about RF coverage.

In contrast, the cell radius estimate is not very sensitive to small errors in the path loss exponent. However, estimating the slope from less than 150 uniformly sampled signal strength measurements produces large errors in the slope estimate and also in the estimated RF coverage. Extensive data analysis has shown that 150 samples corresponds approximately to 3% of the cell area. For these situations we have shown that it is better to predict the slope via Hata's equations than to estimate it from the data.

In future studies, we plan to extend this analysis to include cases where the signal strength measurements are correlated.

ACKNOWLEDGMENTS

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Manuscript 2

On Reliable RF Coverage using Signal Strength Measurements for Cellular Networks *

Kanagalu Manoj, Pete Bernardin and Lakshman Tamil

ABSTRACT

The coverage reliability of a wireless network design depends on the accuracy of the propagation model. For a reliable wireless network design, the propagation models are estimated from the signal strength measurements in the service area. Even though it is known that the accuracy of the propagation models and hence the accuracy of the RF design increases with the increase in the number of uniform signal strength measurements taken in the service area, this is not completely understood. In this paper, it is shown that the RF design accuracy depends on the amount of uniform drive test, standard deviation of the shadow fading and the signal strength correlation distance. Also we show that there is a point of diminishing return in the amount of drive testing, beyond which drive testing will not improve the accuracy of the design. A design procedure for achieving desired coverage reliability using the concept of effective fade margin is proposed which depends on the distance of the drive route, standard deviation of shadow fading and correlation.

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I. INTRODUCTION

There has been an increase in the demand for higher quality networks due the rapid growth and competition for wireless subscribers. Wireless networks are designed for both coverage and capacity requirements. Some common requirements for quality are desired cell edge bit error rate (BER), frame error rate (FER), the reliability (or outage probability) at the cell edge, and grade of service. An accurate network design is necessary to deploy a high quality network. The propagation of radio frequency signals in the service area has a major impact on the design. The propagation models are used to predict the signal strengths at various locations for both coverage and interference studies. Several investigators [1-3] have studied the radio frequency propagation and published empirical formulas for calculating the path loss at various frequencies for different morphologies. These results are limited to approximate path loss calculations and morphology classification. To obtain a more precise propagation model, actual signal strength measurements with their corresponding locations must be taken in the desired service area. In addition, the true signal strength reliability of the network depends on the accuracy of the propagation model, which in turn depends on the number of measurements. Obtaining these signal strength measurements can be a tedious task and in many areas, due to physical limitations, only a small number of the measurements can be taken.

Most of the previous propagation work [2,3] has focused on predicting signal strength for the purpose of cell planning. In [4] it has been shown that cell radius inaccuracy is a more accurate technique for coverage validation rather than area reliability. Some study [5] of the effect of finite signal strength measurements on estimating propagation parameters and hence in estimating coverage has been done. The concept of fade margin to design a system for the desired reliability under lognormal shadowing conditions has been studied in [6,8]. Not much has been done in the determining the effect of a finite number of measurements on the prediction of signal strength. This research is focused on determining the distance of signal strength measurements drive route needed for reliable coverage estimation. We propose the concept of effective standard deviation, which includes the effects of shadowing and inaccuracies in the propagation model. Using this concept we address the required distance of the drive route. If the distance of the drive route is lesser than the minimum required distance, we propose a margin, which depends on the distance of the signal-strength measurements drive route, standard deviation of the lognormal shadowing etc. Using this margin, systems of any desired reliability can be designed, compensating for the insufficient distance of the signal-strength measurement drive route.

The remainder of the paper is organized as follows. Section II gives a brief description of the radio propagation and propagation models. The general principles of the effect of finite sampling on signal strength estimation at the cell edge, the concept of design reliability and margin is explained in Section III. Section IV presents simulation setup and the simulation results. Section V presents some concluding remarks.

II. PROPAGATION MODEL

Several investigators [1-3] have studied mobile radio propagation characteristics. Based on their results, the propagation in the VHF/UHF band is characterized by Rayleigh fading, lognormal shadowing and path loss. The average received power decreases logarithmically with distance. The average large scale path loss $\overline{PL(d)}$ in decibels (dB) for a distance d can be expressed as

$$\overline{PL(d)} = A + B \cdot \log(d) \tag{1}$$

where, A is the path loss at a close in reference distance in dB and B is the propagation slope [7]. The parameters A and B depend on the morphology and the height of the base station transmitter.

When the received signal is measured over the distance of a few tens of wavelengths, the received signal envelope shows rapid and deep fluctuations about the local mean with the movement of the mobile terminal. These fluctuations are caused by multipath propagation and movement of the mobile terminal. The signal envelope can be approximated by the Rayleigh distribution, and the received signal exhibits Rayleigh fading. The statistical properties of multipath fading have been studied extensively in the literature.

When measurements are averaged over more than 40 wavelengths, the path loss *PL(d)* at a distance d is random and log-normally distributed about the mean distance dependent value [3]. This relatively slow variation of the local mean is due to the different environmental clutter in different locations at the same distance from the base station and is called as the slow fading or shadowing. Thus the path loss a distance d can be expressed as

$$PL(d) = A + B \cdot \log(d) + X \tag{2}$$

where, X is a zero mean Gaussian distributed random variable with standard deviation σ (dB) and the probability density function of X is given by,

$$P(X) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(\frac{-x^2}{2\sigma^2}\right) \tag{3}$$

The received power is given by

$$P_R(d) = P_T - PL(d) \tag{4}$$

where, $P_R(d)$ is the received power (dBm) at any point at a distance d from the transmitter, P_T is the base station transmit power (dBm).

A better understanding of the correlation properties of the slow fading process is required in analysis of hand-over schemes, to determine sampling resolution needed for determining the signal strength measurements for propagation modeling purposes, determining the required terrain and clutter resolution for propagation prediction databases. Several researchers have proposed correlation models for shadow fading in mobile communications [9-12].

Typically, the signal strength measurements for modeling the propagation are taken at regular intervals of distance or time. If the number of signal strength measurements in a cell or sector is fixed, then, the error in predicting the mean signal strength at any particular location would be less if the samples are more correlated. To determine the effect of correlation of the measured samples, we use the model proposed in [9].

To model the correlation properties [9] has used a simple decreasing correlation function. It is assumed that the mobile velocity is v and the received analog signal is sampled every T seconds. The model is given by

$$R_{xx}(n) = \sigma^2 a^{|n|} \tag{5}$$

Where, $a = \varepsilon_D^{vT/D}$ (6)

The variance σ^2 is usually in the range between 3 and 10 dB. The correlation coefficient a in equation (5) may be expressed as in (6). Here parameter ε_D is the correlation between two points separated by distance D.

$$R_{xx}(n) = \sigma^2 e^{-\frac{\partial}{D}|n|} \tag{7}$$

Also it has been shown that the model in (7) closely fits the measured data in suburban and urban environments, where ∂ is the distance between the measured samples and D is the de-correlation distance which depends on the morphology, antenna height etc.

III. GENERAL PRINCIPLES

In the first part of this section we investigate the estimation of the mean signal strength at the cell edge using the signal strength measurements taken during a drive test. In the second part we propose the concept of effective margin. Using this the required design reliability can be obtained either by sufficiently drive testing the area or by adding a design margin to compensate for inadequate signal strength measurement drive testing.

A. Effect of finite sampling on signal strength estimation at the cell edge

The signal strength measurements are uniformly sampled at regular intervals in distance in the coverage area. The effect of the antenna pattern is removed to normalize all the sample measurements to a zero gain omni antenna. Using (1-4), the cell radius is given by

$$\hat{R} = 10^{\frac{P_T - P_{MIN} - \hat{A} - Z \cdot \hat{\sigma}}{\hat{B}}} \tag{8}$$

where, P_{MIN} is the minimum required signal strength, the intercept $\hat{A}$, slope $\hat{B}$ and standard deviation $\hat{\sigma}$ are estimated from the measured samples via least squares estimation.

As explained in the earlier section, the distribution of the actual signal strength can be described by $N\left(\overline{\hat{P}_R(d)}, \hat{\sigma}^2\right)$, where $\hat{P}_R(d)$ is the received signal power at a distance d from the base station.

Using the standard deviation $\hat{\sigma}$, systems for desired coverage reliability can be designed. The margin needed to design the system for a desired is given by

$$M\arg in = Z \cdot \hat{\sigma} \tag{9}$$

If P_{MIN} is the minimum required signal strength, then the probability that the received signal x at a distance d will be greater than the minimum signal in terms of the predicted mean signal is given by

$$P(x > P_{MIN}) = \frac{1}{\hat{\sigma}\sqrt{2\pi}} \int_{P_{MIN}}^{\infty} e^{-\frac{(x - \overline{\hat{P}_R(d)})^2}{2\hat{\sigma}^2}} dx \tag{10}$$

This can be expressed as

$$P(x > P_{MIN}) = Q\left(\frac{P_{MIN} - \overline{\hat{P}_R(d)}}{\hat{\sigma}}\right) \tag{11}$$

where,

$$Q(Z) = \frac{1}{\sqrt{2\pi}} \int_{Z}^{\infty} e^{-\frac{x^2}{2}} dx \tag{12}$$

The unknown parameters $\hat{A}$, $\hat{B}$ and $\hat{\sigma}$ are estimated using sample data by method of least squares [13]. Suppose that we have N pairs of measurements, say, (y_1, x_1), (y_2, x_2), …, (y_n, x_n) where y_i represents the path loss in dB and x_i represents the location of the measured sample from the base station. The estimates $\hat{A}$, $\hat{B}$ and $\hat{\sigma}$ are given by,

$$\hat{B} = \frac{\sum_{i=1}^{N} y_i \cdot \log(x_i) - \frac{\left(\sum_{i=1}^{N} y_i\right)\left(\sum_{i=1}^{N} \log(x_i)\right)}{N}}{\sum_{i=1}^{N} (\log x_i)^2 - \frac{\left(\sum_{i=1}^{N} \log(x_i)\right)^2}{N}} \tag{13}$$

$$\hat{A} = \frac{1}{N}\left(\sum_{i=1}^{N} y_i - \hat{B} \cdot \sum_{i=1}^{N} \log x_i\right) \tag{14}$$

$$\hat{\sigma}^2 = \frac{\sum_{i=1}^{N} \left(y_i - (\hat{A} + \hat{B} \cdot \log x_i)\right)^2}{N - 2} \tag{15}$$

The average estimated received power at the cell edge at a distance $\hat{R}$ is given by

$$\hat{P}_R\left(\hat{R}\right) = P_T - \hat{A} - \hat{B} \cdot \log\left(\hat{R}\right) \tag{16}$$

Let A, B, σ and R be correct intercept, the correct slope, be the correct standard deviation, and the correct cell radius if signal strength measurements were taken in 100% of the cell area. Also let the average received power at the actual cell edge be P_{THRESH}. The difference between the actual and the estimated signal strengths Δs at the estimated cell edge due to finite sampling of the cell area is given by

$$\Delta S = P_R(\hat{R}) - \hat{P}_R(\hat{R}) = \hat{A} - A + (\hat{B} - B)\cdot\log\left(\hat{R}\right) \tag{17}$$

The variance of Δs, is given by,

$$Var[\Delta s] = Var\left[\hat{A} - A + (\hat{B} - B)\cdot\log\left(\hat{R}\right)\right] \tag{18}$$

Since the variance of A is zero, (18) becomes,

$$Var[\Delta s] = Var\left[\hat{A} + (\hat{B} - B)\cdot\log\left(\hat{R}\right)\right] \tag{19}$$

To simplify the analysis let us assume that we have estimated the cell radius correctly, then $\hat{R} = R$, (19) becomes,

$$Var[\Delta s] = Var\left[\hat{A} + \hat{B}\cdot\log(R)\right] \tag{20}$$

We know that

$$\hat{A} = \bar{y} - \hat{B}\cdot\overline{\log(x_i)}$$

Therefore (20) becomes,

$$Var[\Delta s] = Var\left[\bar{y} + \hat{B}\cdot\left(\log(R) - \overline{\log(x_i)}\right)\right] \tag{21}$$

Simplifying, we get,

$$Var[\Delta s] = \frac{\sigma^2}{N} + \left(\log(R) - \overline{\log(x_i)}\right)^2 \cdot Var\left[\hat{B}\right] \tag{22}$$

$$Var[\Delta s] = \frac{\sigma^2}{N} + \frac{\sigma^2\cdot\left(\log(R) - \overline{\log x_i}\right)^2}{\sum_{i=1}^{N}\left(\log x_i - \overline{\log x_i}\right)^2} \tag{23}$$

From (23), we can conclude that the variance of ΔS would depend on the number of signal strength measurements, location of the measurements, and standard deviation of slow fading of the signal strength measurements. The above expression gives the lower bound on the variance ΔS. The Var[ΔS] will always be higher, due to the fact that the measurements are never uniformly distributed, we have assumed that $\hat{R} = R$,which is not true. Also the standard deviation is not the same throughout the cell and σ is the standard deviation over the entire cell.

The above result also assumes that the signal strength measurements are uncorrelated. Further, ΔS is zero mean random variable and distribution is approximately Gaussian (Gaussian if $\hat{R} = R$).

Thus the distance of the drive route to attain a certain variance would depend on the de-correlation distance of the signal strength measurements. In section IV we will determine typical values of Var[ΔS] as a function of various parameters and drive route distance via simulation.

b. Design reliability and margin

Cellular systems are designed for certain cell edge or cell area reliability. To achieve this a shadow fade margin is included in the link budget to determine the effective cell radius. But due to the finite number of measurements used to model the propagation, the mean signal strength at the estimated cell edge may be different from the actual desired mean signal strength. To design a system for required coverage reliability in the noise limited case, the design must include the variation of the signal due to lognormal shadowing and the effect of finite signal strength measurements. In the earlier section it was shown that *N* signal strength measurements causes the predicted mean signal strength $\overline{\hat{P}_R(d)}$ at a distance *d* to vary with a Gaussian distribution with a mean $\overline{P_R(d)}$ with a variance, *Var*[ΔS] and standard deviation $\sigma_{\Delta s}$. Due to lognormal shadowing the actual signal strength $\overline{P_R(d)}$ at any point at a distance *d* from the base station varies with a Gaussian distribution about the predicted mean signal strength $\overline{\hat{P}_R(d)}$ with a standard deviation σ. Since both the distributions are Gaussian and independent of each other, the combined distribution for *N* signal strength measurements is also a Gaussian with mean $\overline{\hat{P}_R(d)}$ and standard deviation σ_{eff} given by:

$$\sigma_{eff} = \sqrt{\sigma_{\Delta s}^2 + \sigma^2} \tag{24}$$

Thus the effective distribution of the actual signal strength can be described by $N\left(\overline{\hat{P}_R(d)}, \sigma_{eff}^2\right)$

Using the standard deviation σ_{eff} , systems for desired coverage reliability can be designed. The margin needed to design the system for a desired reliability using N uniformly distributed signal strength measurements in the cell is given by

$$Margin = Z \cdot \sigma_{eff}$$

If P_{MIN} is the minimum required signal strength, then the probability that the received signal x at a distance d will be greater than the minimum signal in terms of the predicted mean signal is given by

$$P(x > P_{MIN}) = \frac{1}{\sigma_{eff}\sqrt{2\pi}} \int_{P_{MIN}}^{\infty} e^{-\frac{(x - \overline{\hat{P}_R(d)})^2}{2\sigma_{eff}^2}} dx \qquad (25)$$

This can be expressed as

$$P(x > P_{MIN}) = Q\left(\frac{P_{MIN} - \overline{\hat{P}_R(d)}}{\sigma_{eff}}\right) \qquad (26)$$

where,

$$Q(Z) = \frac{1}{\sqrt{2\pi}} \int_{Z}^{\infty} e^{-\frac{x^2}{2}} dx \qquad (27)$$

If $\sigma_{\Delta s} << \sigma$ then $\sigma_{eff} \approx \sigma$. Thus in order to minimize the margin the distance of the signal strength measurements drive route is long enough to make $\sigma_{\Delta s} << \sigma$. This would be the minimum recommended drive distance uniformly distributed in the cell to minimize the number of base stations for coverage and provide the required reliability.

IV. SIMULATION SETUP AND RESULTS

From section III it is obvious that solving the problem analytically is extremely complex and only loose bounds can be obtained for certain conditions. We use simulation technique to more precisely study the effect of the design reliability under different environmental conditions and to determine the minimum required distance of the signal strength measurements drive route.

a. Signal Generation

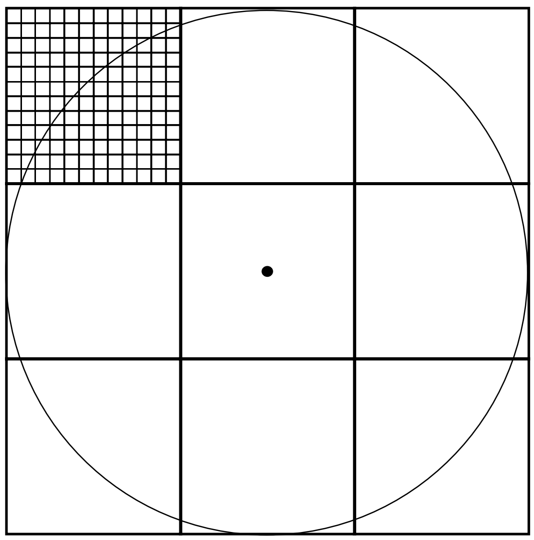

Fig. 1. A schematic diagram showing the structure used for simulation.

To simulate the shadowing conditions in a cell the following simulation setup was used. The entire cell area is divided into a grid of 36 x 36 bins. The bin size would correspond to the sampling resolution of the signal strength measuring equipment. Each block of 12 x 12 bins is modeled to represent a different morphology to simulate the different clutter effects. Thus there are total of 9 different morphology blocks. The base station is assumed to be at the center of the square grid.

SIGNAL GENERATION

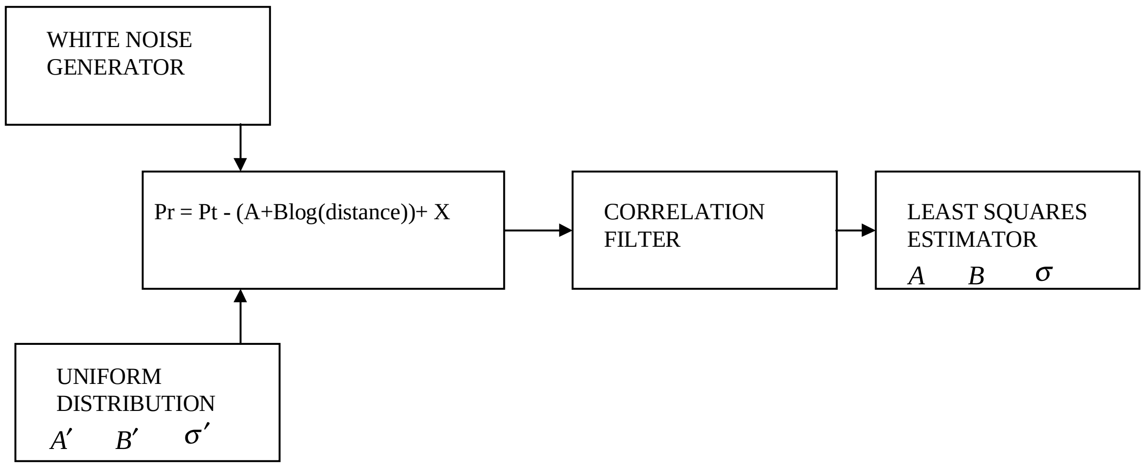

Fig. 2. Block diagram showing the generation of signal in each if the bins and estimation of the parameters A, B and sigma based via least squares under different environmental conditions.

The base station transmit power is P_T. The effect of antenna pattern is normalized and the received signal strength at each bin at a distance *d* from the base station is calculated using

$$P_R = P_T - [A_i + B_i \cdot \log(d) + N(0,\sigma^2)] \quad (28)$$

The A_i, B_i and σ_i are the same in each block, but are different for each of the 9 blocks and are generated using uniform distribution between the limits of 130 and 140 for A, 32 and 38 for B and 6 and 10 dB for σ. To achieve the effect of correlation between the bins, a correlation filter with the desired de-correlation distance was used. The correlation filter is modeled as follows:

A white Gaussian noise process filtered through a first degree filter with a pole at a will produce a log-envelope signal with the required properties. Taking the Fourier transform of $R_{xx}(n)$, the transfer function of the correlation filter is given by

$$H(j\omega) = \frac{\sigma\sqrt{\left(1-\exp\left(-\frac{2\partial}{D}\right)\right)}}{\left(1-\exp\left(-j\omega-\frac{\partial}{D}\right)\right)} \quad (29)$$

The above filter can be used to generate the correlated signal strength measurements from white Gaussian noise. This filter can be implemented by the difference equation (30) where x(n) is the input white noise and y(n) is the correlated output with a de-correlation distance D and standard deviation σ.

$$y(0) = \sigma\sqrt{1-\exp(-2\partial / D)} \cdot x(0) \qquad \text{for } n = 0$$

$$y(n) = \sigma\sqrt{1-\exp(-2\partial / D)} \cdot x(n) + \exp(-\partial / D) \cdot y(n-1) \qquad \text{for } n \geq 1 \quad (30)$$

Using the received signal strength measurements in all the bins, and by applying the method of least squares the correct A, B, σ and correct cell radius R for a desired reliability are evaluated. These correct parameters would be obtained if the measurements were taken in all the bins.

b. Drive Route and Signal Measurement

DRIVE ROUTE GENERATION AND SIGNAL STRENGTH MEASUREMENTS ALONG THE DRIVE ROUTE

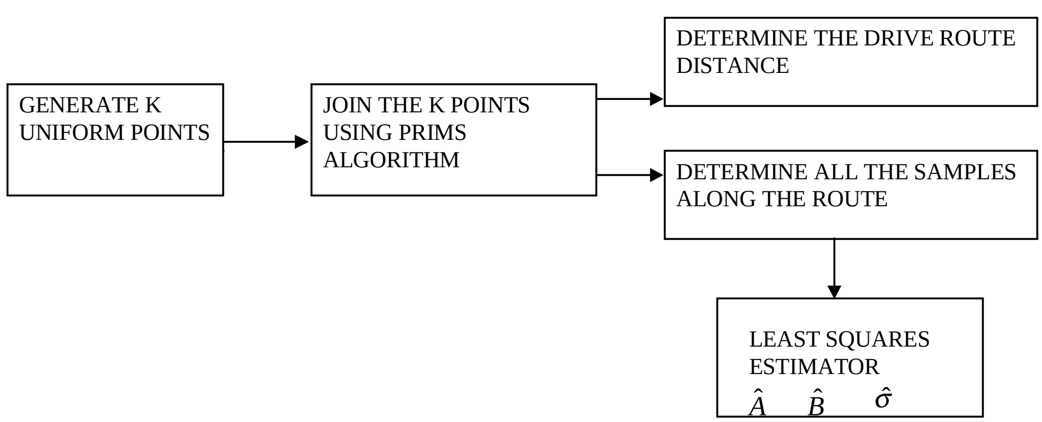

Fig. 3. Block diagram showing the selection of drive route in the simulation and determining the drive distance and the propagation parameters via least squares based on the signal strength measurements in the drive route.

To simulate the random uniform drive route, K uniform points were randomly generated with uniform distribution in the square. The K points were joined using Prim's shortest distance algorithm. Signal strength measurements in all the bins along the route were taken to estimate the propagation parameters $\hat{A}$, $\hat{B}$, $\hat{\sigma}$ and cell radius $\hat{R}$ via the least square method. The distance D of the drive route was also determined.

The effect of the finite number of signal strength samples on the propagation prediction is analyzed as a function of the number of independent samples and standard deviation of the lognormal shadowing. The accuracy of propagation prediction was calculated by determining the standard deviation of the difference in dB between the actual average signal strength and the average predicted signal strength at the estimated cell edge. The average estimated received power at the cell edge at a distance $\hat{R}$ is given by

$$\hat{P}_R\left(\hat{R}\right) = P_T - \hat{A} - \hat{B} \cdot \log\left(\hat{R}\right) \quad (10)$$

The difference between the actual and the estimated signal strengths ΔS is given by

$$\Delta S = P_R(\hat{R}) - \hat{P}_R(\hat{R}) = \hat{A} - A + (\hat{B} - B) \cdot \log\left(\hat{R}\right) \quad (11)$$

ΔS is determined for different sets of distances of the drive routes D and the standard deviation $\sigma_{\Delta S}$ of ΔS are evaluated. This experiment is repeated for different morphology and shadowing conditions standard deviations σ and different distances of the drive route D.

c. Results from Simulation

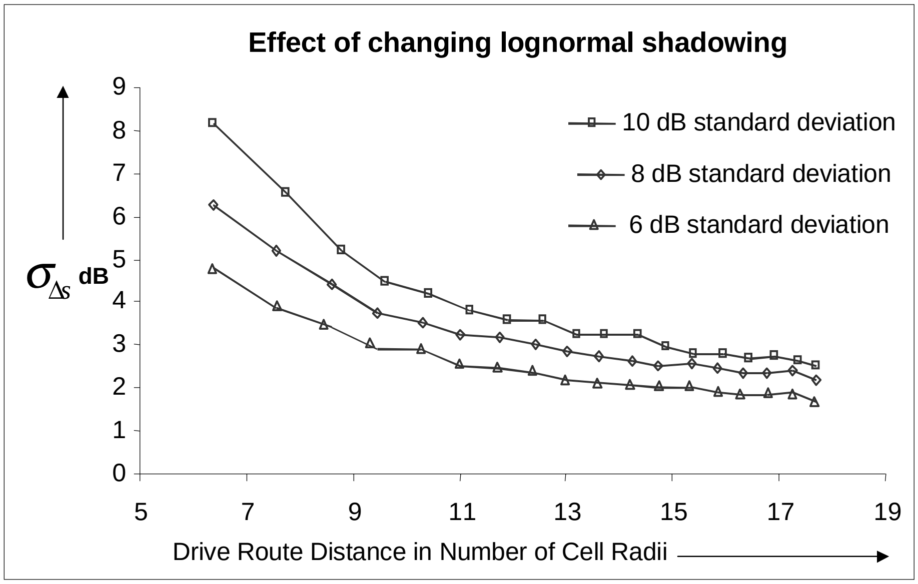

Fig. 4. Standard deviation of the average predicted signal strength about the actual average signal strength at the cell edge versus the average drive distance in radials for different lognormal shadowing conditions for suburban morphology with $\partial / D = 1.0$ in the correlation model.

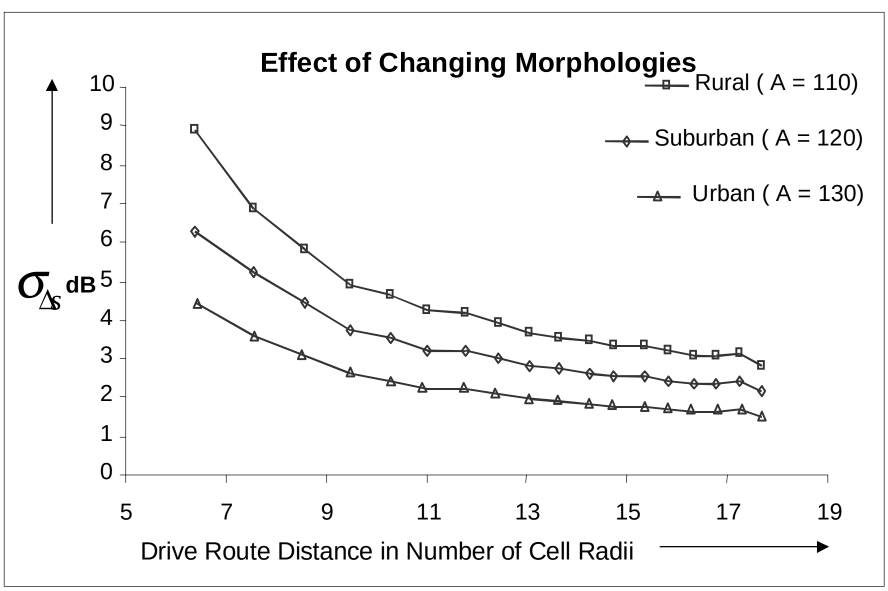

Fig. 5. Standard deviation of the average predicted signal strength about the actual average signal strength at the cell edge versus the average drive distance in radials for 8 dB standard deviation of lognormal shadowing, for different uniform morphologies with $\partial / D = 1.0$ in the correlation model.

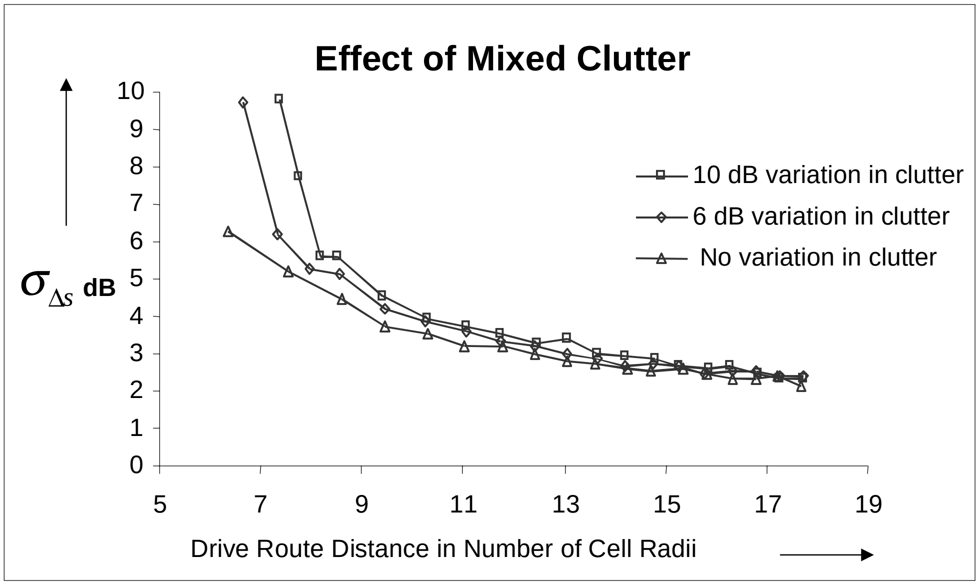

Fig. 6. Standard deviation of the average predicted signal strength about the actual average signal strength at the cell edge versus the average drive distance in radials for 8 dB standard deviation of lognormal shadowing, for suburban morphology with variation in clutter for $\partial / D = 1.0$ in the correlation model.

The mean predicted signal strength at the cell edge varies about the actual mean signal strength at the cell edge, due to the finite number signal strength measurements. This variation exhibits a normal distribution. Examples of the simulation results are shown in Figures 4 - 6.

The analysis indicates that $\sigma_{\Delta s}$ is a function of the uniform drive route distance, the lognormal shadowing standard deviation and the correlation of the measured samples. $\sigma_{\Delta s}$ decreases with the increase in the uniform drive route distance, decrease in lognormal shadowing standard deviation. Also $\sigma_{\Delta s}$ varies with the morphology. The effect of variation of clutter in any particular cell diminishes as the distance of the signal strength measurement drive route increases beyond 8 times the cell radii.

Design Reliability and Margin

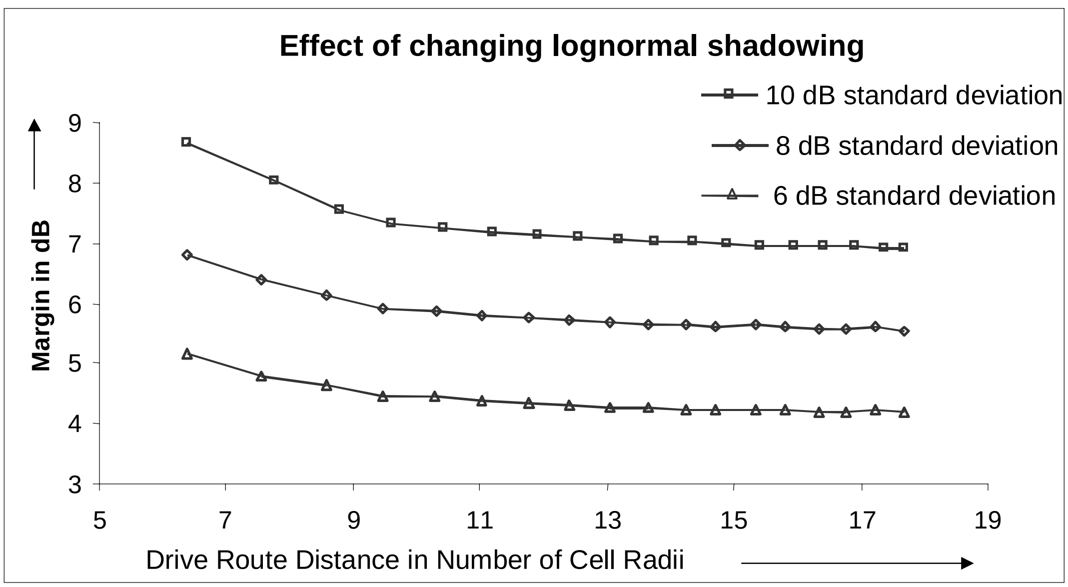

Fig. 7. Margin needed for 75% cell edge reliability of coverage for varying lognormal shadowing standard deviations for suburban morphology versus the distance of the uniform signal strength measurement drive route in number of cell radii for $\partial / D = 1.0$ in the correlation model.

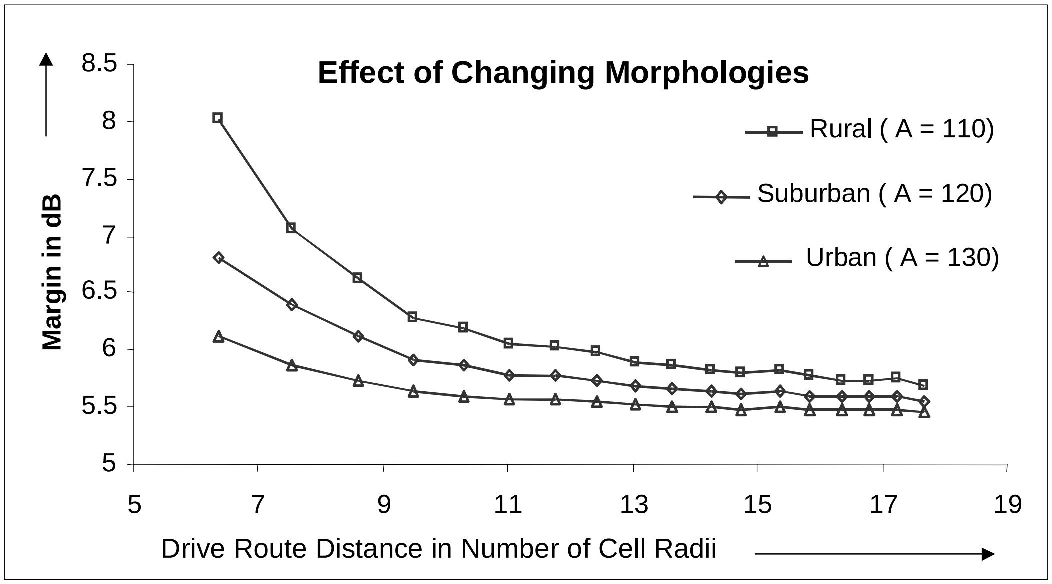

Fig. 8. Margin needed for 75% cell edge reliability of coverage for a standard deviation of 8 dB for different morphologies versus the distance of the uniform signal strength measurement drive route in number of cell radii for $\partial / D = 1.0$ in the correlation model.

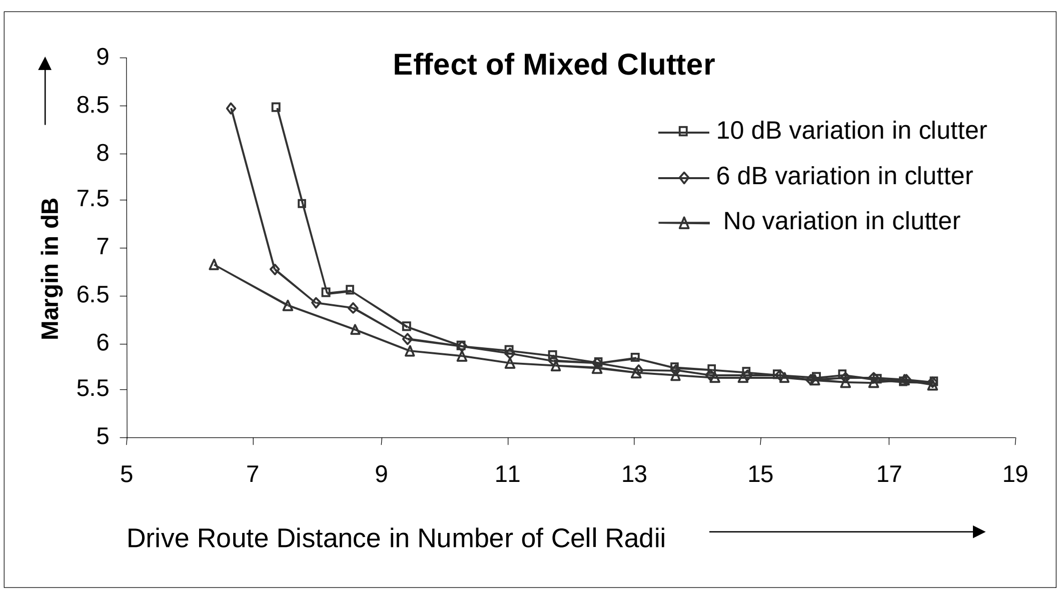

Fig. 9. Margin needed for 75% cell edge reliability of coverage for lognormal shadowing standard deviation of 8 dB for suburban morphology with variation in clutter versus the

distance of the uniform signal strength measurement drive route in number of cell radii for $\partial / D = 1.0$ in the correlation model.

In the Figures 7 - 9, the margin needed for 75 % cell edge reliability of coverage under different conditions is plotted. The slope of the propagation model is chosen to be 35.5. The lognormal shadowing standard deviations chosen are 6 dB, 8 dB and 10 dB. As can be seen, the necessary margin for any reliability decreases with the increase in distance of the uniform signal strength measurement drive route, and converges to the margin needed to for shadow fading. The margin does not reduce much when the distance of the signal strength measurement drive route is more than 8 times the cell radius. Thus, it is recommended that data collection process collect enough signal strength measurement samples by choosing the distance of the drive route to be at least 9 times the cell radius. For example, for a cell edge reliability of 75% in an urban environment with lognormal shadowing of 8 dB, if the distance of uniform signal strength measurement drive route is 6 times the cell radius, then necessary margin is 6.2 dB. This margin should be included in the link budget for calculating the signal strength threshold for coverage and cell radius estimation.

V. CONCLUSION

We have analyzed the effect of processing a finite number of signal strength measurements on the accuracy of coverage prediction. This work has been done using a simulation based on models from literature and extensive drive test measurements.

The mean predicted signal strength at a particular distance varies about the actual mean signal strength at that distance with a Gaussian distribution. The standard deviation of this distribution depends on the distance of uniform signal strength measurements drive route used in determining the propagation model and the standard deviation of the shadow fading. The analysis has shown that the effective distribution of the actual signal strengths about the predicted mean of the propagation model is a Gaussian distribution.

The margin needed in the link budget for desired reliability depends on the number of measured samples and the lognormal standard deviation as shown in the provided curves. Also, the study has indicated that to minimize the margin, a signal strength measurement drive route distance of at least 8 times the cell radius is needed.

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CONCLUSION

This dissertation analyzes and presents techniques improve the accuracy of coverage estimation from signal strength measurements and to design wireless networks for any desired coverage reliability using signal strength measurements. Further, this dissertation addresses the very important question regarding the minimum required signal strength measurements drive route distance, for reliable coverage estimation.

The parameters affecting the accuracy of the coverage estimation using signal strength measurements have been analyzed. A technique has been proposed using which insufficient drive testing can be compensated by additional margin, called the effective fade margin. Determining the margin for required reliability of coverage estimation depending on the distance of the signal strength measurement drive route is extremely useful as it allows the resources needed for drive testing to be traded against the additional cells (margin) needed to cover the area of interest.

Analysis has indicated that the mean predicted signal strength at a particular distance varies about the actual mean signal strength at that distance with a Gaussian distribution. The standard deviation of this distribution depends on the distance of signal strength measurements drive route (uniform distribution of the drive route is assumed) used in determining the propagation model, the standard deviation of the shadow fading and the morphology. Further, the effective distribution of the actual signal strengths about the predicted mean of the propagation model is a Gaussian distribution. The margin needed in the link budget for desired reliability depends on the number of measured samples and the lognormal standard deviation as shown in the provided curves. Also, the simulation results have indicated that to minimize the margin, a signal strength measurement drive route distance of at least 8 times the cell radius is needed.

Improving the accuracy coverage estimation when only a limited number of signal strength measurements are available has been addressed. It is shown through analysis and simulation that a more accurate estimate of the coverage can be obtained with limited number of signal strength measurements if a fixed height dependent propagation slope is used and the intercept of the propagation model is determined by least squares method. The analysis and results explain the reason for the very wide range of path loss slopes (20 to 60 dB/decade), when linear regression is applied to a small number of signal strength measurements.

In a single slope propagation model, the coverage estimation is most sensitive to the intercept of the propagation model. Even a limited number of signal strength samples provides valuable information about the intercept of the propagation model, and thus also about RF coverage. In contrast, the cell radius estimate is not very sensitive to small errors in the path loss exponent. However, estimating the slope from less than 150 uniformly sampled signal strength measurements produces large errors in the slope estimate and also in the estimated RF coverage. For these situations it was shown that it is better to predict the slope via Hata's equations than to estimate it from the data.

Appendix A. Linear Regression Analysis

The simple linear regression model, that is, a model with a single regressor *x* that has a relationship with a response *y* that is a straight line.

The simple linear regression model is

$$y = A + B \cdot x + \varepsilon \tag{1}$$

where, the intercept A and the slope B are unknown constants and ε is a random error component. The errors are assumed to have mean zero and unknown variance σ^2. Additionally we usually assume that the errors are uncorrelated. This means that the value of one error does not depend on the value of any other error.

It is convenient to view the regressor *x* as a controlled variable and measured with negligible error, while the response *y* is a random variable. That is, there is a probability distribution for *y* at each possible value of *x*. The mean of this distribution is

$$E(y \mid x) = A + B \cdot x \tag{2}$$

and the variance is

$$V(y \mid x) = V(A + B \cdot x + \varepsilon) = \sigma^2 \tag{3}$$

Thus the mean of *y* is a linear function of *x* although the variance of *y* does not depend on the value of *x*. Furthermore, because the errors are uncorrelated, the responses are also uncorrelated.

The parameters *A* and *B* are usually regression coefficients. The slope *B* is the change in the mean of the distribution of *y* produced by a unit change in *x*. If the range of data on *x* includes *x* = 0, then the intercept *A* is the mean of the distribution of the response *y* when *x* = 0. If the range of *x* does not include zero, then *A* has no practical interpretation.

Least Squares Estimation of the Parameters

The parameters A and B are unknown and must be estimated using sample data. Suppose that we have n pairs of data, say, (y_1, x_1), (y_2, x_2), …,(y_n, x_n). These data may results form the measurement experiment.

The method of least squares is used to estimate A and B. That is, A and B are estimated so that the sum of the squares of the differences between the observations y_i and the straight line is a minimum. From (1) we may write

$$y_i = A + B \cdot x_i + \varepsilon_i, \quad i = 1,2,\ldots,n \tag{4}$$

Equation (1) may be viewed as a population regression model while (3) is a sample regression model, written in terms of the n pairs of data (y_i, x_i), $i = 1,2,\ldots,n$. Thus the least squares criterion is

$$S(A,B) = \sum_{i=1}^{n} (y_i - A - B \cdot x_i)^2 \tag{5}$$

The least squares estimators of A and B say $\hat{A}$ and $\hat{B}$ must satisfy

$$\frac{\partial S}{\partial A} = -2\sum_{i=1}^{n} (y_i - \hat{A} - \hat{B} \cdot x_i) = 0$$

and

$$\frac{\partial S}{\partial B} = -2\sum_{i=1}^{n} (y_i - \hat{A} - \hat{B} \cdot x_i)x_i = 0 \tag{6}$$

Simplifying these two equations yields

$$n\hat{A} + \hat{B}\sum_{i=1}^{n} x_i = \sum_{i=1}^{n} y_i$$

$$\hat{A}\sum_{i=1}^{n} x_i + \hat{B}\sum_{i=1}^{n} x_i^2 = \sum_{i=1}^{n} y_i x_i \tag{7}$$

Equations (7) are the least squares normal equations. The solution to the normal equations is

$$\hat{A} = \bar{y} - \hat{B} \cdot \bar{x} \tag{8}$$

and

$$\hat{B} = \frac{\sum_{i=1}^{n} y_i x_i - \frac{\left(\sum_{i=1}^{n} y_i\right)\left(\sum_{i=1}^{n} x_i\right)}{n}}{\sum_{i=1}^{n} x_i^2 - \frac{\left(\sum_{i=1}^{n} x_i\right)^2}{n}} \tag{9}$$

where,

$$\bar{y} = \frac{1}{n}\sum_{i=1}^{n} y_i \quad \text{and} \quad \bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$$

are the averages of y_i and x_i respectively Therefore, $\hat{A}$ and $\hat{B}$ in (8) and (9) are the least squares estimators of the intercept and slope. The fitted simple linear regression model is then

$$\hat{y} = \hat{A} + \hat{B} \cdot x \tag{10}$$

Equation (10) gives a point estimate of the mean of y for a particular x.

Since the denominator of (9) is the corrected sum of squares of the x_i and the numerator is the corrected sum of cross products of x_i and y_i, we may write these quantities in a more compact notation as

$$S_{xx} = \sum_{i=1}^{n} x_i^2 - \frac{\left(\sum_{i=1}^{n} x_i\right)^2}{n} = \sum_{i=1}^{n} (x_i - \bar{x})^2 \tag{11}$$

and

$$S_{xy} = \sum_{i=1}^{n} y_i x_i - \frac{\left(\sum_{i=1}^{n} y_i\right)\left(\sum_{i=1}^{n} x_i\right)}{n} = \sum_{i=1}^{n} y_i (x_i - \bar{x}) \tag{12}$$

Thus a convenient way to write (9) is

$$\hat{B} = \frac{S_{xy}}{S_{xx}} \tag{13}$$

The difference between the observed value y_i and the corresponding fitted value $\hat{y}_i$ is a residual. Mathematically the i^{th} residual is

$$e_i = y_i - \hat{y}_i = y_i - \left(\hat{A} + \hat{B} \cdot x_i\right), \qquad i = 1,2,\ldots\ldots,n \tag{14}$$

The least squares estimators A and B have several important statistical properties. First, from (8) and (9) we can infer that $\hat{A}$ and $\hat{B}$ are linear combinations of the observations y_i.

Consider the bias property for $\hat{A}$ and $\hat{B}$. We have for $\hat{B}$,

$$E\left(\hat{B}\right) = E\left(\sum_{i=1}^{n} c_i y_i\right) = \sum_{i=1}^{n} c_i E(y_i) = \sum_{i=1}^{n} c_i (A + B \cdot x_i) = A\sum_{i=1}^{n} c_i + B\sum_{i=1}^{n} c_i x_i \tag{15}$$

where, $c_i = (x_i - \bar{x})/S_{xx}$ since $E(\varepsilon_i) = 0$ by assumption. Also we can show directly that

$\sum_{i=1}^{n} c_i = 0$ and $\sum_{i=1}^{n} c_i x_i = 1$

Therefore

$E\left(\hat{B}\right) = B$

That is assuming that

$E(y_i) = A + Bx_i$

$\hat{B}$ is an unbiased estimator of B. Similarly, $\hat{A}$ is an unbiased estimator of A, or

$E\left(\hat{A}\right) = A$

The variance of $\hat{B}$ is found as

$$V\left(\hat{B}\right) = V\left(\sum_{i=1}^{n} c_i y_i\right) = \sum_{i=1}^{n} c_i^2 V(y_i) \tag{16}$$

because the observations y_i are uncorrelated, and so the variance of the sum is just the sum of the variances. The variance of each term in the sum is $c_i^2 V(y_i)$, and we have assumed that $V(y_i) = \sigma^2$; consequently

$$V\left(\hat{B}\right) = \sigma^2 \sum_{i=1}^{n} c_i^2 = \frac{\sigma^2 \sum_{i=1}^{n} (x_i - \bar{x})^2}{S_{xx}^2} = \frac{\sigma^2}{S_{xx}} \tag{17}$$

The variance of $\hat{A}$ is

$$V(\hat{A}) = V(\bar{y} - \hat{B} \cdot \bar{x}) = V(\bar{y}) + \bar{x}^2 V(\hat{B}) - 2\bar{x}Cov(\bar{y}, \hat{B}) \tag{18}$$

Now the variance of $\bar{y}$ is just $V(\bar{y}) = \sigma^2 / n$, and the covariance between $\bar{y}$ and $\hat{B}$ can be shown to be zero. Thus

$$V(\hat{A}) = V(\bar{y}) + \bar{x}^2 V(\hat{B}) = \sigma^2 \left(\frac{1}{n} + \frac{\bar{x}^2}{S_{xx}} \right) \tag{19}$$

An important result concerning the quality of the least squares estimators $\hat{A}$ and $\hat{B}$ is the Gauss-Markov theorem, which states that for the regression model (1) with the assumptions $E(\varepsilon) = 0$, $V(\varepsilon) = \sigma^2$, and uncorrelated errors, the least squares estimators are unbiased and have minimum variance when compared with all other unbiased estimators that are linear combinations of the y_i. The least squares estimators are unbiased and have minimum variance when compared with all other unbiased estimators that are linear combinations of the y_i. Also the least squares estimators are best linear unbiased estimators, where "best" implies minimum variance.

There are several useful properties of the least squares fit:

The sum of the residuals in any regression model that contains an intercept A is always zero; that is,

$$\sum_{i=1}^{n} (y_i - \hat{y}_i) = \sum_{i=1}^{n} e_i = 0 \tag{20}$$

This property follows directly from the first normal equation in (7). Rounding errors may affect the sum.

The sum of the observed values y_i equals the sum of the fitted values $\hat{y}_i$, or

$$\sum_{i=1}^{n} y_i = \sum_{i=1}^{n} \hat{y}_i \tag{21}$$

The least squares regression line always passes through the centroid of the data.

The sum of the residuals weighted by the corresponding fitted value always equals zero; that is

$$\sum_{i=1}^{n} x_i e_i = 0$$

Estimation of σ^2

The estimate of σ^2 is obtained from the residual or error sum of squares as,

$$SS_E = \sum_{i=1}^{n} e_i^2 = \sum_{i=1}^{n} (y_i - \hat{y}_i)^2 \qquad (22)$$

A convenient computing formula for SS_E may be found by substituting $\hat{y}_i = \hat{A} + \hat{B}x_i$ into (22) and simplifying, yielding

$$SS_E = \sum_{i=1}^{n} y_i^2 - n\bar{y}^2 - \hat{B}S_{xy} \qquad (23)$$

But

$$\sum_{i=1}^{n} y_i^2 - n\bar{y}^2 = \sum_{i=1}^{n} (y_i - \bar{y})^2 = S_{yy} \qquad (24)$$

is just the corrected s of squares of the observations, so

$$SS_E = S_{yy} - \hat{B}S_{xy} \qquad (25)$$

The residual sum of squares has *n - 2* degrees of freedom, because two degrees of freedom are associated with the estimates $\hat{A}$ and $\hat{B}$ involved in obtaining $\hat{y}_i$. Now the expected value of SS_E is $E(SS_E) = (n-2)\sigma^2$, so an unbiased estimator of $\hat{\sigma}^2$ is

$$\hat{\sigma}^2 = \frac{SS_E}{n-2} = MS_E \qquad (26)$$

The quantity MS_E is called the error mean square or the residual mean square. The square root of $\hat{\sigma}^2$ is sometimes called the standard error of regression, and it has the same units as the response variable *y*. Because σ^2 depends on the residual sum of squares, any violation of the assumptions on the model errors or any mis-specification of the model form may seriously damage the usefulness of $\hat{\sigma}^2$ as an estimator of σ^2.

Interval Estimation in Simple Linear Regression

In this section we consider confidence interval estimation of the regression model parameters. We also discuss interval estimation of the mean response *E(y)* for given values of *x*, assuming that the response *y* is normally distributed about the mean value for any given *x*. The confidence intervals on *A*, *B* and σ^2 are derived below.

The width of these confidence intervals is a measure of the overall quality of the regression line. If the errors are normally and independently distributed, then the sampling distribution of both

$$\frac{\hat{B}-B}{\sqrt{MS_E/S_{xx}}} \text{ and } \frac{\hat{A}-A}{\sqrt{MS_E\left(1/n+\bar{x}^2/S_{xx}\right)}} \tag{27}$$

is *t* with *n – 2* degrees of freedom. Therefore a *100(1 - α)* percent confidence interval on the slope *B* is given by

$$\hat{B}-t_{\alpha/2,n-2}\sqrt{\frac{MS_E}{S_{xx}}} \le B \le \hat{B}+t_{\alpha/2,n-2}\sqrt{\frac{MS_E}{S_{xx}}} \tag{28}$$

and a *100(1 - α)* percent confidence interval on the intercept *A* is

$$\hat{A}-t_{\alpha/2,n-2}\sqrt{MS_E\left(\frac{1}{n}+\frac{\bar{x}^2}{S_{xx}}\right)} \le A \le \hat{A}+t_{\alpha/2,n-2}\sqrt{MS_E\left(\frac{1}{n}+\frac{\bar{x}^2}{S_{xx}}\right)} \tag{29}$$

These confidence intervals have the usual frequency interpretation. That is, if we were to take repeated samples of the same size at the same *x* levels and construct, for example, 95 percent confidence intervals on the slope for each sample, then 95 percent of those intervals will contain the true value of *B*.

The quantity

$$se(\hat{B}) = \sqrt{\frac{MS_E}{S_{xx}}} \tag{30}$$

in (28) is called the standard error of the slope $\hat{B}$. It is a measure of how precisely the slope has been estimated. Similarly

$$se(\hat{A}) = \sqrt{MS_E\left(\frac{1}{n} + \frac{\bar{x}^2}{S_{xx}}\right)} \tag{31}$$

in (29) is the standard error of the intercept $\hat{A}$. Regression computer programs usually report the standard errors of the regression coefficients.

If the errors are normally and independently distributed, the sampling distribution of

$$\frac{(n-2)MS_E}{\sigma^2} \tag{32}$$

is chi-square with $n - 2$ degrees of freedom. Thus

$$P\left(\kappa^2_{1-\alpha/2,n-2} \le \frac{(n-2)MS_E}{\sigma^2} \le \kappa^2_{\alpha/2,n-2}\right) = 1-\alpha \tag{33}$$

and consequently a $100(1 - \alpha)$ percent confidence interval on σ^2 is

$$\frac{(n-2)MS_E}{\kappa^2_{\alpha/2,n-2}} \le \sigma^2 \le \frac{(n-2)MS_E}{\kappa^2_{1-\alpha/2,n-2}} \tag{34}$$

Interval Estimation of the Mean Response

A major use of a regression model is to estimate the mean response *E(y)* for a particular value of the regressor variable *x*. Let x_o be any value of the regressor variable within the range of the original data on *x* used to fit the model, for which we wish to estimate the mean response, say *E(y/xo)*. An unbiased point estimator of *E(y/xo)* is found from the fitted model as

$$E(y \mid x_0) = \hat{y}_0 = \hat{A} + \hat{B} \cdot x_0 \qquad (35)$$

To obtain a *100(1 -* α*)* percent confidence interval on $E(y/x_o)$, first note that $\hat{y}_o$ is a normal distributed random variable because it is a linear combination of the observations y_i. The variance of $\hat{y}_o$ is

$$V(\hat{y}_0) = V(\hat{A} + \hat{B} \cdot x_0) = V[\bar{y} + \hat{B}(x_0 - \bar{x})] = \frac{\sigma^2}{n} + \frac{\sigma^2}{S_{xx}} = \sigma^2 \left[\frac{1}{n} + \frac{(x_0 - \bar{x})^2}{S_{xx}} \right] \qquad (36)$$

since $Cov(\bar{y}, \hat{B}) = 0$. The sampling distribution of

$$\frac{\hat{y}_0 - E(y \mid x_0)}{\sqrt{MS_E (1/n + (x_0 - \bar{x})^2 / S_{xx})}} \qquad (37)$$

is *t* with $n - 2$ degrees of freedom. Consequently a *100(1 -* α*)* percent confidence interval on the mean response at the point $x = x_o$ is

$$\hat{y}_0 - t_{\alpha/2, n-2} \cdot \sqrt{MS_E \left[\frac{1}{n} + \frac{(x_0 - \bar{x})^2}{S_{xx}} \right]} \le E(y \mid x_0) \le \hat{y}_0 + t_{\alpha/2, n-2} \cdot \sqrt{MS_E \left[\frac{1}{n} + \frac{(x_0 - \bar{x})^2}{S_{xx}} \right]} \qquad (38)$$

The width of the confidence interval for $E(y/x_o)$ is a function of x_o. The interval width is a minimum for $x_o = \bar{x}$ and widens as $|x_o - \bar{x}|$ increases. Intuitively this is reasonable, as we would expect our best estimates of *y* to be made at *x* values near the center of the data and the precision of estimation to deteriorate as we move to the boundary of the *x*-space.

The probability statement associated with the confidence interval (38) holds only when a single confidence interval on the mean response is to be constructed.

Predictions of New Observations

An important application of the regression model is to predict *y* corresponding to a new specific level of the regressor variable *x*. If x_o is the value of the regressor variable of interest, then

$$\hat{y}_0 = \hat{A} + \hat{B} \cdot x_0 \tag{39}$$

is the point estimate of the new value of the response y_o.

Consider obtaining an interval estimate of this future observation y_o. The confidence interval on the mean response at $x = x_o$ (38) is inappropriate for this problem because it is an interval estimate on the mean of *y* (a parameter), not a probability statement about future observations from that distribution. We will develop a prediction interval for the future observation y_o. The random variable

$$\Psi = y_0 - \hat{y}_0$$

is normally distributed with mean zero and variance

$$V(\Psi) = V(y_0 - \hat{y}_0) = \sigma^2 \left[1 + \frac{1}{n} + \frac{(x_0 - \bar{x})^2}{S_{xx}} \right] \tag{40}$$

because the future observation y_o is independent of $\hat{y}_o$. If we use $\hat{y}_o$ to predict y_o, then the standard error of $\varphi = y_o - \hat{y}_o$ is the appropriate statistic upon which to base a prediction interval. Thus the *100(1 - α)* percent prediction interval on a future observation at x_o is

$$\hat{y}_0 - t_{\alpha/2, n-2} \cdot \sqrt{MS_E \left[1 + \frac{1}{n} + \frac{(x_0 - \bar{x})^2}{S_{xx}} \right]} \le y_0 \le \hat{y}_0 + t_{\alpha/2, n-2} \cdot \sqrt{MS_E \left[1 + \frac{1}{n} + \frac{(x_0 - \bar{x})^2}{S_{xx}} \right]} \tag{41}$$

The prediction interval (41) is of minimum width at $x_o = \bar{x}$ and widens as $|x_o - \bar{x}|$ increases. By comparing (41) and (38) we observe that the prediction interval at x_o is always wider than the confidence interval at x_o because the prediction interval depends on both the error from the fitted model and the error associated with future observations.

We may generalize (41) somewhat to find a *100(1 - α)* percent prediction interval on the mean of m future observations on the response at $x = x_o$. Let $\bar{y}_o$ be the mean of m future observations at $x = x_o$. A point estimator of $\bar{y}_o$ is $\hat{y}_o = \hat{A} + \hat{B}x_o$. The *100(1 - α)* percent prediction interval on $\bar{y}_o$ is

$$\hat{y}_0 - t_{\alpha/2,n-2} \cdot \sqrt{MS_E\left[\frac{1}{m} + \frac{1}{n} + \frac{(x_0 - \bar{x})^2}{S_{xx}}\right]} \le \bar{y}_0 \le \hat{y}_0 + t_{\alpha/2,n-2} \cdot \sqrt{MS_E\left[\frac{1}{m} + \frac{1}{n} + \frac{(x_0 - \bar{x})^2}{S_{xx}}\right]} \quad (42)$$

Appendix B. Simulation Setup and Technique

Introduction

The modeling of RF propagation and coverage estimation process is an extremely complex problem. This complexity increases the time and effort required for analysis and design, the need to insert new technologies into commercial products quickly requires that the design be done in a timely, cost effective and effort-free manner. These demands can be met only through the use of powerful computer aided analysis and design tools.

A large body of computer-aided techniques has been developed in recent years to assist in the process of modeling, analyzing, and designing wireless systems. These computer aided techniques fall into two categories: formula based approaches, where the computer is used to evaluate complex formulas, and simulation based approaches, where the computer used to simulate the waveforms or signals that flow through the system.

Methods of Performance Evaluation

The performance / trade-off involved in estimating the coverage of a wireless systems can be evaluated using formula-based calculations, or waveform level simulation or through hardware prototyping and measurements.

Formula-based techniques which are based on simplified models, provide considerable insight into the relationship between design parameters and system performance, and they are useful in the early stages of the design for broadly exploring the design space. However, except for some idealized and oversimplified cases, it is extremely difficult to evaluate the performance / trade-off involved in the complex systems using analytic techniques along with the degree of accuracy needed for finer exploration of the design process.

Performance evaluation based on measurements obtained from hardware prototypes of designs an accurate and credible method. For our problem this is not a practical solution since it is impossible to obtain signal strength measurements throughout the wireless service area. Further, this approach is very expensive, time consuming and not very flexible.

With simulation based approaches to performance evaluation, systems can be modeled with almost any level of detail desired and the design space can be explored more finely than is possible with formula based approaches or measurements. With a simulation based approach, one can combine mathematical and empirical models easily, and incorporate measured characteristics of devices and actual signals into analysis and design. Simulated waveforms can also be used as test signals for verifying the functionality of hardware.

The primary disadvantage of the simulation approach is the computational burden, which can be reduced by a careful choice of modeling and simulation techniques.

Since performance evaluation and trade-off studies are central issues in the analysis and design of wireless systems, this chapter will focus on the use of simulation for evaluating the trade-off involved in estimating coverage from signal strength measurements. This chapter is organized as follows. In the next section we will briefly discuss the block diagram of simulation setup used in our experiment. Later section will discuss in detail the implementation of each of these blocks.

Simulation Setup

We use simulation technique to more precisely study the effect of the design reliability under different environmental conditions and to determine the minimum required distance of the signal strength measurements drive route. We will discuss the simulation setup below.

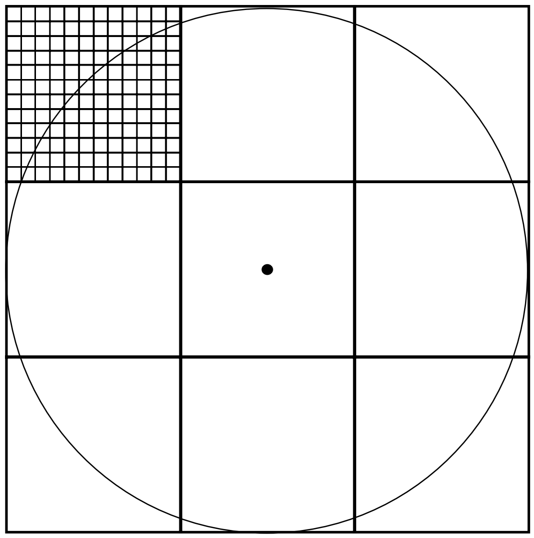

Fig. 1. A schematic diagram showing the structure used for simulation.

To simulate the shadowing conditions in a cell that exists in reality, the following simulation setup was used. The entire cell area is divided into a grid of several bins as shown in the figure. The number of bins was chosen so that one measurement is taken in a bin and the bin size approximately corresponds to the de-correlation distance for urban, suburban and rural cells. Based on the experimental results of several investigators on correlation models [Gudmundson, Giancristofaro and Sorensen], we have chosen a grid of 36 x 36 bins. This would correspond to a bin size of 33 meters in an urban cell (cell radius = 600m), 167 meters in a suburban cell (cell radius = 3 km) and 500 meters in a rural cell (cell radius = 9 km). Further, each block of 12 x 12 bins is modeled to represent a different clutter such as a block of high rise buildings, park, residential block, lake etc. Thus there are total of 9 different clutter blocks. The base station is assumed to be at the center of the square grid.

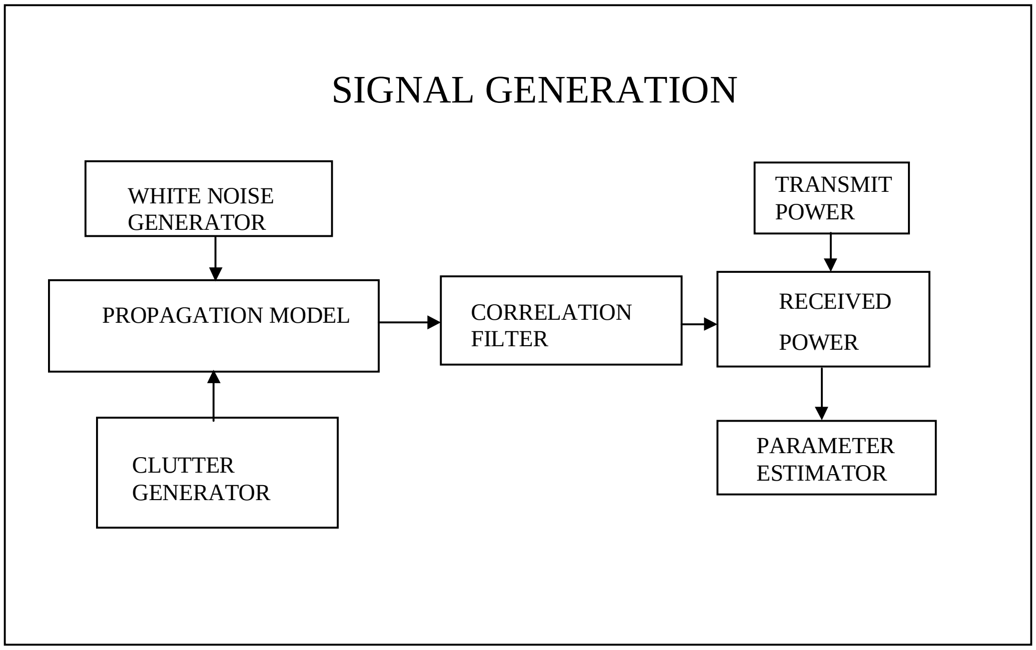

Fig. 2. Block diagram showing the generation of signal in each if the bins and estimation of the parameters A, B and sigma based via least squares under different environmental conditions.

The block diagram for signal generation is shown in figure 2. Each of the blocks is explained below:

Propagation Model

The average large scale path loss PL(d) in decibels (dB) for a distance d can be expressed as

$$PL(d) = A + B \cdot \log(d) \tag{1}$$

The parameters A and B are the intercept and slope of propagation model are based on Hata's empirical results. We have assumed base station antenna height of ** meters, and a mobile station height of 1.5 meters and operating frequency of 800 MHz. Under these assumptions, the value of B is 35.22 dB/decade and the value of A is 130 dB for urban morphology, 120 dB for suburban morphology and 110 dB for rural morphology.

We assume that the received signal strength measurements are averaged over more than 40 wavelengths (λ) (which corresponds to 15 meters at 800 MHz) in measuring equipment to remove the effect of fast fading. Under these conditions, the path loss PL(d) at a distance d is random and log-normally distributed (normally distributed in dB) about the mean distance dependent value. This is called shadowing.

$$P_R = P_T - [A + B \cdot \log(d) + N(0, \sigma^2)] \quad (2)$$

We use a Gaussian random variable generator to achieve this effect. This is explained in detail in a later section.

In real environment the morphology is not uniform and there would variation in clutter, such a block of high rise buildings, residential block, park, lake, etc. To simulate the effect of this we use different values of A_i for different blocks. Published empirical results have shown that the intercept of the propagation model varies with morphology. We assume that A_i is the same in each block, but varies with a uniform distribution in an interval of 8 dB.

Typically, the signal strength measurements are taken at regular intervals of distance or time. These measurements are correlated from one bin to another. Several researchers have proposed models for correlation [ ]. We use the model proposed in []. This model uses a simple decreasing function. It is assumed that the mobile velocity is v and the received analog signal is sampled every T seconds.

The model is given by

$$R_{xx}(n) = \sigma^2 a^{|n|} \quad (3)$$

Where, $a = \varepsilon_D^{vT/D}$ (4)

The variance σ^2 is usually in the range between 3 and 10 dB. The correlation coefficient a in equation (5) may be expressed as in (6). Here parameter ε_D is the correlation between two points separated by distance D.

$$R_{xx}(n) = \sigma^2 e^{-\frac{\partial}{D}|n|} \tag{5}$$

Also it has been shown that the model in (5) closely fits the measured data in suburban and urban environments, where ∂ is the distance between the measured samples and D is the de-correlation distance which depends on the morphology, antenna height etc.

A correlated sequence of random variables can be generated from an uncorrelated sequence by appropriate linear transformation. We have implemented the effect of correlation using a linear filter via difference equations. The filter is applied first on all the bins in horizontal direction and then on all the bins in the vertical direction. The derivation is explained in a later section.

The base station transmit power is P_T is assumed to be 50 dBm which is typical for cellular systems with omni antenna configuration. For our study we are assuming omni antennas though the results are applicable for sectored antennas also. The effect of antenna pattern is normalized and the received signal strength at any bin (m, n) is calculated using

$$P_R(m,n) = P_T - PL(m,n) \tag{6}$$

Parameter Estimation

The parameters A, B, σ are estimated by method of least squares using all the signal strength from all the bins The method of least square estimation is explained in Appendix A. These values of A, B, σ represent the truth or the reference. Using these parameters the cell radius R is estimated using

$$R = 10^{\frac{P_T - P_{MIN} - A - Z\sigma}{B}} \tag{7}$$

where, Z as explained in chapter ** is determined for the required cell edge reliability. For cell edge reliability of 75%, Z = 0.67 and for cell edge reliability of 90%, Z = 1.26. P_{MIN} is the minimum required ground level signal strength. This depends on the receiver sensitivity and the penetration margins. We have assumed a fading receiver sensitivity of ** dBm, which is typical for a 30 KHz TDMA system (IS 54). A body penetration loss of 3 dB and car penetration loss of 7 dB is assumed. Under these conditions, P_{MIN} = ** dBm.

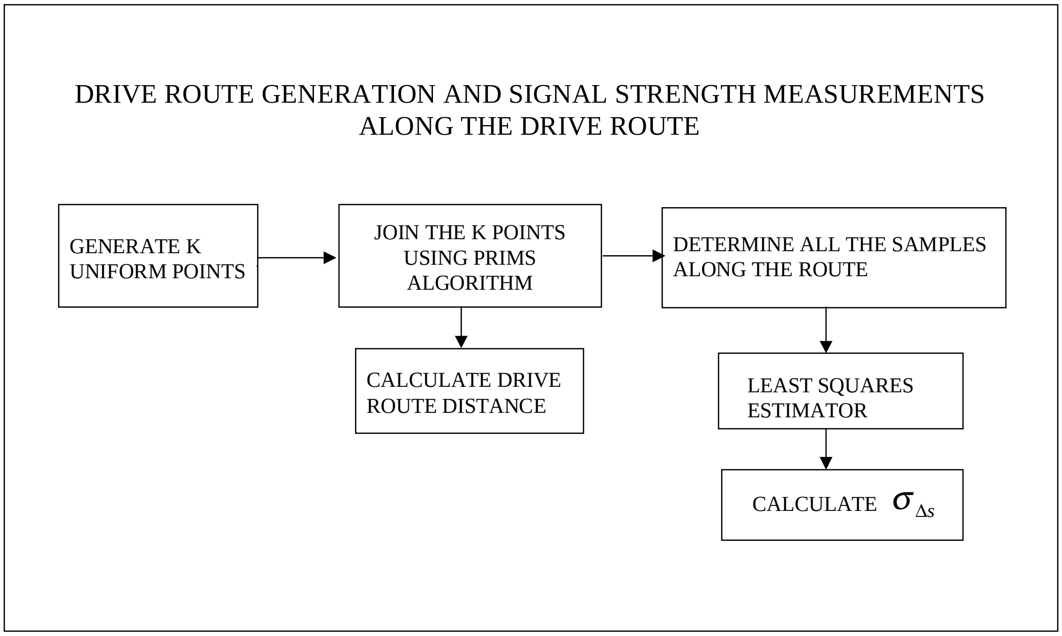

Fig. 3. Block diagram showing the selection of drive route in the simulation and determining the drive distance and the propagation parameters via least squares based on the signal strength measurements in the drive route.

The block diagram for determining the uniform drive route and estimating $\sigma_{\Delta S}$ is shown in figure 3. To simulate the random uniform drive route, K uniform points were randomly generated with uniform distribution in the square. The K points were joined such that the total distance needed to cove these points is the shortest. For this we have used Prim’s minimum spanning tree algorithm explained in detail later.

Signal strength measurements from all the bins along the route are determined. The propagation parameters $\hat{A}$, $\hat{B}$ and $\hat{\sigma}$ are estimated from these signal strength measurements. The distance D of the drive route is also determined. The estimated cell radius is given by

$$\hat{R} = 10^{\frac{P_T - P_{MIN} - \hat{A} - Z\hat{\sigma}}{\hat{B}}} \qquad (8)$$

The accuracy of propagation prediction was calculated by determining the standard deviation of the difference in dB between the actual average signal strength and the average predicted signal strength at the estimated cell edge. The average estimated received power at the cell edge at a distance $\hat{R}$ is given by

$$\hat{P}_R\left(\hat{R}\right) = P_T - \hat{A} - \hat{B} \cdot \log\left(\hat{R}\right) \qquad (9)$$

The difference between the actual and the estimated signal strengths ΔS is given by

$$\Delta S = P_R(\hat{R}) - \hat{P}_R(\hat{R}) = \hat{A} - A + (\hat{B} - B) \cdot \log\left(\hat{R}\right) \qquad (10)$$

ΔS is determined for different sets of distances of the drive routes D and the standard deviation $\sigma_{\Delta S}$ of ΔS is evaluated. This experiment is repeated for different morphology and shadowing conditions standard deviations σ and different distances of the drive route D.

Generating Gaussian Random Variables

The simplest method of generating random numbers with a standard Gaussian probability density function is via the use of central limit theorem according to the formula

$$Y = \sum_{k=1}^{12} U(k) - 6.0 \qquad (11)$$

where $U(k)$, k= 1,2,…., 12 is a set of independent, identically distributed random variables with uniform distribution in the interval [0,1]. Since $U(k)$ has a mean of 0.5 and variance of 1/12, it is easy to verify that the right-hand side of () yields a mean of 0 and variance of 1.

It should be noted that whereas a Gaussian random variable has values ranging from -∞ to ∞, equation () produces values of Y in the interval [-6.0, 6.0]. The number 12 is traditional and represents some compromise between speed and accuracy but there is no reason to limit *k* to 12.

<u>Testing of Random Number Generators</u>

Outputs of random number generators (RNGs) used to drive long Monte Carlo simulations were tested to ensure that they have the right statistical properties.
There are two general sets of tests were performed on the random number generators. The first set of tests checked the stationarity and independence of the outputs of RNGs. The second test verified distributional properties. This set of tests included simple checks on the values of parameters such as means and variances of the output sequences to "goodness of fit" tests, which provide indications of how closely the pdfs of the actual outputs of the RNGs fit the distributions they are supposed to produce.

Implementing Correlation Filter

A correlated sequence of random variables can be generated from an uncorrelated sequence by appropriate linear transformation. A white Gaussian noise process filtered through a first degree filter with a pole at *a* will produce a log-envelope signal with the required properties. From the correlation model [] we know that the auto-correlation function is of the signal strength measurements is given by

$$R_{xx}(n) = \sigma^2 a^{|n|} \qquad (12)$$

Taking the Discrete Fourier transform of $R_{xx}(n)$, we get,

$$S_{XX}(j\omega) = \sum_{-\infty}^{\infty} R_{xx}(n) e^{-j\omega n} \qquad (13)$$

This can be simplified as shown below,

$$S_{XX}(j\omega)=\sum_{-\infty}^{\infty}\sigma^2 a^{|n|}e^{-j\omega n}=\sum_{-\infty}^{0}\sigma^2 a^{|n|}e^{-j\omega n}+\sum_{0}^{\infty}\sigma^2 a^{|n|}e^{-j\omega n}-\sigma^2 \tag{14}$$

$$S_{XX}(j\omega)=\sum_{0}^{\infty}\sigma^2 a^{n}e^{j\omega n}+\sum_{0}^{\infty}\sigma^2 a^{n}e^{-j\omega n}-\sigma^2 \tag{15}$$

$$S_{XX}(j\omega)=\sigma^2\sum_{0}^{\infty}\left(ae^{j\omega}\right)^n+\sigma^2\sum_{0}^{\infty}\left(ae^{-j\omega}\right)^n-\sigma^2 \tag{16}$$

$$S_{XX}(j\omega)=\frac{\sigma^2}{1-ae^{j\omega}}+\frac{\sigma^2}{1-ae^{-j\omega}}-\sigma^2 \tag{17}$$

$$S_{XX}(j\omega)=\frac{\sigma^2\left(1-ae^{-j\omega}\right)+\sigma^2\left(1-ae^{j\omega}\right)-\sigma^2\left(1-ae^{j\omega}\right)\left(1-ae^{-j\omega}\right)}{\left(1-ae^{j\omega}\right)\left(1-ae^{-j\omega}\right)} \tag{18}$$

$$S_{XX}(j\omega)=\frac{\sigma^2\left(\left(2-ae^{-j\omega}-ae^{j\omega}\right)-\left(1-ae^{-j\omega}-ae^{j\omega}+a^2\right)\right)}{\left(1-ae^{j\omega}\right)\left(1-ae^{-j\omega}\right)} \tag{19}$$

$$S_{XX}(j\omega)=\frac{\sigma^2\left(1-a^2\right)}{\left(1-ae^{j\omega}\right)\left(1-ae^{-j\omega}\right)}=H(j\omega)H^*(j\omega) \tag{20}$$

The causal part is

$$H(j\omega)=\frac{\sigma\sqrt{\left(1-a^2\right)}}{\left(1-ae^{-j\omega}\right)} \tag{21}$$

Further from [] we know that

$$a=\exp\left(\frac{-\partial}{D}\right) \tag{22}$$

where, ∂ is the distance between the measured samples and D is de-correlation distance which depends on the morphology, height, etc. Substituting this in () we get,

$$H(j\omega) = \frac{\sigma\sqrt{\left(1-\exp\left(-\frac{2\partial}{D}\right)\right)}}{\left(1-\exp\left(-j\omega-\frac{\partial}{D}\right)\right)} \tag{23}$$

The above filter can be used to generate the correlated signal strength measurements from white Gaussian noise. This filter can be implemented with a difference equation. This is derived as follows.

$$\frac{Y(j\omega)}{X(j\omega)} = \frac{\sigma\sqrt{\left(1-\exp\left(-\frac{2\partial}{D}\right)\right)}}{\left(1-\exp\left(-j\omega-\frac{\partial}{D}\right)\right)} \tag{24}$$

$$Y(j\omega) - Y(j\omega)\exp(-j\omega)\exp\left(\frac{-\partial}{D}\right) = X(j\omega)\sigma\sqrt{1-\exp\left(\frac{-2\partial}{D}\right)} \tag{25}$$

Taking inverse Fourier transform,

$$y(n) = \sigma\sqrt{1-\exp(-2\partial/D)}\cdot x(n) + \exp(-\partial/D)\cdot y(n-1) \tag{26}$$

This filter can be implemented by the difference equation () where x(n) is the input white noise and y(n) is the correlated output with a de-correlation distance D and standard deviation σ.

$$y(0) = \sigma\sqrt{1-\exp(-2\partial/D)}\cdot x(0) \qquad \text{for } n = 0$$

$$y(n) = \sigma\sqrt{1-\exp(-2\partial/D)}\cdot x(n) + \exp(-\partial/D)\cdot y(n-1) \qquad \text{for } n \geq 1 \tag{27}$$

This filter can be implemented by the difference equation () where x(n) is the input white noise and y(n) is the correlated output with a de-correlation distance D and standard deviation σ.

Determining the Shortest Path

The shortest distance connecting several points can be determined using the algorithms developed in graph theory. Given a connected weighted graph G, we want to create a spanning tree T for G such that the sum of the weights the tree edges in T is as small as possible. Such a tree is called a minimum spanning tree and represents the cheapest way of connecting all the nodes in G.

There are a number of techniques for creating a minimum spanning tree for a weighted graph. The first of these, Prim's algorithm, discovered independently by Prim and Dijkstra, is very much like Dijkstra's algorithm for finding shortest paths. An arbitrary node is chosen initially as the tree root (note that in an undirected graph and its spanning tree, any node can be considered the tree root and the nodes adjacent to it as its sons). The nodes of graph are then appended to the tree one at a time until all nodes of the graph are included.

The node of the graph added to the tree at each point is that node adjacent to a node of the tree by an arc of minimum weight. The arc of minimum weight becomes a tree arc connecting the new node to the tree. When all the nodes of the graph have been added to the tree, a minimum spanning tree has been constructed for the graph.

Estimation of Performance Measures from Simulation

A Monte Carlo simulation run can be considered as a statistical experiment. The objective of the experiment is to allow us to make inferences about one or more performance parameters. The observations (measurements) consist of discretely spaced values of a finite duration segment of a random process at some point in the system. Hence, these measurements are inherently random. Therefore, the inferences made can only be statistical.

Random Process Model: Stationarity and Ergodicity

Ergodic property basically means that we can equate a time average over a particular sample function with an ensemble average. Ergodicity requires that the process be stationary. This means that the statistical properties of estimators are time invariant.

The quality parameters of an estimator are bias, variance, confidence interval and time reliability product.

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Vita

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